BASIC MATHEMATICAL AND STATISTICAL TABLES FOR PSYCHOLOGY AND EDUCATION

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662,596	539,353,144	28.5307	9.3370	.001228501	2,
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672,400	551,368,000	28.6356	9.3599	.001219512	2,
674,041	553,387,661	28.6531	9.3637	.001218027	2,
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683,929	565,609,283	28.7576	9.3865	.001209190	2,
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687,241	569,722,789	28.7924	9.3940	.001206273	2,
688,900	571,787,000	28.8097	9.3978	.001204819	2,
690,561	573,856,191	28.8271	9.4016	.001203369	2,
692,224	575,930,368	28.8444	9.4053	.001201923	2,
693,889	578,009,537	28.8617	9.4091	.001200480	2,
695,556	580,093,704	28.8791	9.4129	.001199041	2,
697,225	582,182,875	28.8964	9.4166	.001197605	2,
698,896	584,277,056	28.9137	9.4204	.001196172	2,
700,569	586,376,253	28.9310	9.4241	.001194743	2,
702,244	588,480,472	28.9482	9.4279	.001193317	2,
703,921	590,589,719	28.9655	9.4316	.001191895	2,
705,600	592,704,000	28.9828	9.4354	.001190476	2,
707,281	594,823,321	29.0000	9.4391	.001189061	2,
708,964	596,947,688	29.0172	9.4429	.001187648	2,
710,649	599,077,107	29.0345	9.4466	.001186240	2,
712,336	601,211,584	29.0517	9.4503	.001184834	2,
714,025	603,351,125	29.0689	9.4541	.001183432	2,
715,716	605,495,736	29.0861	9.4578	.001182033	2,
717,409	607,645,423	29.1033	9.4615	.001180638	2,
719,104	609,800,192	29.1204	9.4652	.001179245	2,
720.801	611,960,049	29.1376	9.4690	.001177856	2,
722,500	614,125,000	29.1548	9.4727	.001176471	2,
724,201	616,295,051	29.1719	9.4764	.001175088	2,
725,904	618,470,208	29.1890	9.4801	.001173709	2,

WILLIAM M. MEREDITH

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editor's preface

These *Basic Mathematical and Statistical Tables for Psychology and Education* represent an attempt to replace the usual portable collections of general-purpose mathematical tables with a small reference book designed specifically for a broad specialty area. It appears reasonable to omit, for instance, standard mortality tables from a reference work intended for electrical engineers and elliptic integrals from tables for education majors. Such moderate specialization has made it possible to present, instead, more useful and interesting material in each specialty area. The publisher's permission to reprint material from the massive array of McGraw-Hill handbooks has made it possible to offer a very comprehensive collection of useful reference material at a moderate price. Similar tables in business and economics, electrical engineering, chemistry, and physics are companion volumes.

Considerable effort has been expended to select a format, paper color, and flat-opening book cover which would make it possible to consult tables with a maximum of convenience.

This handbook, designed for students and professionals in education, psychology, sociology, and many related fields, comprises, in addition to tables of numerical functions and useful probability distributions, a considerable collection of formulas for reference. A brief discussion section, including suitable definitions, precedes each group of tables; the reader is referred to standard texts for more detailed information.

Section 1 presents *useful formulas*, including a 438-item *integral table, units, conversion factors, and physical and numerical constants*. Sections 2 through 4 are standard *four- and five-figure logarithm and function tables*, with the frequently used four-figure tables collected in one place for convenient reference.

Section 5 comprises practical *tables of the standardized normal, binomial, Poisson, gamma, and beta distributions;* Section 6 contains critical values, power functions, and confidence intervals for *the classical tests based on normal samples*.

Section 7 has nine tables for frequently useful *nonparametric tests*, and Section 8 refers to *tests of correlation and association*, both classical and nonparametric. Section 9 includes data for *simplified statistics based on the sample range* rather than on sample variance and presents *tables of random numbers* to aid in statistical experiments. Coefficients of orthogonal polynomials, a table of the arcsin transformation, an entropy table, and a list of references complete the volume.

The editor and the publishers owe special thanks to the many McGraw-Hill authors who have permitted tabular material to be reprinted here and thus have made this small manual possible.

This volume of tables and formulas is, in a sense, an experiment, and the editor will always be grateful for comments and suggestions relating to possible improvements in this series.

Granino A. Korn

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BASIC MATHEMATICAL AND STATISTICAL TABLES FOR PSYCHOLOGY AND EDUCATION

section

1

GENERAL INFORMATION

introduction to the use of tables, linear interpolation, and logarithms

FUNCTION TABLES AND LINEAR INTERPOLATION

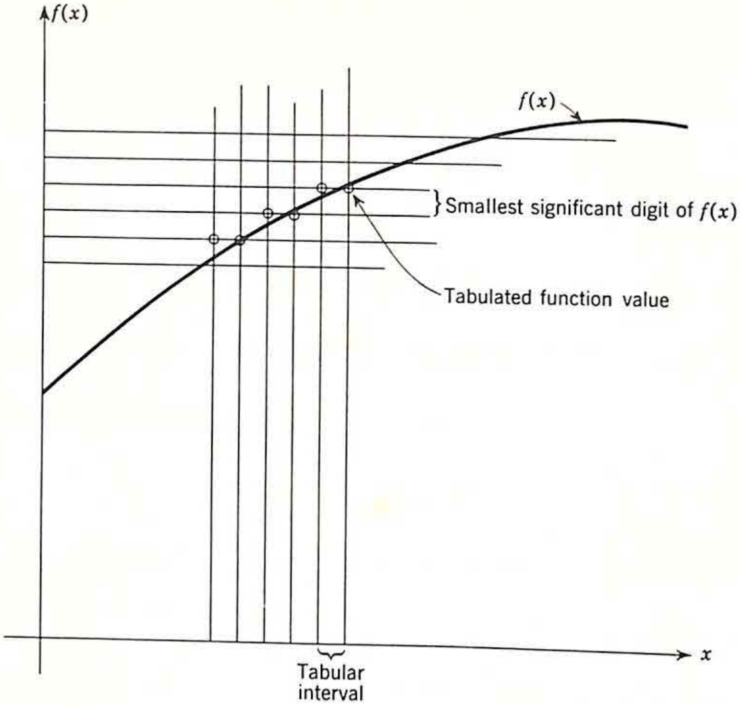
Numerical tabulation of a continuous function $f(x)$ of the numerical argument x to within m significant figures implies a *round-off*, or *quantization, error*. The absolute value of the round-off error will be less than or equal to one-half the value of the least significant digit tabulated. Greater accuracy necessarily requires a table with more significant figures.

Table entries of $f(x)$ can, of necessity, be made only for a discrete set of values of the continuous independent variable x, say for $x = x_1$, $x = x_2$, . . . (Fig. 1-1). For reasonably "smooth" functions $f(x)$ we can, however, approximate function values $f(x)$ for arguments x between x_k and x_{k+1} by values of a simple continuous curve (straight line, parabola) which passes through the tabulated points. The simplest approach employs a straight line through the points $[x_k, f(x_k)]$, $[x_{k+1}, f(x_{k+1})]$ as an approximation function:

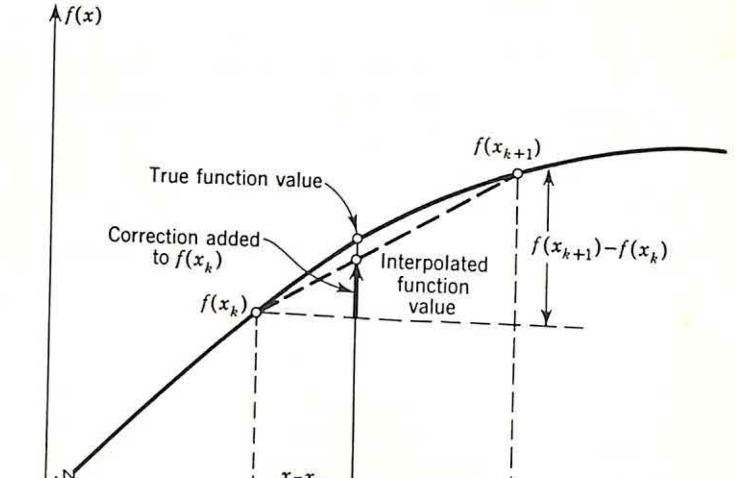
$$f(x) \approx f(x_k) + \frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k} (x - x_k) \tag{1-1}$$

(*linear interpolation*, Fig. 1-2). The tables in this volume are designed for linear interpolation, i.e., we have chosen the constant tabular interval $x_{k+1} - x_k$ sufficiently small to make the added absolute error due to the approximation (1-1) less than the round-off error in each table. Any exception to this rule, say in a region where $f(x)$ has a large curvature, is noted in the individual tables.

Linear interpolation is greatly simplified by tables of *proportional parts*. Tables of proportional parts are small multiplication tables listing



Graph of a continuous function $f(x)$ showing tabulated values and round-off errors (errors are exaggerated for clarity).



Linear interpolation. No round-off error is shown; in practice, interpolation errors are smaller than round-off errors.

integral multiples of

$$\frac{f(x_{k+1}) - f(x_k)}{x_{k+1} - x_k} = {}^{1}\!/_{10}[f(x_{k+1}) - f(x_k)] \tag{1-2}$$

in units of the last significant digit of $f(x)$. The resulting corrections are then simply added to $f(x_k)$ to yield the desired approximation (1-1). The correction will be negative for decreasing functions.

Example

To find $\log_{10} 7.0123$, we read

$$\begin{array}{lll} f(x_k) & = \log_{10} 7.0120 & = 0.84584 \\ \underline{f(x_{k+1})} & \underline{= \log_{10} 7.0130} & \underline{= 0.84590} \\ f(x_{k+1}) - f(x_k) & & = (0.0000)6 \end{array}$$

The proportional-parts table yields the correction $3({}^{6}\!/_{10}) = 1.8 \approx 2$, which is added to $\log_{10} 7.0120 = 0.84584$ to yield $\log_{10} 7.0123 \approx 0.84586$.

INVERSE INTERPOLATION

In order to find the argument value x corresponding to a given function value $f(x)$ from a table of $f(x)$, we note the arguments x_k, $x_{k+1} > x_k$ corresponding to the nearest tabulated function values $f(x_k)$, $f({}_{k+1})$ and employ Eq. (1-1) to obtain the approximation

$$x \approx x_k + \frac{x_{k+1} - x_k}{f(x_{k+1}) - f(x_k)}[f(x) - f(x_k)] \tag{1-3}$$

The desired correction

$$\frac{x_{k+1} - x_k}{f(x_{k+1}) - f(x_k)}[f(x) - f(x_k)] \approx \frac{10}{f(x_{k+1}) - f(x_k)}[f(x) - f(x_k)] \tag{1-4}$$

can be obtained with the aid of proportional-parts tables; the correction will be negative for decreasing functions.

Example

To find $\text{antilog}_{10}\, 0.84586 = 10^{0.84586}$ from the logarithm table, we read

$$\begin{aligned} \text{antilog}_{10}\, 0.84584 &= 7.0120 \\ \text{antilog}_{10}\, 0.84590 &= 7.0130 \end{aligned}$$

and Eq. (1-3) yields

$$x = 7.0120 + {}^{10}\!/_{6}(2) \approx 7.0120 + 3 = 7.0123$$

LOGARITHMS

The *common (Briggsian) logarithm* $\log_{10} x$ is defined by $10^{\log_{10}(x)} = x$ $(x > 0)$. The logarithmic function has the properties

$$\left.\begin{aligned} \log_{10} ab &= \log_{10} a + \log_{10} b \\ \log_{10} \frac{a}{b} &= \log_{10} a - \log_{10} b \\ \log_{10} a^p &= p \log_{10} a \\ \log_{10} \sqrt[q]{a} &= \frac{1}{q} \log_{10} a \end{aligned}\right\} \qquad (a > 0,\ b > 0,\ q \neq 0) \qquad \text{(1-5)}$$

which permit very convenient computation of products, quotients, powers, and roots with the aid of logarithm tables.

To find the common logarithm $\log_{10} x$ of a given positive number x, we always write x in the form

$$x = N \times 10^n \qquad (n = 0, \pm 1, \pm 2, \ldots) \qquad \text{(1-6)}$$

where N is a number between 1 and 10. It follows that

$$\log_{10} x = \log_{10} N + n \qquad (n = 0, \pm 1, \pm 2, \ldots) \qquad \text{(1-7)}$$

where $\log_{10} N$ is a number between 0 and 1 (*mantissa of the logarithm*); the integer N is called the *characteristic* of $\log_{10} x$.

Examples

$$\begin{aligned} \log_{10} 7.4010 &= 0.87929 \\ \log_{10} 740.10 &= 0.87929 + 2 = 2.87929 \\ \log_{10} 0.074010 &= 0.87929 - 2 = 8.87929 - 10 \end{aligned}$$

(sometimes written $\overline{2}.87929$)

To find the number x (*antilogarithm*) corresponding to a given value of $y = \log_{10} x$, we always rewrite y in the form (1-7) and then employ the logarithm table to find the number N corresponding to the mantissa.

Examples of computations

(*a*)
$$\begin{aligned} \log_{10} 2 &= 0.30103 \\ \log_{10} 8 &= 0.90309 \\ \log_{10} (2 \times 8) &= 0.30103 + 0.90309 = 1.20412 \\ \therefore 2 \times 8 &= 1.600 \times 10 = 16.00 \end{aligned}$$

(*b*)
$$\begin{aligned} \log_{10} 8/2 &= 0.90309 - 0.30103 = 0.60206 \\ \therefore 8/2 &= 4.000 \end{aligned}$$

(*c*)
$$\begin{aligned} \log_{10} \sqrt[3]{8} &= 1/3(0.90309) = 0.30103 \\ \therefore \sqrt[3]{8} &= 2.000 \end{aligned}$$

(*d*)
$$\begin{aligned}
\log_{10}\sqrt{2} &= \tfrac{1}{2}(0.30103) \approx 0.15052\\
\therefore \sqrt{2} &= 1.4142\\
\log_{10}\sqrt{20} &= \tfrac{1}{2}(1.30103) \approx 0.65052\\
\therefore \sqrt{20} &\approx 4.4722 \qquad \text{(correct value is 4.4721)}\\
\log_{10}\sqrt{0.2} &= \tfrac{1}{2}(0.30103 - 1) = \tfrac{1}{2}(1.30103 - 2)\\
\therefore \sqrt{0.2} &\approx 0.44722 \qquad \text{(correct value is 0.44721)}
\end{aligned}$$

NATURAL LOGARITHMS

The *natural* (or *Naperian*) *logarithm* $\log_e x$ is defined by

$$e^{\log_e x} = x \qquad e \approx 2.71828 \tag{1-8}$$

Note

$$\left.\begin{aligned}
\log_e x &= \frac{1}{\log_{10} e}\log_{10} x = \log_e 10 \log_{10} x\\
M &= \log_{10} e \approx 0.43429
\end{aligned}\right\} \tag{1-9}$$

LOGARITHMS OF TRIGONOMETRICAL FUNCTIONS

In view of the relations

$$\cos x = \sin (90^\circ - x^\circ) \qquad \cot x = \tan (90^\circ - x^\circ)$$

the number of degrees and the name of each trigonometric function is read from the top of the page for angles between 0° and 45°; the corresponding information for angles between 45° and 90° is found at the bottom of the page and in the right-hand columns. Following common usage, logarithms of trigonometrical functions are tabulated in the form 9. . . . −10 instead of 0. . . . −1, and the −10 portion of the characteristic is not printed in the table.

Five-figure tables of $\log_{10} \sin x$ and $\log_{10} \tan x$ do not permit linear interpolation for small angles between 0° and 3°; the same is necessarily true for $\log_{10} \cos x$ and $\log_{10} \cot x$ for angles between 87° and 90°. Table 3-2 gives special correction functions permitting interpolation in these regions (refer to p. 89).

To find trigonometric functions or their logarithms for angles not between 0° and 90°, one employs the formulas

$$\begin{aligned}
\sin (x + n360^\circ) &= \sin x & \cos (x + n360^\circ) &= \cos x\\
\tan (x + n180^\circ) &= \tan x & \cot (x + n180^\circ) &= \cot x
\end{aligned}\Bigg\}\ (n = \pm 1, \pm 2, \ldots)$$

$$\begin{aligned}
\sin (180^\circ - x) &= \sin x & \cos (180^\circ - x) &= -\cos x\\
\tan (180^\circ - x) &= -\tan x & \cot (180^\circ - x) &= -\cot x\\
\sin (-x) &= -\sin x & \cos (-x) &= -\cos x\\
\tan (-x) &= -\tan x & \cot (-x) &= -\cot x
\end{aligned}$$

Trigonometric identities*

(An identity is an equality which is true for every value of the variable.)

$$\sin x \csc x = 1 \qquad \cos x \sec x = 1 \qquad \tan x \cot x = 1$$
$$\sin^2 x + \cos^2 x = 1 \qquad \sec^2 x = 1 + \tan^2 x \qquad \csc^2 x = 1 + \cot^2 x$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$
$$\cot(-x) = -\cot x \qquad \sec(-x) = +\sec x \qquad \csc(-x) = -\csc x$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin\left(\frac{\pi}{2} \pm x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} \pm x\right) = \mp \sin x$$

$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}} \qquad \cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

where the sign is chosen to agree with the sign of the function on the left side of the equation.

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\sin x - \sin y = 2 \cos \frac{x + y}{2} \sin \frac{x - y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}$$

Relations among Sides and Angles of Any Plane Triangle

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

---

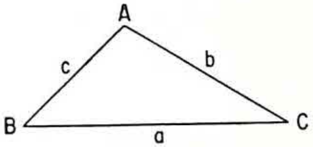
* By I. Maddaus, Jr., from W. D. Cockrell, "Industrial Electronics Handbook," McGraw-Hill Book Company, New York, 1958.

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$$

$$\frac{a-c}{a+c}=\frac{\tan \frac{1}{2}(A-C)}{\tan \frac{1}{2}(A+C)}$$

$$\frac{b-c}{b+c}=\frac{\tan \frac{1}{2}(B-C)}{\tan \frac{1}{2}(B+C)}$$



Half-angle Formulas

$$\tan\frac{A}{2}=\frac{\gamma}{s-a} \qquad \tan\frac{B}{2}=\frac{\gamma}{s-b} \qquad \tan\frac{C}{2}=\frac{\gamma}{s-c}$$

where $s = \frac{1}{2}(a+b+c)$

$$\gamma=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

Mollweide's Equations

$$\frac{a+b}{c}=\frac{\cos \frac{1}{2}(A-B)}{\sin \frac{1}{2}C} \qquad \frac{b+c}{a}=\frac{\cos \frac{1}{2}(B-C)}{\sin \frac{1}{2}A} \qquad \frac{c+a}{b}=\frac{\cos \frac{1}{2}(C-A)}{\sin \frac{1}{2}B}$$

$$\frac{a-b}{c}=\frac{\sin \frac{1}{2}(A-B)}{\cos \frac{1}{2}C} \qquad \frac{b-c}{a}=\frac{\sin \frac{1}{2}(B-C)}{\cos \frac{1}{2}A} \qquad \frac{c-a}{b}=\frac{\sin \frac{1}{2}(C-A)}{\cos \frac{1}{2}B}$$

$$c\cos B+b\cos C=a \qquad c\cos A+a\cos C=b \qquad b\cos A+a\cos B=c$$

where $A + B + C = 180°$

Relations among Sides and Angles of Any Spherical Triangle

Law of Sines

$$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$$

Law of Cosines for Sides

$$\begin{aligned}\cos a &= \cos b\cos c+\sin b\sin c\cos A\\ \cos b &= \cos a\cos c+\sin a\sin c\cos B\\ \cos c &= \cos a\cos b+\sin a\sin b\cos C\end{aligned}$$

Law of Cosines for Angles

$$\begin{aligned}\cos A &= -\cos B\cos C+\sin B\sin C\cos a\\ \cos B &= -\cos A\cos C+\sin A\sin C\cos b\\ \cos C &= -\cos A\cos B+\sin A\sin B\cos c\end{aligned}$$

Half-angle Formulas

$$\tan \tfrac{1}{2}A=\frac{\gamma}{\sin (s-a)} \qquad \tan \tfrac{1}{2}B=\frac{\gamma}{\sin (s-b)} \qquad \tan \tfrac{1}{2}C=\frac{\gamma}{\sin (s-c)}$$

where $s = \frac{1}{2}(a+b+c)$

$$\gamma=\sqrt{\frac{\sin (s-a)\sin (s-b)\sin (s-c)}{\sin s}}$$

Trigonometric identities (continued)

Half-side Formulas

$\tan \tfrac{1}{2}a = R \cos (S - A) \qquad \tan \tfrac{1}{2}b = R \cos (S - B) \qquad \tan \tfrac{1}{2}c = R \cos (S - C)$

where $S = \tfrac{1}{2}(A + B + C)$

$$R = \sqrt{\frac{-\cos S}{\cos (S - A) \cos (S - B) \cos (S - C)}}$$

Napier's Analogies

$$\frac{\sin \tfrac{1}{2}(A - B)}{\sin \tfrac{1}{2}(A + B)} = \frac{\tan \tfrac{1}{2}(a - b)}{\tan \tfrac{1}{2}C}$$

$$\frac{\cos \tfrac{1}{2}(A - B)}{\cos \tfrac{1}{2}(A + B)} = \frac{\tan \tfrac{1}{2}(a + b)}{\tan \tfrac{1}{2}C}$$

$$\frac{\sin \tfrac{1}{2}(a - b)}{\sin \tfrac{1}{2}(a + b)} = \frac{\tan \tfrac{1}{2}(A - B)}{\cot \tfrac{1}{2}C}$$

$$\frac{\cos \tfrac{1}{2}(a - b)}{\cos \tfrac{1}{2}(a + b)} = \frac{\tan \tfrac{1}{2}(A + B)}{\cot \tfrac{1}{2}C}$$

Other forms of Napier's analogies may be obtained by a cyclic interchange of the letters.

Relations among the Sides and Angles of a Right Spherical Triangle When $C = 90°$

1. $\sin a = \sin c \sin A$
2. $\tan b = \tan c \cos A$
3. $\tan a = \sin b \tan A$
4. $\cos A = \sin B \cos a$
5. $\cos c = \cos a \cos b$
6. $\sin b = \sin c \sin B$
7. $\tan a = \tan c \cos B$
8. $\tan b = \sin a \tan B$
9. $\cos B = \cos b \sin A$
10. $\cos c = \cot A \cot B$

Differentiation formulas*

(c and n are constants. u, v, and w are functions of x. $\log_e = \ln$, $e = 2.7182818285$, $M = \log_{10} e = 0.4342944819$)

No.	$f(x)$	$\frac{d}{dx} f(x) = f'(x)$
1	c	0
2	cu	$cu' = c\frac{du}{dx}$
3	$u + v + w$	$u' + v' + w'$
4	uv	$uv' + vu'$
5	$\frac{u}{v}$	$\frac{vu' - uv'}{v^2}$
6	u^n	$nu^{n-1}u'$
7	u^v	$vu^{v-1}u' + u^v v' \ln u$
8	$f(u)$	$\frac{df(u)}{du} u'$
9	e^u	$e^u u'$
10	$\ln u$	$\frac{1}{u} u'$
11	$\sin u$	$\cos u \cdot u'$
12	$\cos u$	$- \sin u \cdot u'$
13	$\tan u$	$\sec^2 u \cdot u'$
14	$\cot u$	$- \csc^2 u \cdot u'$
15	$\sec u$	$\sec u \tan u \cdot u'$
16	$\csc u$	$- \csc u \cot u \cdot u'$
17	$\sin^{-1} u$	$(1 - u^2)^{-\frac{1}{2}} u'$
18	$\cos^{-1} u$	$-(1 - u^2)^{-\frac{1}{2}} u'$
19	$\tan^{-1} u$	$(1 + u^2)^{-1} u'$
20	$\cot^{-1} u$	$-(1 + u^2)^{-1} u'$
21	$\sec^{-1} u$	$u^{-1}(u^2 - 1)^{-\frac{1}{2}} u'$
22	$\csc^{-1} u$	$-u^{-1}(u^2 - 1)^{-\frac{1}{2}} u'$
23	$\sinh u$	$\cosh u \cdot u'$
24	$\cosh u$	$\sinh u \cdot u'$
25	$\tanh u$	$\operatorname{sech}^2 u \cdot u'$
26	$\coth u$	$- \operatorname{csch}^2 u \cdot u'$
27	$\operatorname{sech} u$	$- \operatorname{sech} u \tanh u \cdot u'$
28	$\operatorname{csch} u$	$- \operatorname{csch} u \coth u \cdot u'$
29	$\sinh^{-1} u$	$(1 + u^2)^{-\frac{1}{2}} u'$
30	$\cosh^{-1} u$	$(u^2 - 1)^{-\frac{1}{2}} u'$
31	$\tanh^{-1} u$	$(1 - u^2)^{-1} u'$
32	$\coth^{-1} u$	$-(u^2 - 1)^{-1} u'$
33	$\operatorname{sech}^{-1} u$	$-u^{-1}(1 - u^2)^{-\frac{1}{2}} u'$
34	$\operatorname{csch}^{-1} u$	$-u^{-1}(1 + u^2)^{-\frac{1}{2}} u'$
35	$\ln \sin u$	$\cot u \cdot u'$
36	$\ln \cos u$	$- \tan u \cdot u'$
37	$\ln \sinh u$	$\tanh u \cdot u'$
38	$\ln \cosh u$	$\coth u \cdot u'$

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* From H. Etherington, "Nuclear Engineering Handbook," McGraw-Hill Book Company, New York, 1958.

Power-series expansions*

$$\frac{1}{1-z} = 1 + z + z^2 + \cdots \qquad (|z| < 1; \text{ GEOMETRIC SERIES})$$

$$(1+z)^p = 1 + \binom{p}{1} z + \binom{p}{2} z^2 + \cdots \qquad (|z| < 1; \quad \text{BINOMIAL SERIES})$$

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots \qquad (z \neq \infty)$$

$$\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} \mp \cdots \qquad \cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} \mp \cdots \qquad (z \neq \infty)$$

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \cdots \qquad \cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \cdots \qquad (z \neq \infty)$$

$$\log_e (1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} \pm \cdots \qquad (|z| < 1)$$

$$\left.\begin{aligned} \arcsin z &= z + \frac{1}{2}\cdot\frac{z^3}{3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{z^5}{5} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{z^7}{7} + \cdots \\ \sinh^{-1} z &= z - \frac{1}{2}\cdot\frac{z^3}{3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{z^5}{5} - \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{z^7}{7} \pm \cdots \end{aligned}\right\} (|z| < 1)$$

$$\left.\begin{aligned} \arctan z &= z - \frac{z^3}{3} + \frac{z^5}{5} \mp \cdots \\ \tanh^{-1} z &= \frac{1}{2}\log_e \frac{1+z}{1-z} = z + \frac{z^3}{3} + \frac{z^5}{5} + \cdots \end{aligned}\right\} (|z| < 1)$$

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* From G. A. Korn and T. M. Korn, "Mathematical Handbook for Scientists and Engineers," McGraw-Hill Book Company, New York, 1961.

Tables of indefinite and definite integrals*

Fundamental Theorems on Integrals. Short Table of Integrals. (u and v denote functions of x; a, b, and C denote constants.)

1. $\int df(x) = f(x) + C.$

2. $d\int f(x)dx = f(x)dx.$

3. $\int 0 \cdot dx = C.$

4. $\int a\, f(x)\, dx = a\int f(x)\, dx.$

5. $\int (u \pm v)dx = \int udx \pm \int vdx.$

6. $\int u\, dv = uv - \int v\, du.$

7. $\int \frac{u\, dv}{dx}\, dx = uv - \int v\, \frac{du}{dx}\, dx.$

8. $\int f(y)dx = \int \frac{f(y)dy}{\frac{dy}{dx}}.$

9. $\int u^n\, du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1.$

10. $\int \frac{du}{u} = \log_e u + C,\ u > 0;$ or, $\log_e |u| + C,\ u \neq 0.$

11. $\int e^u\, du = e^u + C.$

12. $\int b^u\, du = \frac{b^u}{\log_e b} + C,\ b > 0,\ b \neq 1.$

13. $\int \sin u\, du = -\cos u + C.$

14. $\int \cos u\, du = \sin u + C.$

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* From R. S. Burington, "Handbook of Mathematical Tables and Formulas," 3d ed., McGraw-Hill Book Company, New York, 1948.

Tables of indefinite and definite integrals (continued)

15. $$\int \tan u\,du = \log_e \sec u + C = -\log_e \cos u + C.$$

16. $$\int \operatorname{ctn} u\,du = \log_e \sin u + C = -\log_e \csc u + C.$$

17. $$\int \sec u\,du = \log_e (\sec u + \tan u) + C = \log_e \tan\left(\frac{u}{2} + \frac{\pi}{4}\right) + C.$$

18. $$\int \csc u\,du = \log_e(\csc u - \operatorname{ctn} u) + C = \log_e \tan \frac{u}{2} + C.$$

19. $$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{2}\sin u \cos u + C.$$

20. $$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{2}\sin u \cos u + C.$$

21. $$\int \sec^2 u\,du = \tan u + C.$$

22. $$\int \csc^2 u\,du = -\operatorname{ctn} u + C.$$

23. $$\int \tan^2 u\,du = \tan u - u + C.$$

24. $$\int \operatorname{ctn}^2 u\,du = -\operatorname{ctn} u - u + C.$$

25. $$\int \frac{du}{u^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C.$$

26. $$\int \frac{du}{u^2 - a^2} = \frac{1}{2a}\log_e\left(\frac{u-a}{u+a}\right) + C = -\frac{1}{a}\operatorname{ctnh}^{-1}\left(\frac{u}{a}\right) + C, \text{ if } u^2 > a^2,$$
$$= \frac{1}{2a}\log_e\left(\frac{a-u}{a+u}\right) + C = -\frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, \text{ if } u^2 < a^2.$$

27. $$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, a > 0; \text{or, } \sin^{-1}\left(\frac{u}{|a|}\right) + C, |a| \neq 0.$$

28. $$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \log_e\left(u + \sqrt{u^2 \pm a^2}\right)^{*} + C.$$

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* See footnote on page 15.

Tables of indefinite and definite integrals (continued)

29. $$\int \frac{du}{\sqrt{2\,au - u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C.$$

30. $$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C = \frac{1}{a}\cos^{-1}\frac{a}{u} + C.$$

31. $$\int \frac{du}{u\sqrt{a^2 \pm u^2}} = -\frac{1}{a}\log_e\left(\frac{a + \sqrt{a^2 \pm u^2}}{u}\right)^{*} + C.$$

32. $$\int \sqrt{a^2 - u^2}\cdot du = \frac{1}{2}\left(u\sqrt{a^2 - u^2} + a^2\sin^{-1}\frac{u}{a}\right) + C.$$

33. $$\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}\left[u\sqrt{u^2 \pm a^2} \pm a^2\log_e\left(u + \sqrt{u^2 \pm a^2}\right)\right]^{*} + C.$$

34. $$\int \sinh u\,du = \cosh u + C.$$

35. $$\int \cosh u\,du = \sinh u + C.$$

36. $$\int \tanh u\,du = \log_e(\cosh u) + C.$$

37. $$\int \operatorname{ctnh} u\,du = \log_e(\sinh u) + C.$$

38. $$\int \operatorname{sech} u\,du = \sin^{-1}(\tanh u) + C.$$

39. $$\int \operatorname{csch} u\,du = \log_e\left(\tanh\frac{u}{2}\right) + C.$$

40. $$\int \operatorname{sech} u\cdot\tanh u\cdot du = -\operatorname{sech} u + C.$$

41. $$\int \operatorname{csch} u\cdot\operatorname{ctnh} u\cdot du = -\operatorname{csch} u + C.$$

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* $\log_e\left(\frac{u+\sqrt{u^2+a^2}}{a}\right) = \sinh^{-1}\left(\frac{u}{a}\right)$; $\log_e\left(\frac{a+\sqrt{a^2-u^2}}{u}\right) = \operatorname{sech}^{-1}\left(\frac{u}{a}\right)$;

$\log_e\left(\frac{u+\sqrt{u^2-a^2}}{a}\right) = \cosh^{-1}\left(\frac{u}{a}\right)$; $\log_e\left(\frac{a+\sqrt{a^2+u^2}}{u}\right) = \operatorname{csch}^{-1}\left(\frac{u}{a}\right)$.

Tables of indefinite and definite integrals (continued)

In the following table, the constant of integration, C, is omitted but should be added to the result of every integration. The letter x represents any variable; u represents any function of x; the remaining letters represent arbitrary constants, unless otherwise indicated; all angles are in radians. **Unless otherwise mentioned $\log_e u \equiv \log u$.**

Expressions Containing $(ax + b)$.

42. $$\int (ax+b)^n\,dx = \frac{1}{a\,(n+1)}\,(ax+b)^{n+1},\quad n \neq -1.$$

43. $$\int \frac{dx}{ax+b} = \frac{1}{a}\log_e(ax+b).$$

44. $$\int \frac{dx}{(ax+b)^2} = -\frac{1}{a(ax+b)}.$$

45. $$\int \frac{dx}{(ax+b)^3} = -\frac{1}{2a(ax+b)^2}.$$

46. $$\int x(ax+b)^n\,dx = \frac{1}{a^2(n+2)}\,(ax+b)^{n+2} - \frac{b}{a^2(n+1)}\,(ax+b)^{n+1},\quad n \neq -1, -2.$$

47. $$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\log(ax+b).$$

48. $$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\log(ax+b).$$

49. $$\int \frac{x\,dx}{(ax+b)^3} = \frac{b}{2a^2(ax+b)^2} - \frac{1}{a^2(ax+b)}.$$

50. $$\int x^2(ax+b)^n dx = \frac{1}{a^3}\left[\frac{(ax+b)^{n+3}}{n+3} - 2b\,\frac{(ax+b)^{n+2}}{n+2} + b^2\frac{(ax+b)^{n+1}}{n+1}\right], \quad n \neq -1, -2, -3.$$

51. $$\int \frac{x^2dx}{ax+b} = \frac{1}{a^3}\left[\frac{1}{2}(ax+b)^2 - 2b(ax+b) + b^2\log(ax+b)\right].$$

52. $$\int \frac{x^2dx}{(ax+b)^2} = \frac{1}{a^3}\left[(ax+b) - 2b\log(ax+b) - \frac{b^2}{ax+b}\right].$$

53. $$\int \frac{x^2\,dx}{(ax+b)^3} = \frac{1}{a^3}\left[\log(ax+b) + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right].$$

54. $$\int x^m(ax+b)^n\,dx$$
$$= \frac{1}{a(m+n+1)}\left[x^m(ax+b)^{n+1} - mb\int x^{m-1}(ax+b)^n dx\right],$$
$$= \frac{1}{m+n+1}\left[x^{m+1}(ax+b)^n + nb\int x^m(ax+b)^{n-1}dx\right],$$
$$m > 0,\ m+n+1 \neq 0.$$

55. $$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\log\frac{x}{ax+b}.$$

56. $$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\log\frac{ax+b}{x}.$$

57. $$\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\log\frac{x}{ax+b}.$$

58. $$\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2}\log\frac{ax+b}{x}.$$

59. $$\int \frac{dx}{x(ax+b)^3} = \frac{1}{b^3}\left[\frac{1}{2}\left(\frac{ax+2b}{ax+b}\right)^2 + \log\frac{x}{ax+b}\right].$$

60. $$\int \frac{dx}{x^2(ax+b)^2} = -\frac{b+2ax}{b^2x(ax+b)} + \frac{2a}{b^3}\log\frac{ax+b}{x}.$$

Tables of indefinite and definite integrals (continued)

61. $$\int \sqrt{ax+b}\,dx = \frac{2}{3a}\sqrt{(ax+b)^3}.$$

62. $$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}.$$

63. $$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)\sqrt{(ax+b)^3}}{105a^3}.$$

64. $$\int x^3\sqrt{ax+b}\,dx$$
$$= \frac{2(35a^3x^3-30a^2bx^2+24ab^2x-16b^3)\sqrt{(ax+b)^3}}{315a^4}.$$

65. $$\int x^n\sqrt{ax+b}\,dx = \frac{2}{a^{n+1}}\int u^2(u^2-b)^n du, \quad u = \sqrt{ax+b}.$$

66. $$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}.$$

67. $$\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}.$$

68. $$\int \frac{xdx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}.$$

69. $$\int \frac{x^2dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^3}\sqrt{ax+b}.$$

70. $$\int \frac{x^3dx}{\sqrt{ax+b}} = \frac{2(5a^3x^3-6a^2bx^2+8ab^2x-16b^3)}{35a^4}\sqrt{ax+b}.$$

71. $$\int \frac{x^ndx}{\sqrt{ax+b}} = \frac{2}{a^{n+1}}\int (u^2-b)^n du, \quad u = \sqrt{ax+b}.$$

72. $$\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}}\log\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}, \quad \text{for } b > 0.$$

73. $$\int \frac{dx}{x\sqrt{ax+b}} = \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{ax+b}{-b}},\ b<0;\ \frac{-2}{\sqrt{b}}\tanh^{-1}\sqrt{\frac{ax+b}{b}},\ b>0.$$

Tables of indefinite and definite integrals (continued)

74. $$\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}.$$

75. $$\int \frac{dx}{x^3\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{2bx^2} + \frac{3a\sqrt{ax+b}}{4b^2x} + \frac{3a^2}{8b^2}\int \frac{dx}{x\sqrt{ax+b}}.$$

76. $$\int \frac{dx}{x^n(ax+b)^m} = -\frac{1}{b^{m+n-1}}\int \frac{(u-a)^{m+n-2}du}{u^m}, \quad u = \frac{ax+b}{x}.$$

77. $$\int (ax+b)^{\pm\frac{n}{2}}\,dx = \frac{2(ax+b)^{\frac{2\pm n}{2}}}{a(2\pm n)}.$$

78. $$\int x(ax+b)^{\pm\frac{n}{2}}\,dx = \frac{2}{a^2}\left[\frac{(ax+b)^{\frac{4\pm n}{2}}}{4\pm n} - \frac{b(ax+b)^{\frac{2\pm n}{2}}}{2\pm n}\right].$$

79. $$\int \frac{dx}{x(ax+b)^{\frac{n}{2}}} = \frac{1}{b}\int \frac{dx}{x(ax+b)^{\frac{n-2}{2}}} - \frac{a}{b}\int \frac{dx}{(ax+b)^{\frac{n}{2}}}.$$

80. $$\int \frac{x^m dx}{\sqrt{ax+b}} = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a}\int \frac{x^{m-1}dx}{\sqrt{ax+b}}.$$

81. $$\int \frac{dx}{x^n\sqrt{ax+b}} = \frac{-\sqrt{ax+b}}{(n-1)bx^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{dx}{x^{n-1}\sqrt{ax+b}}.$$

82. $$\int \frac{(ax+b)^{\frac{n}{2}}}{x}\,dx = a\int (ax+b)^{\frac{n-2}{2}}\,dx + b\int \frac{(ax+b)^{\frac{n-2}{2}}}{x}\,dx.$$

83. $$\int \frac{dx}{(ax+b)(cx+d)} = \frac{1}{bc-ad}\log\frac{cx+d}{ax+b}, \quad bc-ad \neq 0.$$

84. $$\int \frac{dx}{(ax+b)^2(cx+d)}$$
$$= \frac{1}{bc-ad}\left[\frac{1}{ax+b} + \frac{c}{bc-ad}\log\left(\frac{cx+d}{ax+b}\right)\right], \quad bc-ad \neq 0.$$

85. $$\int (ax+b)^n(cx+d)^m\,dx = \frac{1}{(m+n+1)a}\left[(ax+b)^{n+1}(cx+d)^m - m(bc-ad)\int (ax+b)^n(cx+d)^{m-1}\,dx\right].$$

Tables of indefinite and definite integrals (continued)

$$86.\ \int \frac{dx}{(ax+b)^n\,(cx+d)^m} = \frac{-1}{(m-1)\,(bc-ad)}\left[\frac{1}{(ax+b)^{n-1}(cx+d)^{m-1}}\right.$$

$$\left.+\,a(m+n-2)\int\frac{dx}{(ax+b)^n(cx+d)^{m-1}}\right],\ m>1,\ n>0,\ bc-ad\neq 0.$$

$$87.\ \int\frac{(ax+b)^n}{(cx+d)^m}\,dx$$

$$= -\frac{1}{(m-1)(bc-ad)}\left[\frac{(ax+b)^{n+1}}{(cx+d)^{m-1}}+(m-n-2)a\int\frac{(ax+b)^n dx}{(cx+d)^{m-1}}\right],$$

$$= \frac{-1}{(m-n-1)c}\left[\frac{(ax+b)^n}{(cx+d)^{m-1}}+n\,(bc-ad)\int\frac{(ax+b)^{n-1}}{(cx+d)^m}\,dx\right].$$

$$88.\ \int\frac{x\,dx}{(ax+b)\,(cx+d)} = \frac{1}{bc-ad}\left[\frac{b}{a}\log\,(ax+b)\right.$$

$$\left.-\,\frac{d}{c}\log\,(cx+d)\right],\ bc-ad\neq 0.$$

$$89.\ \int\frac{x\,dx}{(ax+b)^2(cx+d)} = \frac{1}{bc-ad}\left[-\frac{b}{a(ax+b)}\right.$$

$$\left.-\,\frac{d}{bc-ad}\log\frac{cx+d}{ax+b}\right],\ bc-ad\neq 0.$$

$$90.\ \int\frac{cx+d}{\sqrt{ax+b}}\,dx = \frac{2}{3a^2}\,(3ad-2bc+acx)\,\sqrt{ax+b}.$$

$$91.\ \int\frac{\sqrt{ax+b}}{cx+d}\,dx = \frac{2\sqrt{ax+b}}{c}$$

$$-\,\frac{2}{c}\sqrt{\frac{ad-bc}{c}}\,\tan^{-1}\sqrt{\frac{c(ax+b)}{ad-bc}},\ c>0,\ ad>bc.$$

$$92.\ \int\frac{\sqrt{ax+b}}{cx+d}\,dx = \frac{2\sqrt{ax+b}}{c}$$

$$+\,\frac{1}{c}\sqrt{\frac{bc-ad}{c}}\,\log\frac{\sqrt{c(ax+b)}-\sqrt{bc-ad}}{\sqrt{c(ax+b)}+\sqrt{bc-ad}},\ c>0,\ bc>ad.$$

$$93.\ \int\frac{dx}{(cx+d)\sqrt{ax+b}} = \frac{2}{\sqrt{c}\sqrt{ad-bc}}\,\tan^{-1}\sqrt{\frac{c(ax+b)}{ad-bc}},$$

$$c>0,\ ad>bc.$$

Tables of indefinite and definite integrals (continued)

94. $$\int \frac{dx}{(cx+d)\sqrt{ax+b}} = \frac{1}{\sqrt{c}\sqrt{bc-ad}} \log \frac{\sqrt{c(ax+b)} - \sqrt{bc-ad}}{\sqrt{c(ax+b)} + \sqrt{bc-ad}}, \; c > 0,\; bc > ad.$$

Expressions Containing $ax^2 + c$, $ax^n + c$, $x^2 \pm p^2$, *and* $p^2 - x^2$.

95. $$\int \frac{dx}{p^2 + x^2} = \frac{1}{p} \tan^{-1} \frac{x}{p}, \quad \text{or} \quad -\frac{1}{p} \operatorname{ctn}^{-1}\left(\frac{x}{p}\right).$$

96. $$\int \frac{dx}{p^2 - x^2} = \frac{1}{2p} \log \frac{p+x}{p-x}, \quad \text{or} \quad \frac{1}{p} \tanh^{-1}\left(\frac{x}{p}\right).$$

97. $$\int \frac{dx}{ax^2 + c} = \frac{1}{\sqrt{ac}} \tan^{-1}\left(x\sqrt{\frac{a}{c}}\right), \quad a \text{ and } c > 0.$$

98. $$\int \frac{dx}{ax^2 + c} = \frac{1}{2\sqrt{-ac}} \log \frac{x\sqrt{a} - \sqrt{-c}}{x\sqrt{a} + \sqrt{-c}}, \; a > 0,\; c < 0.$$

$$= \frac{1}{2\sqrt{-ac}} \log \frac{\sqrt{c} + x\sqrt{-a}}{\sqrt{c} - x\sqrt{-a}}, \; a < 0,\; c > 0.$$

99. $$\int \frac{dx}{(ax^2+c)^n} = \frac{1}{2(n-1)c} \cdot \frac{x}{(ax^2+c)^{n-1}} + \frac{2n-3}{2(n-1)c} \int \frac{dx}{(ax^2+c)^{n-1}}, \quad n > 1.$$

100. $$\int x(ax^2+c)^n \, dx = \frac{1}{2a} \frac{(ax^2+c)^{n+1}}{n+1}, \quad n \neq -1.$$

101. $$\int \frac{x}{ax^2+c} \, dx = \frac{1}{2a} \log (ax^2 + c).$$

102. $$\int \frac{dx}{x(ax^2+c)} = \frac{1}{2c} \log \frac{x^2}{ax^2+c}.$$

103. $$\int \frac{dx}{x^2(ax^2+c)} = -\frac{1}{cx} - \frac{a}{c} \int \frac{dx}{ax^2+c}.$$

104. $$\int \frac{x^2\,dx}{ax^2+c} = \frac{x}{a} - \frac{c}{a} \int \frac{dx}{ax^2+c}.$$

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Tables of indefinite and definite integrals (continued)

105. $$\int \frac{x^n dx}{ax^2 + c} = \frac{x^{n-1}}{a(n-1)} - \frac{c}{a}\int \frac{x^{n-2} dx}{ax^2 + c}, \quad n \neq 1.$$

106. $$\int \frac{x^2 dx}{(ax^2 + c)^n} = -\frac{1}{2(n-1)a} \cdot \frac{x}{(ax^2 + c)^{n-1}} + \frac{1}{2(n-1)a} \int \frac{dx}{(ax^2 + c)^{n-1}}.$$

107. $$\int \frac{dx}{x^2(ax^2 + c)^n} = \frac{1}{c}\int \frac{dx}{x^2(ax^2 + c)^{n-1}} - \frac{a}{c}\int \frac{dx}{(ax^2 + c)^n}$$

108. $$\int \sqrt{x^2 \pm p^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm p^2} \pm p^2 \log\left(x + \sqrt{x^2 \pm p^2}\right)\right].$$

109. $$\int \sqrt{p^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{p^2 - x^2} + p^2 \sin^{-1}\left(\frac{x}{p}\right)\right].$$

110. $$\int \frac{dx}{\sqrt{x^2 \pm p^2}} = \log\left(x + \sqrt{x^2 \pm p^2}\right).$$

111. $$\int \frac{dx}{\sqrt{p^2 - x^2}} = \sin^{-1}\left(\frac{x}{p}\right) \quad \text{or} \quad -\cos^{-1}\left(\frac{x}{p}\right).$$

112. $$\int \sqrt{ax^2 + c}\, dx = \frac{x}{2}\sqrt{ax^2 + c} + \frac{c}{2\sqrt{a}} \log\left(x\sqrt{a} + \sqrt{ax^2 + c}\right), \quad a > 0.$$

113. $$\int \sqrt{ax^2 + c}\, dx = \frac{x}{2}\sqrt{ax^2 + c} + \frac{c}{2\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right), \quad a < 0.$$

114. $$\int \frac{dx}{\sqrt{ax^2 + c}} = \frac{1}{\sqrt{a}} \log\left(x\sqrt{a} + \sqrt{ax^2 + c}\right), \quad a > 0.$$

115. $$\int \frac{dx}{\sqrt{ax^2 + c}} = \frac{1}{\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right), \quad a < 0.$$

116. $$\int x\sqrt{ax^2 + c}\, dx = \frac{1}{3a}(ax^2 + c)^{\frac{3}{2}}.$$

Tables of indefinite and definite integrals (continued)

$$117.\ \int x^2\sqrt{ax^2+c}\;dx = \frac{x}{4a}\sqrt{(ax^2+c)^3} - \frac{cx}{8a}\sqrt{ax^2+c} - \frac{c^2}{8\sqrt{a^3}}\log\left(x\sqrt{a}+\sqrt{ax^2+c}\right),\quad a>0.$$

$$118.\ \int x^2\sqrt{ax^2+c}\;dx = \frac{x}{4a}\sqrt{(ax^2+c)^3} - \frac{cx}{8a}\sqrt{ax^2+c} - \frac{c^2}{8a\sqrt{-a}}\sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right),\quad a<0.$$

$$119.\ \int \frac{x\,dx}{\sqrt{ax^2+c}} = \frac{1}{a}\sqrt{ax^2+c}\,.$$

$$120.\ \int \frac{x^2dx}{\sqrt{ax^2+c}} = \frac{x}{a}\sqrt{ax^2+c} - \frac{1}{a}\int\sqrt{ax^2+c}\;dx.$$

$$121.\ \int \frac{\sqrt{ax^2+c}}{x}\;dx = \sqrt{ax^2+c} + \sqrt{c}\;\log\frac{\sqrt{ax^2+c}-\sqrt{c}}{x},\quad c>0.$$

$$122.\ \int \frac{\sqrt{ax^2+c}}{x}\;dx = \sqrt{ax^2+c} - \sqrt{-c}\;\tan^{-1}\frac{\sqrt{ax^2+c}}{\sqrt{-c}},\quad c<0.$$

$$123.\ \int \frac{dx}{x\sqrt{p^2\pm x^2}} = -\frac{1}{p}\log\left(\frac{p+\sqrt{p^2\pm x^2}}{x}\right).$$

$$124.\ \int \frac{dx}{x\sqrt{x^2-p^2}} = \frac{1}{p}\cos^{-1}\left(\frac{p}{x}\right),\ \text{or}\ -\frac{1}{p}\sin^{-1}\left(\frac{p}{x}\right).$$

$$125.\ \int \frac{dx}{x\sqrt{ax^2+c}} = \frac{1}{\sqrt{c}}\log\frac{\sqrt{ax^2+c}-\sqrt{c}}{x},\quad c>0.$$

$$126.\ \int \frac{dx}{x\sqrt{ax^2+c}} = \frac{1}{\sqrt{-c}}\sec^{-1}\left(x\sqrt{-\frac{a}{c}}\right),\quad c<0.$$

Tables of indefinite and definite integrals (continued)

127. $$\int \frac{dx}{x^2\sqrt{ax^2+c}} = -\frac{\sqrt{ax^2+c}}{cx}.$$

128. $$\int \frac{x^n dx}{\sqrt{ax^2+c}} = \frac{x^{n-1}\sqrt{ax^2+c}}{na} - \frac{(n-1)c}{na}\int \frac{x^{n-2}dx}{\sqrt{ax^2+c}},\ n>0.$$

129. $$\int x^n\sqrt{ax^2+c}\ dx = \frac{x^{n-1}(ax^2+c)^{\frac{3}{2}}}{(n+2)a} - \frac{(n-1)c}{(n+2)a}\int x^{n-2}\sqrt{ax^2+c}\ dx,\ n>0.$$

130. $$\int \frac{\sqrt{ax^2+c}}{x^n}\,dx = -\frac{(ax^2+c)^{\frac{3}{2}}}{c(n-1)x^{n-1}} - \frac{(n-4)a}{(n-1)c}\int \frac{\sqrt{ax^2+c}}{x^{n-2}}\,dx,\ n>1.$$

131. $$\int \frac{dx}{x^n\sqrt{ax^2+c}} = -\frac{\sqrt{ax^2+c}}{c(n-1)x^{n-1}} - \frac{(n-2)a}{(n-1)c}\int \frac{dx}{x^{n-2}\sqrt{ax^2+c}},\ n>1.$$

132. $$\int (ax^2+c)^{\frac{3}{2}}\,dx = \frac{x}{8}(2ax^2+5c)\sqrt{ax^2+c} + \frac{3c^2}{8\sqrt{a}}\log\left(x\sqrt{a}+\sqrt{ax^2+c}\right),\ a>0.$$

133. $$\int (ax^2+c)^{\frac{3}{2}}\,dx = \frac{x}{8}(2ax^2+5c)\sqrt{ax^2+c} + \frac{3c^2}{8\sqrt{-a}}\sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right),\ a<0.$$

134. $$\int \frac{dx}{(ax^2+c)^{\frac{3}{2}}} = \frac{x}{c\sqrt{ax^2+c}}.$$

135. $$\int x(ax^2+c)^{\frac{3}{2}}\,dx = \frac{1}{5a}(ax^2+c)^{\frac{5}{2}}.$$

136. $$\int x^2(ax^2+c)^{\frac{3}{2}}\,dx = \frac{x^3}{6}(ax^2+c)^{\frac{3}{2}} + \frac{c}{2}\int x^2\sqrt{ax^2+c}\,dx.$$

137. $$\int x^n(ax^2+c)^{\frac{3}{2}}\,dx = \frac{x^{n+1}(ax^2+c)^{\frac{3}{2}}}{n+4} + \frac{3c}{n+4}\int x^n\sqrt{ax^2+c}\,dx.$$

138. $$\int \frac{x\,dx}{(ax^2+c)^{\frac{3}{2}}} = -\frac{1}{a\sqrt{ax^2+c}}.$$

139. $$\int \frac{x^2dx}{(ax^2+c)^{\frac{3}{2}}} = -\frac{x}{a\sqrt{ax^2+c}} + \frac{1}{a\sqrt{a}} \log\left(x\sqrt{a}+\sqrt{ax^2+c}\right), \quad a > 0.$$

140. $$\int \frac{x^2dx}{(ax^2+c)^{\frac{3}{2}}} = -\frac{x}{a\sqrt{ax^2+c}} + \frac{1}{a\sqrt{-a}} \sin^{-1}\left(x\sqrt{\frac{-a}{c}}\right), \quad a < 0.$$

141. $$\int \frac{x^3dx}{(ax^2+c)^{\frac{3}{2}}} = -\frac{x^2}{a\sqrt{ax^2+c}} + \frac{2}{a^2}\sqrt{ax^2+c}.$$

142. $$\int \frac{dx}{x(ax^n+c)} = \frac{1}{cn} \log \frac{x^n}{ax^n+c}.$$

143. $$\int \frac{dx}{(ax^n+c)^m} = \frac{1}{c}\int \frac{dx}{(ax^n+c)^{m-1}} - \frac{a}{c}\int \frac{x^ndx}{(ax^n+c)^m}.$$

144. $$\int \frac{dx}{x\sqrt{ax^n+c}} = \frac{1}{n\sqrt{c}} \log \frac{\sqrt{ax^n+c}-\sqrt{c}}{\sqrt{ax^n+c}+\sqrt{c}}, \quad c > 0.$$

145. $$\int \frac{dx}{x\sqrt{ax^n+c}} = \frac{2}{n\sqrt{-c}} \sec^{-1}\sqrt{\frac{-ax^n}{c}}, \quad c < 0.$$

146. $$\int x^{m-1}(ax^n+c)^p dx$$
$$= \frac{1}{m+np}\left[x^m(ax^n+c)^p + npc\int x^{m-1}(ax^n+c)^{p-1}dx\right].$$
$$= \frac{1}{cn(p+1)}\left[-x^m(ax^n+c)^{p+1}+(m+np+n)\int x^{m-1}(ax^n+c)^{p+1}dx\right].$$
$$= \frac{1}{a(m+np)}\left[x^{m-n}(ax^n+c)^{p+1}-(m-n)c\int x^{m-n-1}(ax^n+c)^p dx\right].$$
$$= \frac{1}{mc}\left[x^m(ax^n+c)^{p+1}-(m+np+n)\,a\int x^{m+n-1}(ax^n+c)^p dx\right].$$

147. $$\int \frac{x^mdx}{(ax^n+c)^p} = \frac{1}{a}\int \frac{x^{m-n}dx}{(ax^n+c)^{p-1}} - \frac{c}{a}\int \frac{x^{m-n}dx}{(ax^n+c)^p}.$$

148. $$\int \frac{dx}{x^m(ax^n + c)^p} = \frac{1}{c}\int \frac{dx}{x^m(ax^n + c)^{p-1}} - \frac{a}{c}\int \frac{dx}{x^{m-n}(ax^n + c)^p}.$$

Expressions Containing $(ax^2 + bx + c)$.

149. $$\int \frac{dx}{ax^2+bx+c} = \frac{1}{\sqrt{b^2 - 4ac}} \log \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}, \quad b^2 > 4ac.$$

150. $$\int \frac{dx}{ax^2 + bx + c} = \frac{2}{\sqrt{4ac - b^2}} \tan^{-1}\frac{2ax + b}{\sqrt{4ac - b^2}}, \quad b^2 < 4ac.$$

151. $$\int \frac{dx}{ax^2 + bx + c} = -\frac{2}{2ax + b}, \quad b^2 = 4ac.$$

152. $$\int \frac{dx}{(ax^2 + bx + c)^{n+1}} = \frac{2ax + b}{n(4ac - b^2)(ax^2 + bx + c)^n} + \frac{2(2n - 1)a}{n(4ac - b^2)}\int \frac{dx}{(ax^2 + bx + c)^n}.$$

153. $$\int \frac{xdx}{ax^2+bx+c} = \frac{1}{2a} \log (ax^2+bx+c) - \frac{b}{2a}\int \frac{dx}{ax^2 + bx + c}.$$

154. $$\int \frac{x^2dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2} \log(ax^2 + bx + c) + \frac{b^2 - 2\,ac}{2a^2}\int \frac{dx}{ax^2 + bx + c}.$$

155. $$\int \frac{x^n dx}{ax^2 + bx + c} = \frac{x^{n-1}}{(n - 1)a} - \frac{c}{a}\int \frac{x^{n-2}dx}{ax^2 + bx + c} - \frac{b}{a}\int \frac{x^{n-1}dx}{ax^2 + bx + c}.$$

156. $$\int \frac{xdx}{(ax^2 + bx + c)^{n+1}} = \frac{-(2c + bx)}{n(4ac - b^2)(ax^2 + bx + c)^n} - \frac{b(2n - 1)}{n(4\,ac - b^2)}\int \frac{dx}{(ax^2 + bx + c)^n}.$$

157. $$\int \frac{x^m\,dx}{(ax^2 + bx + c)^{n+1}} = -\frac{x^{m-1}}{a(2n - m + 1)(ax^2 + bx + c)^n} - \frac{(n - m + 1)}{(2n - m + 1)}\cdot\frac{b}{a}\int \frac{x^{m-1}dx}{(ax^2 + bx + c)^{n+1}} + \frac{(m - 1)}{(2n - m + 1)}\cdot\frac{c}{a}\int \frac{x^{m-2}dx}{(ax^2 + bx + c)^{n+1}}.$$

Tables of indefinite and definite integrals (continued)

158. $$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\log\left(\frac{x^2}{ax^2+bx+c}\right) - \frac{b}{2c}\int\frac{dx}{(ax^2+bx+c)}.$$

159. $$\int \frac{dx}{x^2(ax^2+bx+c)} = \frac{b}{2c^2}\log\left(\frac{ax^2+bx+c}{x^2}\right) - \frac{1}{cx} + \left(\frac{b^2}{2c^2} - \frac{a}{c}\right)\int\frac{dx}{(ax^2+bx+c)}.$$

160. $$\int \frac{dx}{x^m(ax^2+bx+c)^{n+1}} = -\frac{1}{(m-1)cx^{m-1}(ax^2+bx+c)^n} - \frac{(n+m-1)}{m-1}\cdot\frac{b}{c}\int\frac{dx}{x^{m-1}(ax^2+bx+c)^{n+1}} - \frac{(2n+m-1)}{m-1}\cdot\frac{a}{c}\int\frac{dx}{x^{m-2}(ax^2+bx+c)^{n+1}}.$$

161. $$\int \frac{dx}{x(ax^2+bx+c)^n} = \frac{1}{2c(n-1)(ax^2+bx+c)^{n-1}} - \frac{b}{2c}\int\frac{dx}{(ax^2+bx+c)^n} + \frac{1}{c}\int\frac{dx}{x(ax^2+bx+c)^{n-1}}.$$

162. $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}}\log(2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c}),\quad a > 0.$$

163. $$\int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{-a}}\sin^{-1}\frac{-2ax-b}{\sqrt{b^2-4ac}},\quad a < 0.$$

164. $$\int \frac{xdx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int\frac{dx}{\sqrt{ax^2+bx+c}}.$$

165. $$\int \frac{x^n dx}{\sqrt{ax^2+bx+c}} = \frac{x^{n-1}}{an}\sqrt{ax^2+bx+c} - \frac{b(2n-1)}{2an}\int\frac{x^{n-1}dx}{\sqrt{ax^2+bx+c}} - \frac{c(n-1)}{an}\int\frac{x^{n-2}dx}{\sqrt{ax^2+bx+c}}.$$

166. $$\int \sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a}\int\frac{dx}{\sqrt{ax^2+bx+c}}.$$

167. $$\int x\sqrt{ax^2+bx+c}\,dx = \frac{(ax^2+bx+c)^{\frac{3}{2}}}{3a} - \frac{b}{2a}\int\sqrt{ax^2+bx+c}\,dx.$$

Tables of indefinite and definite integrals (continued)

168. $$\int x^2\sqrt{ax^2+bx+c}\,dx = \left(x-\frac{5b}{6a}\right)\frac{(ax^2+bx+c)^{\frac{3}{2}}}{4a} + \frac{(5b^2-4ac)}{16a^2}\int\sqrt{ax^2+bx+c}\,dx.$$

169. $$\int\frac{dx}{x\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{c}}\log\left(\frac{\sqrt{ax^2+bx+c}+\sqrt{c}}{x}+\frac{b}{2\sqrt{c}}\right),\ c>0.$$

170. $$\int\frac{dx}{x\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{-c}}\sin^{-1}\frac{bx+2c}{x\sqrt{b^2-4ac}},\quad c<0.$$

171. $$\int\frac{dx}{x\sqrt{ax^2+bx}} = -\frac{2}{bx}\sqrt{ax^2+bx},\quad c=0.$$

172. $$\int\frac{dx}{x^n\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{c(n-1)x^{n-1}} + \frac{b(3-2n)}{2c(n-1)}\int\frac{dx}{x^{n-1}\sqrt{ax^2+bx+c}} + \frac{a(2-n)}{c(n-1)}\int\frac{dx}{x^{n-2}\sqrt{ax^2+bx+c}}.$$

173. $$\int\frac{dx}{(ax^2+bx+c)^{\frac{3}{2}}} = -\frac{2(2ax+b)}{(b^2-4ac)\sqrt{ax^2+bx+c}},\quad b^2\neq 4ac.$$

174. $$\int\frac{dx}{(ax^2+bx+c)^{\frac{3}{2}}} = -\frac{1}{2\sqrt{a^3}(x+b/2a)^2},\quad b^2=4ac.$$

Miscellaneous Algebraic Expressions.

175. $$\int\sqrt{2px-x^2}\,dx = \frac{1}{2}\left[(x-p)\sqrt{2px-x^2}+p^2\sin^{-1}[(x-p)/p]\right].$$

176. $$\int\frac{dx}{\sqrt{2px-x^2}} = \cos^{-1}\left(\frac{p-x}{p}\right).$$

177. $$\int\frac{dx}{\sqrt{ax+b}\cdot\sqrt{cx+d}} = \frac{2}{\sqrt{-ac}}\tan^{-1}\sqrt{\frac{-c(ax+b)}{a(cx+d)}},$$
$$\text{or}\quad \frac{2}{\sqrt{ac}}\tanh^{-1}\sqrt{\frac{c(ax+b)}{a(cx+d)}}.$$

178. $$\int\sqrt{ax+b}\cdot\sqrt{cx+d}\,dx = \frac{(2acx+bc+ad)\sqrt{ax+b}\cdot\sqrt{cx+d}}{4ac} - \frac{(ad-bc)^2}{8ac}\int\frac{dx}{\sqrt{ax+b}\cdot\sqrt{cx+d}}.$$

179. $$\int \sqrt{\frac{cx+d}{ax+b}}\,dx = \frac{\sqrt{ax+b}\cdot\sqrt{cx+d}}{a} + \frac{(ad-bc)}{2a}\int \frac{dx}{\sqrt{ax+b}\cdot\sqrt{cx+d}}.$$

180. $$\int \sqrt{\frac{x+b}{x+d}}\,dx = \sqrt{x+d}\cdot\sqrt{x+b} + (b-d)\log\left[\sqrt{x+d}+\sqrt{x+b}\right].$$

181. $$\int \sqrt{\frac{1+x}{1-x}}\,dx = \sin^{-1} x - \sqrt{1-x^2}.$$

182. $$\int \sqrt{\frac{p-x}{q+x}}\,dx = \sqrt{p-x}\cdot\sqrt{q+x} + (p+q)\sin^{-1}\sqrt{\frac{x+q}{p+q}}.$$

183. $$\int \sqrt{\frac{p+x}{q-x}}\,dx = -\sqrt{p+x}\cdot\sqrt{q-x} - (p+q)\sin^{-1}\sqrt{\frac{q-x}{p+q}}.$$

184. $$\int \frac{dx}{\sqrt{x-p}\cdot\sqrt{q-x}} = 2\sin^{-1}\sqrt{\frac{x-p}{q-p}}.$$

Expressions Containing **sin** ***ax.***

185. $\int \sin u\,du = -\cos u$, where u is any function of x.

186. $$\int \sin ax\,dx = -\frac{1}{a}\cos ax.$$

187. $$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a}.$$

188. $$\int \sin^3 ax\,dx = -\frac{1}{a}\cos ax + \frac{1}{3a}\cos^3 ax.$$

189. $$\int \sin^4 ax\,dx = \frac{3x}{8} - \frac{3\sin 2ax}{16a} - \frac{\sin^3 ax\cos ax}{4a}.$$

190. $$\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax\cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax\,dx,$$ (n pos. integer).

Tables of indefinite and definite integrals (continued)

191. $\int \frac{dx}{\sin ax} = \frac{1}{a} \log \tan \frac{ax}{2} = \frac{1}{a} \log (\csc ax - \operatorname{ctn} ax).$

192. $\int \frac{dx}{\sin^2 ax} = \int \csc^2 ax\, dx = -\frac{1}{a} \operatorname{ctn} ax.$

193. $\int \frac{dx}{\sin^n ax} = -\frac{1}{a(n-1)} \frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\sin^{n-2} ax},$ n integer > 1.

194. $\int \frac{dx}{1 \pm \sin ax} = \mp \frac{1}{a} \tan \left(\frac{\pi}{4} \mp \frac{ax}{2}\right).$

195. $\int \frac{dx}{b + c \sin ax} = \frac{-2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\frac{b-c}{b+c}} \tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right], \; b^2 > c^2.$

196. $\int \frac{dx}{b + c \sin ax} = \frac{-1}{a\sqrt{c^2 - b^2}} \log \frac{c + b \sin ax + \sqrt{c^2 - b^2} \cos ax}{b + c \sin ax}, \; c^2 > b^2.$

197. $\int \sin ax \sin bx\, dx = \frac{\sin (a-b)x}{2(a-b)} - \frac{\sin (a+b)x}{2(a+b)}, \quad a^2 \neq b^2.$

198. $\int \sqrt{1 + \sin x}\, dx = \pm 2\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)$; use + sign when $(8k-1)\frac{\pi}{2} < x \leqq (8k+3)\frac{\pi}{2}$, otherwise −, k an integer.

199. $\int \sqrt{1 - \sin x}\, dx = \pm 2\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)$; use + sign when $(8k-3)\frac{\pi}{2} < x \leqq (8k+1)\frac{\pi}{2}$, otherwise −, k an integer.

Expressions Involving cos ax.

200. $\int \cos u\, du = \sin u$, where u is any function of x.

201. $\int \cos ax\, dx = \frac{1}{a} \sin ax.$

202. $\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a}.$

203. $\int \cos^3 ax\, dx = \frac{1}{a} \sin ax - \frac{1}{3a} \sin^3 ax.$

204. $\int \cos^4 ax\, dx = \frac{3x}{8} + \frac{3 \sin 2ax}{16a} + \frac{\cos^3 ax \sin ax}{4a}.$

205. $\int \cos^n ax\, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax\, dx.$

206. $$\int \frac{dx}{\cos ax} = \frac{1}{a} \log \tan\left(\frac{ax}{2} + \frac{\pi}{4}\right) = \frac{1}{a} \log (\tan ax + \sec ax).$$

207. $$\int \frac{dx}{\cos^2 ax} = \frac{1}{a} \tan ax.$$

208. $$\int \frac{dx}{\cos^n ax} = \frac{1}{a(n-1)} \frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} ax},$$
n integer > 1.

209. $$\int \frac{dx}{1 + \cos ax} = \frac{1}{a} \tan \frac{ax}{2}.$$ 210. $$\int \frac{dx}{1 - \cos ax} = -\frac{1}{a} \operatorname{ctn} \frac{ax}{2}.$$

211. $$\int \sqrt{1 + \cos x}\, dx = \pm \sqrt{2} \int \cos \frac{x}{2}\, dx = \pm 2\sqrt{2} \sin \frac{x}{2}.$$

Use $+$ when $(4k - 1)\pi < x \leqq (4k + 1)\pi$, otherwise $-$, k an integer.

212. $$\int \sqrt{1 - \cos x}\, dx = \pm\sqrt{2} \int \sin \frac{x}{2}\, dx = \mp 2\sqrt{2} \cos \frac{x}{2}.$$

Use top signs when $4k\pi < x \leqq (4k + 2)\pi$, otherwise bottom signs.

213. $$\int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{b^2 - c^2}} \tan^{-1}\left(\frac{\sqrt{b^2 - c^2} \cdot \sin ax}{c + b \cos ax}\right), \quad b^2 > c^2.$$

214. $$\int \frac{dx}{b + c \cos ax} = \frac{1}{a\sqrt{c^2 - b^2}} \tanh^{-1}\left[\frac{\sqrt{c^2 - b^2} \cdot \sin ax}{c + b \cos ax}\right], \quad c^2 > b^2.$$

215. $$\int \cos ax \cdot \cos bx\, dx = \frac{\sin (a - b)x}{2(a - b)} + \frac{\sin (a + b)x}{2(a + b)}, \quad a^2 \neq b^2.$$

Expressions Containing **sin** ax *and* **cos** ax.

216. $$\int \sin ax \cos bx\, dx = -\tfrac{1}{2}\left[\frac{\cos(a-b)x}{a-b} + \frac{\cos(a+b)x}{a+b}\right], \quad a^2 \neq b^2.$$

217. $$\int \sin^n ax \cos ax\, dx = \frac{1}{a(n + 1)} \sin^{n+1} ax, \quad n \neq -1.$$

218. $$\int \cos^n ax \sin ax\, dx = -\frac{1}{a(n + 1)} \cos^{n+1} ax, \quad n \neq -1.$$

219. $$\int \frac{\sin ax}{\cos ax}\, dx = -\frac{1}{a} \log \cos ax.$$

Tables of indefinite and definite integrals (continued).

$$220.\ \int \frac{\cos ax}{\sin ax}\,dx = \frac{1}{a}\log\sin ax.$$

$$221.\ \int (b+c\sin ax)^n \cos ax\,dx = \frac{1}{ac(n+1)}(b+c\sin ax)^{n+1},\ \ n\neq -1.$$

$$222.\ \int (b+c\cos ax)^n \sin ax\,dx = -\frac{1}{ac(n+1)}(b+c\cos ax)^{n+1},\ n\neq -1.$$

$$223.\ \int \frac{\cos ax\,dx}{b+c\sin ax} = \frac{1}{ac}\log\,(b+c\sin ax).$$

$$224.\ \int \frac{\sin ax}{b+c\cos ax}\,dx = -\frac{1}{ac}\log\,(b+c\cos ax).$$

$$225.\ \int \frac{dx}{b\sin ax + c\cos ax} = \frac{1}{a\sqrt{b^2+c^2}}\left[\log\tan\tfrac{1}{2}\left(ax+\tan^{-1}\frac{c}{b}\right)\right].$$

$$226.\ \int \frac{dx}{b+c\cos ax+d\sin ax} = \frac{-1}{a\sqrt{b^2-c^2-d^2}}\sin^{-1}\ U,$$

$$U \equiv \left[\frac{c^2+d^2+b(c\cos ax+d\sin ax)}{\sqrt{c^2+d^2}\,(b+c\cos ax+d\sin ax)}\right];\ \text{or} = \frac{1}{a\sqrt{c^2+d^2-b^2}}\log V,$$

$$V \equiv \left[\frac{c^2+d^2+b(c\cos ax+d\sin ax)+\sqrt{c^2+d^2-b^2}\,(c\sin ax-d\cos ax)}{\sqrt{c^2+d^2}\,(b+c\cos ax+d\sin ax)}\right],$$

$$b^2\neq c^2+d^2,\ -\pi < ax < \pi.$$

$$227.\ \int \frac{dx}{b+c\cos ax+d\sin ax}$$

$$= \frac{1}{ab}\left[\frac{b-(c+d)\cos ax+(c-d)\sin ax}{b+(c-d)\cos ax+(c+d)\sin ax}\right],\ b^2 = c^2+d^2.$$

Tables of indefinite and definite integrals (continued)

228. $$\int \frac{\sin^2 ax\,dx}{b + c\cos^2 ax} = \frac{1}{ac}\sqrt{\frac{b+c}{b}}\tan^{-1}\left(\sqrt{\frac{b}{b+c}}\cdot\tan ax\right) - \frac{x}{c}.$$

229. $$\int \frac{\sin ax\cos ax\,dx}{b\cos^2 ax + c\sin^2 ax} = \frac{1}{2a(c-b)}\log\,(b\cos^2 ax + c\sin^2 ax).$$

230. $$\int \frac{dx}{b^2\cos^2 ax - c^2\sin^2 ax} = \frac{1}{2abc}\log\frac{b\cos ax + c\sin ax}{b\cos ax - c\sin ax}.$$

231. $$\int \frac{dx}{b^2\cos^2 ax + c^2\sin^2 ax} = \frac{1}{abc}\tan^{-1}\left(\frac{c\tan ax}{b}\right).$$

232. $$\int \sin^2 ax\cos^2 ax\,dx = \frac{x}{8} - \frac{\sin 4\,ax}{32a}.$$

233. $$\int \frac{dx}{\sin ax\cos ax} = \frac{1}{a}\log\tan ax.$$

234. $$\int \frac{dx}{\sin^2 ax\cos^2 ax} = \frac{1}{a}\,(\tan ax - \operatorname{ctn} ax).$$

235. $$\int \frac{\sin^2 ax}{\cos ax}\,dx = \frac{1}{a}\left[-\sin ax + \log\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right].$$

236. $$\int \frac{\cos^2 ax}{\sin ax}\,dx = \frac{1}{a}\left[\cos ax + \log\tan\frac{ax}{2}\right].$$

237. $$\int \sin^m ax\cos^n ax\,dx = -\frac{\sin^{m-1} ax\cos^{n+1} ax}{a(m+n)} + \frac{m-1}{m+n}\int \sin^{m-2} ax\cos^n ax\,dx, \quad m, n > 0.$$

238. $$\int \sin^m ax\cos^n ax\,dx = \frac{\sin^{m+1} ax\cos^{n-1} ax}{a(m+n)} + \frac{n-1}{m+n}\int \sin^m ax\cos^{n-2} ax\,dx, \quad m, n > 0.$$

239. $$\int \frac{\sin^m ax}{\cos^n ax}\,dx = \frac{\sin^{m+1} ax}{a(n-1)\cos^{n-1} ax} - \frac{m-n+2}{n-1}\int \frac{\sin^m ax}{\cos^{n-2} ax}\,dx, \quad m, n > 0,\ n \neq 1.$$

240. $$\int \frac{\cos^n ax}{\sin^m ax}\,dx = \frac{-\cos^{n+1} ax}{a(m-1)\sin^{m-1} ax} + \frac{m-n-2}{(m-1)}\int \frac{\cos^n ax}{\sin^{m-2} ax}\,dx,\quad m, n, > 0, m \neq 1.$$

241. $$\int \frac{dx}{\sin^m ax \cos^n ax} = \frac{1}{a(n-1)}\,\frac{1}{\sin^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{(n-1)}\int \frac{dx}{\sin^m ax \cos^{n-2} ax}.$$

242. $$\int \frac{dx}{\sin^m ax \cos^n ax} = -\frac{1}{a(m-1)}\,\frac{1}{\sin^{m-1} ax \cos^{n-1} ax} + \frac{m+n-2}{(m-1)}\int \frac{dx}{\sin^{m-2} ax \cos^n ax}.$$

243. $$\int \frac{\sin^{2n} ax}{\cos ax}\,dx = \int \frac{(1-\cos^2 ax)^n}{\cos ax}\,dx.$$ (Expand, divide, and use 205).

244. $$\int \frac{\cos^{2n} ax}{\sin ax}\,dx = \int \frac{(1-\sin^2 ax)^n}{\sin ax}\,dx.$$ (Expand, divide, and use 190).

245. $$\int \frac{\sin^{2n+1} ax}{\cos ax}\,dx = \int \frac{(1-\cos^2 ax)^n}{\cos ax}\sin ax\,dx.$$ (Expand, divide, and use 218).

246. $$\int \frac{\cos^{2n+1} ax}{\sin ax}\,dx = \int \frac{(1-\sin^2 ax)^n}{\sin ax}\cos ax\,dx.$$ (Expand, divide, and use 217).

Expressions Containing **tan *ax* or ctn *ax*** ($\tan ax = 1/\text{ctn } ax$)

247. $$\int \tan u\,du = -\log\cos u, \quad\text{or}\quad \log\sec u,$$ where u is any function of x.

248. $$\int \tan ax\,dx = -\frac{1}{a}\log\cos ax.$$

249. $$\int \tan^2 ax\,dx = \frac{1}{a}\tan ax - x.$$

250. $$\int \tan^3 ax\,dx = \frac{1}{2a}\tan^2 ax + \frac{1}{a}\log\cos ax.$$

Tables of indefinite and definite integrals (continued)

251. $$\int \tan^n ax\,dx = \frac{1}{a(n-1)}\tan^{n-1} ax - \int \tan^{n-2} ax\,dx, \quad n \text{ integer} > 1.$$

252. $$\int \operatorname{ctn} u\,du = \log \sin u, \quad \text{or} \quad -\log \csc u, \quad \text{where } u \text{ is any function of } x.$$

253. $$\int \operatorname{ctn}^2 ax\,dx = \int \frac{dx}{\tan^2 ax} = -\frac{1}{a}\operatorname{ctn} ax - x.$$

254. $$\int \operatorname{ctn}^3 ax\,dx = -\frac{1}{2a}\operatorname{ctn}^2 ax - \frac{1}{a}\log \sin ax.$$

255. $$\int \operatorname{ctn}^n ax\,dx = \int \frac{dx}{\tan^n ax} = -\frac{1}{a(n-1)}\operatorname{ctn}^{n-1} ax - \int \operatorname{ctn}^{n-2} ax\,dx, \quad n \text{ integer} > 1.$$

256. $$\int \frac{dx}{b + c\tan ax} = \int \frac{\operatorname{ctn} ax\,dx}{b\operatorname{ctn} ax + c} = \frac{1}{b^2 + c^2}\left[bx + \frac{c}{a}\log(b\cos ax + c\sin ax)\right].$$

257. $$\int \frac{dx}{b + c\operatorname{ctn} ax} = \int \frac{\tan ax\,dx}{b\tan ax + c} = \frac{1}{b^2 + c^2}\left[bx - \frac{c}{a}\log(c\cos ax + b\sin ax)\right].$$

258. $$\int \frac{dx}{\sqrt{b + c\tan^2 ax}} = \frac{1}{a\sqrt{b-c}}\sin^{-1}\left(\sqrt{\frac{b-c}{b}}\sin ax\right), \quad b \text{ pos.}, \ b^2 > c^2.$$

***Expressions Containing* $\sec ax = 1/\cos ax$ *or* $\csc ax = 1/\sin ax$.**

259. $$\int \sec u\,du = \log(\sec u + \tan u) = \log\tan\left(\frac{u}{2} + \frac{\pi}{4}\right), \quad \text{where } u \text{ is any function of } x.$$

260. $$\int \sec ax\,dx = \frac{1}{a}\log\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right).$$

Tables of indefinite and definite integrals (continued)

261. $$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax.$$

262. $$\int \sec^3 ax\,dx = \frac{1}{2a}\left[\tan ax \sec ax + \log\tan\left(\frac{ax}{2}+\frac{\pi}{4}\right)\right].$$

263. $$\int \sec^n ax\,dx = \frac{1}{a(n-1)}\frac{\sin ax}{\cos^{n-1} ax} + \frac{n-2}{n-1}\int \sec^{n-2} ax\,dx, \quad n \text{ integer} > 1.$$

264. $$\int \csc u\,du = \log(\csc u - \operatorname{ctn} u) = \log\tan\frac{u}{2},$$ where u is any function of x.

265. $$\int \csc ax\,dx = \frac{1}{a}\log\tan\frac{ax}{2}.$$

266. $$\int \csc^2 ax\,dx = -\frac{1}{a}\operatorname{ctn} ax.$$

267. $$\int \csc^3 ax\,dx = \frac{1}{2a}\left[-\operatorname{ctn} ax \csc ax + \log\tan\frac{ax}{2}\right].$$

268. $$\int \csc^n ax\,dx = -\frac{1}{a(n-1)}\frac{\cos ax}{\sin^{n-1} ax} + \frac{n-2}{n-1}\int \csc^{n-2} ax\,dx, \quad n \text{ integer} > 1.$$

***Expressions Containing* tan *ax and* sec *ax or* ctn *ax and* csc *ax*.**

269. $$\int \tan u \sec u\,du = \sec u,$$ where u is any function of x.

270. $$\int \tan ax \sec ax\,dx = \frac{1}{a}\sec ax.$$

271. $$\int \tan^n ax \sec^2 ax\,dx = \frac{1}{a(n+1)}\tan^{n+1} ax, \quad n \neq -1.$$

272. $$\int \tan ax \sec^n ax\,dx = \frac{1}{an}\sec^n ax, \quad n \neq 0.$$

273. $$\int \operatorname{ctn} u \csc u\,du = -\csc u,$$ where u is any function of x.

Tables of indefinite and definite integrals (continued)

274. $\int \operatorname{ctn} ax \csc ax\, dx = -\frac{1}{a} \csc ax.$

275. $\int \operatorname{ctn}^n ax \csc^2 ax\, dx = -\frac{1}{a(n+1)} \operatorname{ctn}^{n+1} ax, \quad n \neq -1.$

276. $\int \operatorname{ctn} ax \csc^n ax\, dx = -\frac{1}{an} \csc^n ax, \quad n \neq 0.$

277. $\int \frac{\csc^2 ax\, dx}{\operatorname{ctn} ax} = -\frac{1}{a} \log \operatorname{ctn} ax.$

Expressions Containing Algebraic and Trigonometric Functions.

278. $\int x \sin ax\, dx = \frac{1}{a^2} \sin ax - \frac{1}{a} x \cos ax.$

279. $\int x^2 \sin ax\, dx = \frac{2x}{a^2} \sin ax + \frac{2}{a^3} \cos ax - \frac{x^2}{a} \cos ax.$

280. $\int x^3 \sin ax\, dx = \frac{3x^2}{a^2} \sin ax - \frac{6}{a^4} \sin ax - \frac{x^3}{a} \cos ax + \frac{6x}{a^3} \cos ax$

281. $\int x \sin^2 ax\, dx = \frac{x^2}{4} - \frac{x \sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}.$

282. $\int x^2 \sin^2 ax\, dx = \frac{x^3}{6} - \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right) \sin 2ax - \frac{x \cos 2ax}{4a^2}.$

283. $\int x^3 \sin^2 ax\, dx = \frac{x^4}{8} - \left(\frac{x^3}{4a} - \frac{3x}{8a^3}\right) \sin 2ax$
$- \left(\frac{3x^2}{8a^2} - \frac{3}{16a^4}\right) \cos 2ax.$

284. $\int x \sin^3 ax\, dx = \frac{x \cos 3ax}{12a} - \frac{\sin 3ax}{36a^2} - \frac{3x \cos ax}{4a} + \frac{3 \sin ax}{4a^2}.$

285. $\int x^n \sin ax\, dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax\, dx.$

286. $\int \frac{\sin ax\, dx}{x} = ax - \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \cdots.$

287. $\int \frac{\sin ax\, dx}{x^m} = \frac{-1}{(m-1)} \frac{\sin ax}{x^{m-1}} + \frac{a}{(m-1)} \int \frac{\cos ax\, dx}{x^{m-1}}.$

288. $$\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{1}{a} x \sin ax.$$

289. $$\int x^2 \cos ax \, dx = \frac{2x}{a^2} \cos ax - \frac{2}{a^3} \sin ax + \frac{x^2}{a} \sin ax.$$

290. $$\int x^3 \cos ax \, dx = \frac{(3a^2x^2 - 6) \cos ax}{a^4} + \frac{(a^2x^3 - 6x) \sin ax}{a^3}.$$

291. $$\int x \cos^2 ax \, dx = \frac{x^2}{4} + \frac{x \sin 2ax}{4a} + \frac{\cos 2ax}{8a^2}.$$

292. $$\int x^2 \cos^2 ax \, dx = \frac{x^3}{6} + \left(\frac{x^2}{4a} - \frac{1}{8a^3}\right) \sin 2ax + \frac{x \cos 2ax}{4a^2}.$$

293. $$\int x^3 \cos^2 ax \, dx = \frac{x^4}{8} + \left(\frac{x^3}{4a} - \frac{3x}{8a^3}\right) \sin 2ax + \left(\frac{3x^2}{8a^2} - \frac{3}{16a^4}\right) \cos 2ax.$$

294. $$\int x \cos^3 ax \, dx = \frac{x \sin 3ax}{12a} + \frac{\cos 3ax}{36a^2} + \frac{3x \sin ax}{4a} + \frac{3 \cos ax}{4a^2}.$$

295. $$\int x^n \cos ax \, dx = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx, \quad n \text{ pos.}$$

296. $$\int \frac{\cos ax \, dx}{x} = \log ax - \frac{(ax)^2}{2 \cdot 2\,!} + \frac{(ax)^4}{4 \cdot 4\,!} - \cdots.$$

297. $$\int \frac{\cos ax}{x^m} \, dx = - \frac{1}{(m-1)} \cdot \frac{\cos ax}{x^{m-1}} - \frac{a}{(m-1)} \int \frac{\sin ax \, dx}{x^{m-1}}.$$

Expressions Containing Exponential and Logarithmic Functions.

298. $$\int e^u \, du = e^u, \quad \text{where } u \text{ is any function of } x.$$

299. $$\int b^u \, du = \frac{b^u}{\log b}, \quad \text{where } u \text{ is any function of } x.$$

300. $$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \qquad \int b^{ax} \, dx = \frac{b^{ax}}{a \log b}.$$

301. $$\int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1), \int x b^{ax} \, dx = \frac{x b^{ax}}{a \log b} - \frac{b^{ax}}{a^2 (\log b)^2}.$$

Tables of indefinite and definite integrals (continued)

302. $$\int x^2 e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2).$$

303. $$\int x^n e^{ax}\,dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx, \quad n \text{ pos.}$$

304. $$\int x^n e^{ax}\,dx = \frac{e^{ax}}{a^{n+1}}\Big[(ax)^n - n(ax)^{n-1} + n(n-1)(ax)^{n-2} - \cdots + (-1)^n n!\Big], \quad n \text{ pos. integ.}$$

305. $$\int x^n e^{-ax}\,dx = -\frac{e^{-ax}}{a^{n+1}}\Big[(ax)^n + n(ax)^{n-1} + n(n-1)(ax)^{n-2} + \cdots + n!\Big], \quad n \text{ pos. integ.}$$

306. $$\int x^n b^{ax}\,dx = \frac{x^n b^{ax}}{a\log b} - \frac{n}{a\log b}\int x^{n-1}b^{ax}\,dx, \quad n \text{ pos.}$$

307. $$\int \frac{e^{ax}}{x}\,dx = \log x + ax + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^3}{3\cdot 3!} + \cdots.$$

308. $$\int \frac{e^{ax}}{x^n}\,dx = \frac{1}{n-1}\Big[-\frac{e^{ax}}{x^{n-1}} + a\int \frac{e^{ax}}{x^{n-1}}\,dx\Big], \quad n \text{ integ.} > 1.$$

309. $$\int \frac{dx}{b + ce^{ax}} = \frac{1}{ab}[ax - \log(b + ce^{ax}$$

310. $$\int \frac{e^{ax}\,dx}{b + ce^{ax}} = \frac{1}{ac}\log(b + ce^{ax}).$$

311. $$\int \frac{dx}{be^{ax} + ce^{-ax}} = \frac{1}{a\sqrt{bc}}\tan^{-1}\left(e^{ax}\sqrt{\frac{b}{c}}\right), \quad b \text{ and } c \text{ pos.}$$

312. $$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx).$$

313. $$\int e^{ax}\sin bx\sin cx\,dx = \frac{e^{ax}[(b-c)\sin(b-c)x + a\cos(b-c)x]}{2[a^2 + (b-c)^2]} - \frac{e^{ax}[(b+c)\sin(b+c)x + a\cos(b+c)x]}{2[a^2 + (b+c)^2]}.$$

Tables of indefinite and definite integrals (continued)

314. $$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx).$$

315. $$\int e^{ax}\cos bx\cos cx\,dx = \frac{e^{ax}[(b-c)\sin(b-c)x + a\cos(b-c)x]}{2[a^2+(b-c)^2]} + \frac{e^{ax}[(b+c)\sin(b+c)x + a\cos(b+c)x]}{2[a^2+(b+c)^2]}.$$

316. $$\int e^{ax}\sin bx\cos cx\,dx = \frac{e^{ax}[a\sin(b-c)x - (b-c)\cos(b-c)x]}{2[a^2+(b-c)^2]} + \frac{e^{ax}[a\sin(b+c)x - (b+c)\cos(b+c)x]}{2[a^2+(b+c)^2]}.$$

317. $$\int e^{ax}\sin bx\sin(bx+c)\,dx = \frac{e^{ax}\cos c}{2a} - \frac{e^{ax}[a\cos(2bx+c) + 2b\sin(2bx+c)]}{2(a^2+4b^2)}.$$

318. $$\int e^{ax}\cos bx\cos(bx+c)\,dx = \frac{e^{ax}\cos c}{2a} + \frac{e^{ax}\,[a\cos(2bx+c) + 2b\sin(2bx+c)]}{2(a^2+4b^2)}.$$

319. $$\int e^{ax}\sin bx\cos(bx+c)\,dx = -\frac{e^{ax}\sin c}{2a} + \frac{e^{ax}\,[a\sin(2bx+c) - 2b\cos(2bx+c)]}{2(a^2+4b^2)}.$$

320. $$\int e^{ax}\cos bx\sin(bx+c)\,dx = \frac{e^{ax}\sin c}{2a} + \frac{e^{ax}\,[a\sin(2bx+c) - 2b\cos(2bx+c)]}{2(a^2+4b^2)}.$$

321. $$\int xe^{ax}\sin bx\,dx = \frac{xe^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) - \frac{e^{ax}}{(a^2+b^2)^2}[(a^2-b^2)\sin bx - 2\,ab\cos bx].$$

Tables of indefinite and definite integrals (continued)

322. $$\int xe^{ax}\cos bx\,dx = \frac{xe^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) - \frac{e^{ax}}{(a^2+b^2)^2}[(a^2-b^2)\cos bx + 2\,ab\sin bx].$$

323. $$\int e^{ax}\cos^n bx\,dx = \frac{e^{ax}\cos^{n-1}bx\,(a\cos bx + nb\sin bx)}{a^2+n^2b^2} + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{ax}\cos^{n-2}bx\,dx.$$

324. $$\int e^{ax}\sin^n bx\,dx = \frac{e^{ax}\sin^{n-1}bx\,(a\sin bx - nb\cos bx)}{a^2+n^2b^2} + \frac{n(n-1)b^2}{a^2+n^2b^2}\int e^{ax}\sin^{n-2}bx\,dx.$$

325. $$\int \log ax\,dx = x\log ax - x.$$

326. $$\int x\log ax\,dx = \frac{x^2}{2}\log ax - \frac{x^2}{4}.$$

327. $$\int x^2\log ax\,dx = \frac{x^3}{3}\log ax - \frac{x^3}{9}.$$

328. $$\int (\log ax)^2\,dx = x(\log ax)^2 - 2x\log ax + 2x.$$

329. $$\int (\log ax)^n\,dx = x\,(\log ax)^n - n\int(\log ax)^{n-1}\,dx,\quad n\text{ pos.}$$

330. $$\int x^n\log ax\,dx = x^{n+1}\left[\frac{\log ax}{n+1} - \frac{1}{(n+1)^2}\right],\quad n \neq -1.$$

331. $$\int x^n(\log ax)^m\,dx = \frac{x^{n+1}}{n+1}(\log ax)^m - \frac{m}{n+1}\int x^n(\log ax)^{m-1}\,dx.$$

332. $$\int\frac{(\log ax)^n}{x}\,dx = \frac{(\log ax)^{n+1}}{n+1},\quad n \neq -1.$$

333. $$\int\frac{dx}{x\log ax} = \log(\log ax).$$

334. $$\int\frac{dx}{x(\log ax)^n} = -\frac{1}{(n-1)(\log ax)^{n-1}}.$$

Tables of indefinite and definite integrals (continued)

335. $$\int \frac{x^n dx}{(\log ax)^m} = \frac{-x^{n+1}}{(m-1)(\log ax)^{m-1}} + \frac{n+1}{m-1}\int \frac{x^n\,dx}{(\log ax)^{m-1}}, \quad m \neq 1.$$

336. $$\int \frac{x^n\,dx}{\log ax} = \frac{1}{a^{n+1}}\int \frac{e^y\,dy}{y}, \quad y = (n+1)\log ax.$$

337. $$\int \frac{x^n\,dx}{\log ax} = \frac{1}{a^{n+1}}\left[\log|\log ax| + (n+1)\log ax + \frac{(n+1)^2(\log ax)^2}{2\cdot 2!} + \frac{(n+1)^3(\log ax)^3}{3\cdot 3!} + \cdots\right].$$

338. $$\int \frac{dx}{\log ax} = \frac{1}{a}\left[\log|\log ax| + \log ax + \frac{(\log ax)^2}{2\cdot 2!} + \frac{(\log ax)^3}{3\cdot 3!} + \cdots\right].$$

339. $$\int \sin(\log ax)\,dx = \frac{x}{2}[\sin(\log ax) - \cos(\log ax)].$$

340. $$\int \cos(\log ax)\,dx = \frac{x}{2}[\sin(\log ax) + \cos(\log ax)].$$

341. $$\int e^{ax}\log bx\,dx = \frac{1}{a}e^{ax}\log bx - \frac{1}{a}\int \frac{e^{ax}}{x}\,dx.$$

Expressions Containing Inverse Trigonometric Functions

342. $$\int \sin^{-1} ax\,dx = x\sin^{-1} ax + \frac{1}{a}\sqrt{1-a^2x^2}.$$

343. $$\int (\sin^{-1} ax)^2\,dx = x(\sin^{-1} ax)^2 - 2x + \frac{2}{a}\sqrt{1-a^2x^2}\sin^{-1} ax.$$

344. $$\int x\sin^{-1} ax\,dx = \frac{x^2}{2}\sin^{-1} ax - \frac{1}{4a^2}\sin^{-1} ax + \frac{x}{4a}\sqrt{1-a^2x^2}.$$

Tables of indefinite and definite integrals (continued)

345. $$\int x^n \sin^{-1} ax\, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1}\int \frac{x^{n+1}\, dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1.$$

346. $$\int \frac{\sin^{-1} ax\, dx}{x} = ax + \frac{1}{2\cdot 3\cdot 3}(ax)^3 + \frac{1\cdot 3}{2\cdot 4\cdot 5\cdot 5}(ax)^5 + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7\cdot 7}(ax)^7 + \cdots, \quad a^2x^2<1.$$

347. $$\int \frac{\sin^{-1} ax\, dx}{x^2} = -\frac{1}{x}\sin^{-1} ax - a \log\left|\frac{1+\sqrt{1-a^2x^2}}{ax}\right|.$$

348. $$\int \cos^{-1} ax\, dx = x\cos^{-1} ax - \frac{1}{a}\sqrt{1-a^2x^2}.$$

349. $$\int (\cos^{-1} ax)^2\, dx = x(\cos^{-1} ax)^2 - 2x - \frac{2}{a}\sqrt{1-a^2x^2}\cos^{-1} ax.$$

350. $$\int x\cos^{-1} ax\, dx = \frac{x^2}{2}\cos^{-1} ax - \frac{1}{4a^2}\cos^{-1} ax - \frac{x}{4a}\sqrt{1-a^2x^2}.$$

351. $$\int x^n \cos^{-1} ax\, dx = \frac{x^{n+1}}{n+1}\cos^{-1} ax + \frac{a}{n+1}\int \frac{x^{n+1}dx}{\sqrt{1-a^2x^2}}, \quad n\neq -1.$$

352. $$\int \frac{\cos^{-1} ax\, dx}{x} = \frac{\pi}{2}\log|ax| - ax - \frac{1}{2\cdot 3\cdot 3}(ax)^3 - \frac{1\cdot 3}{2\cdot 4\cdot 5\cdot 5}(ax)^5 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7\cdot 7}(ax)^7 - \cdots, \quad a^2x^2<1.$$

353. $$\int \frac{\cos^{-1} ax\, dx}{x^2} = -\frac{1}{x}\cos^{-1} ax + a\log\left|\frac{1+\sqrt{1-a^2x^2}}{ax}\right|.$$

354. $$\int \tan^{-1} ax\, dx = x\tan^{-1} ax - \frac{1}{2a}\log(1+a^2x^2).$$

355. $$\int x^n \tan^{-1} ax\, dx = \frac{x^{n+1}}{n+1}\tan^{-1} ax - \frac{a}{n+1}\int \frac{x^{n+1}\, dx}{1+a^2x^2}, \quad n\neq -1.$$

356. $$\int \frac{\tan^{-1} ax\,dx}{x^2} = -\frac{1}{x}\tan^{-1} ax - \frac{a}{2}\log\left(\frac{1+a^2x^2}{a^2x^2}\right).$$

357. $$\int \operatorname{ctn}^{-1} ax\,dx = x\operatorname{ctn}^{-1} ax + \frac{1}{2a}\log(1+a^2x^2).$$

358. $$\int x^n \operatorname{ctn}^{-1} ax\,dx = \frac{x^{n+1}}{n+1}\operatorname{ctn}^{-1} ax + \frac{a}{n+1}\int \frac{x^{n+1}\,dx}{1+a^2x^2},\quad n \neq -1.$$

359. $$\int \frac{\operatorname{ctn}^{-1} ax\,dx}{x^2} = -\frac{1}{x}\operatorname{ctn}^{-1} ax + \frac{a}{2}\log\left(\frac{1+a^2x^2}{a^2x^2}\right).$$

360. $$\int \sec^{-1} ax\,dx = x\sec^{-1} ax - \frac{1}{a}\log\left(ax + \sqrt{a^2x^2-1}\right).$$

361. $$\int x^n \sec^{-1} ax\,dx = \frac{x^{n+1}}{n+1}\sec^{-1} ax \pm \frac{1}{n+1}\int \frac{x^n\,dx}{\sqrt{a^2x^2-1}},\quad n \neq -1.$$

Use + sign when $\frac{\pi}{2} < \sec^{-1} ax < \pi$; − sign when $0 < \sec^{-1} ax < \frac{\pi}{2}$.

362. $$\int \csc^{-1} ax\,dx = x\csc^{-1} ax + \frac{1}{a}\log\left(ax + \sqrt{a^2x^2-1}\right).$$

363. $$\int x^n \csc^{-1} ax\,dx = \frac{x^{n+1}}{n+1}\csc^{-1} ax \pm \frac{1}{n+1}\int \frac{x^n\,dx}{\sqrt{a^2x^2-1}},\quad n \neq -1.$$

Use + sign when $0 < \csc^{-1} ax < \frac{\pi}{2}$; − sign when $-\frac{\pi}{2} < \csc^{-1} ax < 0$.

Tables of indefinite and definite integrals (continued)

Definite Integrals

364. $\int_0^{\infty} \frac{a\,dx}{a^2 + x^2} = \frac{\pi}{2}$, if $a > 0$; 0, if $a = 0$; $\frac{-\pi}{2}$, if $a < 0$.

365. $\int_0^{\infty} x^{n-1} e^{-x} dx = \int_0^1 \left[\log_e (1/x)\right]^{n-1} dx = \Gamma(n)$.

$\Gamma(n + 1) = n \cdot \Gamma(n)$, if $n > 0$. $\Gamma(2) = \Gamma(1) = 1$.

$\Gamma(n + 1) = n!$, if n is an integer. $\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$.

$\Gamma(n) = \Pi(n - 1)$. $Z(y) = D_y\,[\log_e \Gamma(y)]$.

$Z(1) = -0.5772157 \cdots$. (See integral 418.)

366. $\int_0^{\infty} e^{-zx} \cdot z^n \cdot x^{n-1}\,dx = \Gamma(n), \quad z > 0$.

367. $\int_0^1 x^{m-1}(1 - x)^{n-1}\,dx = \int_0^{\infty} \frac{x^{m-1}\,dx}{(1 + x)^{m+n}} = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m + n)}$.

368. $\int_0^{\infty} \frac{x^{n-1}}{1 + x}\,dx = \frac{\pi}{\sin n\pi}, \quad 0 < n < 1$.

369. $\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx$

$$= \frac{1}{2}\sqrt{\pi} \cdot \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{n}{2} + 1\right)}, \quad \text{if } n > -1;$$

$$= \frac{1 \cdot 3 \cdot 5 \cdots (n - 1)}{2 \cdot 4 \cdot 6 \cdots (n)} \cdot \frac{\pi}{2}, \quad \text{if } n \text{ is an even integer;}$$

$$= \frac{2 \cdot 4 \cdot 6 \cdots (n - 1)}{1 \cdot 3 \cdot 5 \cdot 7 \cdots n}, \quad \text{if } n \text{ is an odd integer.}$$

370. $\int_0^{\infty} \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}$.

Tables of indefinite and definite integrals (continued)

371. $$\int_0^\infty \frac{\sin ax}{x}\,dx = \frac{\pi}{2}, \quad \text{if } a > 0.$$

372. $$\int_0^\infty \frac{\sin x \cos ax}{x}\,dx = 0, \quad \text{if } a < -1, \text{ or } a > 1;$$
$$= \frac{\pi}{4}, \quad \text{if } a = -1, \text{ or } a = 1;$$
$$= \frac{\pi}{2}, \quad \text{if } -1 < a < 1.$$

373. $$\int_0^\pi \sin^2 ax\,dx = \int_0^\pi \cos^2 ax\,dx = \frac{\pi}{2}.$$

374. $$\int_0^{\pi/a} \sin ax \cdot \cos ax\,dx = \int_0^\pi \sin ax \cdot \cos ax\,dx = 0.$$

375. $$\int_0^\pi \sin ax \sin bx\,dx = \int_0^\pi \cos ax \cos bx\,dx = 0, \quad a \neq b.$$

376. $$\int_0^\pi \sin ax \cos bx\,dx = \frac{2a}{a^2 - b^2}, \quad \text{if } a - b \text{ is odd};$$
$$= 0, \quad \text{if } a - b \text{ is even}.$$

377. $$\int_0^\infty \frac{\sin ax \sin bx}{x^2}\,dx = \frac{1}{2}\pi a, \quad \text{if } a < b.$$

378. $$\int_0^\infty \cos(x^2)\,dx = \int_0^\infty \sin(x^2)\,dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}.$$

379. $$\int_0^\infty e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{2a} = \frac{1}{2a}\Gamma\left(\frac{1}{2}\right), \quad \text{if } a > 0.$$

380. $$\int_0^\infty x^n \cdot e^{-ax}\,dx = \frac{\Gamma(n+1)}{a^{n+1}},$$
$$= \frac{n!}{a^{n+1}}, \quad \text{if } n \text{ is a positive integer}, \quad a > 0.$$

381. $$\int_0^\infty x^{2n} e^{-ax^2}\,dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n}\sqrt{\frac{\pi}{a}}.$$

382. $$\int_0^\infty \sqrt{x}\; e^{-ax}\,dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}.$$

Tables of indefinite and definite integrals (continued)

383. $$\int_0^\infty \frac{e^{-ax}}{\sqrt{x}}\,dx = \sqrt{\frac{\pi}{a}}.$$

384. $$\int_0^\infty e^{(-x^2-a^2/x^2)}\,dx = \frac{1}{2}\,e^{-2a}\,\sqrt{\pi}, \quad \text{if } a > 0.$$

385. $$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}, \quad \text{if } a > 0.$$

386. $$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}, \quad \text{if } a > 0.$$

387. $$\int_0^\infty \frac{e^{-ax}\sin x}{x}\,dx = \operatorname{ctn}^{-1} a, \quad a > 0.$$

388. $$\int_0^\infty e^{-a^2x^2}\cos bx\,dx = \frac{\sqrt{\pi}\cdot e^{-b^2/4a^2}}{2a}, \quad \text{if } a > 0.$$

389. $$\int_0^1 (\log x)^n\,dx = (-1)^n\cdot n!, \quad n \text{ pos. integ.}$$

390. $$\int_0^1 \frac{\log x}{1-x}\,dx = -\frac{\pi^2}{6}.$$

391. $$\int_0^1 \frac{\log x}{1+x}\,dx = -\frac{\pi^2}{12}.$$

392. $$\int_0^1 \frac{\log x}{1-x^2}\,dx = -\frac{\pi^2}{8}.$$

393. $$\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\,dx = -\frac{\pi}{2}\log 2.$$

394. $$\int_0^1 \log\left(\frac{1+x}{1-x}\right)\cdot\frac{dx}{x} = \frac{\pi^2}{4}.$$

395. $$\int_0^\infty \log\left(\frac{e^x+1}{e^x-1}\right)dx = \frac{\pi^2}{4}.$$

396. $$\int_0^1 \frac{dx}{\sqrt{\log(1/x)}} = \sqrt{\pi}.$$

Tables of indefinite and definite integrals (continued)

397. $$\int_0^1 \log|\log x|\,dx = \int_0^\infty e^{-x}\log x\,dx = -\gamma = -0.577\,2157\cdots$$

398. $$\int_0^{\frac{\pi}{2}} \log\sin x\,dx = \int_0^{\frac{\pi}{2}} \log\cos x\,dx = -\frac{\pi}{2}\log_e 2.$$

399. $$\int_0^{\pi} x\log\sin x\,dx = -\frac{\pi^2}{2}\log_e 2.$$

400. $$\int_0^1 \left(\log\frac{1}{x}\right)^{\frac{1}{2}} dx = \frac{\sqrt{\pi}}{2}.$$

401. $$\int_0^1 \left(\log\frac{1}{x}\right)^{-\frac{1}{2}} dx = \sqrt{\pi}.$$

402. $$\int_0^1 x^m\left(\log\frac{1}{x}\right)^n dx = \frac{\Gamma(n+1)}{(m+1)^{n+1}},\ \text{if } m+1>0,\ \ n+1>0.$$

403. $$\int_0^{\pi} \log(a\pm b\cos x)\,dx = \pi\log\left(\frac{a+\sqrt{a^2-b^2}}{2}\right),\quad a\geqq b.$$

404. $$\int_0^{\pi} \frac{\log(1+\sin a\cos x)}{\cos x}\,dx = \pi a.$$

405. $$\int_0^1 \frac{x^b-x^a}{\log x}\,dx = \log\frac{1+b}{1+a}.$$

406. $$\int_0^{\pi} \frac{dx}{a+b\cos x} = \frac{\pi}{\sqrt{a^2-b^2}},\quad \text{if } a>b>0.$$

407. $$\int_0^{\frac{\pi}{2}} \frac{dx}{a+b\cos x} = \frac{\cos^{-1}\left(\frac{b}{a}\right)}{\sqrt{a^2-b^2}},\quad a>b.$$

408. $$\int_0^\infty \frac{\cos ax\,dx}{1+x^2} = \frac{\pi}{2}e^{-a},\ \text{if } a>0;\ \ = \frac{\pi}{2}e^{a},\ \text{if } a<0.$$

409. $$\int_0^\infty \frac{\cos x\,dx}{\sqrt{x}} = \int_0^\infty \frac{\sin x\,dx}{\sqrt{x}} = \sqrt{\frac{\pi}{2}}.$$

Tables of indefinite and definite integrals (continued)

410. $$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x}\,dx = \log\frac{b}{a}.$$

411. $$\int_0^{\infty} \frac{\tan^{-1} ax - \tan^{-1} bx}{x}\,dx = \frac{\pi}{2}\log\frac{a}{b}.$$

412. $$\int_0^{\infty} \frac{\cos ax - \cos bx}{x}\,dx = \log\frac{b}{a}.$$

413. $$\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi}{2ab}.$$

414. $$\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2\cos^2 x + b^2\sin^2 x)^2} = \frac{\pi(a^2 + b^2)}{4a^3b^3}.$$

415. $$\begin{aligned}\int_0^{\pi} \frac{(a - b\cos x)dx}{a^2 - 2ab\cos x + b^2} &= 0, \quad \text{if } a^2 < b^2;\\ &= \frac{\pi}{a}, \quad \text{if } a^2 > b^2;\\ &= \frac{\pi}{2a}, \quad \text{if } a = b.\end{aligned}$$

416. $$\int_0^{1} \frac{1 + x^2}{1 + x^4}\,dx = \frac{\pi}{4}\sqrt{2}.$$

417. $$\int_0^{1} \frac{\log(1 + x)}{x}\,dx = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}.$$

418. $$\int_{+\infty}^{1} \frac{e^{-xu}}{u}\,du = \gamma + \log x - x + \frac{x^2}{2\cdot 2!} - \frac{x^3}{3\cdot 3!} + \frac{x^4}{4\cdot 4!} - \cdots,$$

where $\gamma = \lim_{t\to\infty}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{t} - \log t\right) = 0.5772157\cdots, 0 < x < \infty.$

419. $$\int_{+\infty}^{1} \frac{\cos xu}{u}\,du = \gamma + \log x - \frac{x^2}{2\cdot 2!} + \frac{x^4}{4\cdot 4!} - \frac{x^6}{6\cdot 6!} + \cdots,$$

where $\gamma = 0.5772157\cdots, \quad 0 < x < \infty.$

420. $$\int_0^{1} \frac{e^{xu} - e^{-xu}}{u}\,du = 2\left(x + \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!} + \cdots\right), \ 0 < x < \infty.$$

421. $$\int_0^{1} \frac{1 - e^{-xu}}{u}\,du = x - \frac{x^2}{2\cdot 2!} + \frac{x^3}{3\cdot 3!} - \frac{x^4}{4\cdot 4!} + \cdots, 0 < x < \infty.$$

Tables of indefinite and definite integrals (continued)

422. $$\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1-K^2\sin^2 x}} = \frac{\pi}{2}\left[1+\left(\frac{1}{2}\right)^2K^2+\left(\frac{1\cdot 3}{2\cdot 4}\right)^2K^4+\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2K^6+\cdots\right], \quad \text{if } K^2<1.$$

423. $$\int_0^{\frac{\pi}{2}} \sqrt{1-K^2\sin^2 x}\,dx = \frac{\pi}{2}\left[1-\left(\frac{1}{2}\right)^2K^2 - \left(\frac{1\cdot 3}{2\cdot 4}\right)^2\frac{K^4}{3} - \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2\frac{K^6}{5} - \cdots\right], \quad \text{if } K^2<1.$$

424. $$f(x) = \tfrac{1}{2}a_0 + a_1\cos\frac{\pi x}{c} + a_2\cos\frac{2\pi x}{c} + \cdots + b_1\sin\frac{\pi x}{c} + b_2\sin\frac{2\pi x}{c} + \cdots, \quad -c < x < +c,$$

where $$a_m = \frac{1}{c}\int_{-c}^{+c} f(x)\cos\frac{m\pi x}{c}\,dx,$$

$$b_m = \frac{1}{c}\int_{-c}^{+c} f(x)\sin\frac{m\pi x}{c}\,dx.$$ (Fourier Series)

425. $$\int_0^{\infty} e^{-ax}\cosh bx\,dx = \frac{a}{a^2-b^2}, \quad 0 \leqq |b| < a.$$

426. $$\int_0^{\infty} e^{-ax}\sinh bx\,dx = \frac{b}{a^2-b^2}, \quad 0 \leqq |b| < a.$$

427. $$\int_0^{\infty} xe^{-ax}\sin bx\,dx = \frac{2ab}{(a^2+b^2)^2}, \quad a > 0.$$

428. $$\int_0^{\infty} xe^{-ax}\cos bx\,dx = \frac{a^2-b^2}{(a^2+b^2)^2}, \quad a > 0.$$

Tables of indefinite and definite integrals (continued)

429. $$\int_0^\infty x^2 e^{-ax} \sin bx\, dx = \frac{2b(3a^2 - b^2)}{(a^2 + b^2)^3}, \quad a > 0.$$

430. $$\int_0^\infty x^2 e^{-ax} \cos bx\, dx = \frac{2a(a^2 - 3b^2)}{(a^2 + b^2)^3}, \quad a > 0.$$

431. $$\int_0^\infty x^3 e^{-ax} \sin bx\, dx = \frac{24ab(a^2 - b^2)}{(a^2 + b^2)^4}, \quad a > 0.$$

432. $$\int_0^\infty x^3 e^{-ax} \cos bx\, dx = \frac{6(a^4 - 6a^2b^2 + b^4)}{(a^2 + b^2)^4}, \quad a > 0.$$

433. $$\int_0^\infty x^n e^{-ax} \sin bx\, dx = \frac{i \cdot n!\,[(a - ib)^{n+1} - (a + ib)^{n+1}]}{2(a^2 + b^2)^{n+1}}, \quad a > 0$$

434. $$\int_0^\infty x^n e^{-ax} \cos bx\, dx = \frac{n!\,[(a - ib)^{n+1} + (a + ib)^{n+1}]}{2(a^2 + b^2)^{n+1}}, \quad a > 0.$$

435. $$\int_0^\infty e^{-x} \log x\, dx = -\gamma = -0.577\,2157\cdots. \quad \text{(See 418.)}$$

436. $$\int_0^\infty \left(\frac{1}{1 - e^{-x}} - \frac{1}{x}\right) e^{-x}\, dx = \gamma = 0.577\,2157\cdots.$$

437. $$\int_0^\infty \frac{1}{x}\left(\frac{1}{1 + x} - e^{-x}\right) dx = \gamma = 0.577\,2157\cdots.$$

438. $$\int_0^1 \frac{(1 - e^{-x} - e^{-1/x})}{x}\, dx = \gamma = 0.577\,2157\cdots.$$

Units, conversion factors, and physical constants*

Length

1 meter = 39.3700 in. = 3.280833 ft = 1.093611 yd
1 kilometer = 0.62137 mile = 0.53996 nautical mile (International)
1 micron (μ) = 10^{-6} meter = 10^{-4} cm = 10^{4} angstrom units = 39.3700×10^{-6} in.
1 angstrom unit = 10^{-8} cm = 10^{-10} meter = 10^{-4} micron = 3.937×10^{-9} in.
1 inch = 1,000 mils = 2.540005 cm
1 foot = 30.48006 cm = 1.89394×10^{-4} mile
1 yard = 0.91440183 meter
1 fathom = 6 ft = 1.828804 meters
1 chain (Gunter's) = 4 rods = 66 ft = 100 links = 0.1 furlong = 20.1168 meters
1 statute mile = 1,760 yd = 5,280 ft = 6.3360×10^{4} in. = 1.60935 km = 0.868979 nautical mile (International)
1 light year = 5.8804×10^{12} miles = 9.4637×10^{12} km = 0.307 parsec
1 nautical mile (International) = 1,852 meters = 6,076.10333 ft = 1.15078 miles

Area

1 sq cm = 10^{-4} sq meter = 10.764×10^{-4} sq ft = 0.15500 sq in.
1 sq km = 0.3861006 sq mile = 247.1044 acres
1 cir mil = area of circle 1 mil in diameter = 0.78540 sq mil = 10^{-6} cir in.
1 sq in. = 6.4516258 sq cm = 1.27324×10^{6} cir mils
1 sq yd = 9 sq ft = 0.83613 sq meter
1 sq mile = 3.0976×10^{6} sq yd = 640 acres = 2.589998×10^{6} sq meters
1 acre = 43,560 sq ft = 10 sq chains = 4,046.873 sq meters

Volume

1 cu cm = 0.061023 cu in. = 0.03381 fl oz
1 liter = 1,000.027 cu cm = 33.8147 fl oz = 1.05671 qt (liq) = 0.26418 gal
1 cu ft = 1,728 cu in. = 28.317 liters = 7.481 gal = 0.80357 bushel
1 gal = 231 cu in. = 0.13368 cu ft = 3.78533 liters
1 qt = 32 fl oz = 946.33 ml
1 pt (liq) = 0.5 qt = 28.875 cu in. = 1.042 lb water at 62°F
1 fl oz = 1.042 oz (avdp) water at 62°F

Time

1 week = 7 days = 168 hr = 10,080 min = 604,800 sec
1 mean solar day = 1,440 min = 86,400 sec
1 calendar year = 365 days = 8,760 hr = 5.256×10^{5} min = 3.1536×10^{7} sec
1 sidereal year = 365.256 days (mean solar) = 8,766.14 hr (mean solar)

Velocity

1 meter/sec = 2.2369 mph
1 mph = 1.4667 ft/sec = 0.8690 knot = 1.6093 km/hr = 0.447041 meter/sec
1 knot = 1 nautical mile (International)/hr = 1.1508 statute miles/hr = 1.6878 ft/sec

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* By A. H. Canada, et al. from William D. Cockrell, "Industrial Electronics Handbook," McGraw-Hill Book Company, New York, 1958.

Units, conversion factors and physical constants (continued)

Angular Velocity

1 radian/sec = 0.1592 rps = 9.549 rpm = 57.296 deg/sec
1 rpm = 0.01667 rps = 0.10472 radian/sec = 6 deg/sec

Acceleration

1 ft/sec^2 = 30.4801 cm/sec^2 = 0.6818 mile/hr-sec

Angle

1 radian = 360°/2π = 57.29578 deg = 10^3 angular mils
1 deg = 60 min = 0.01745 radian
1 min = 60 sec = 2.90888×10^{-4} radian
1 military mil = 360/6,400 deg = 0.05625 deg
1 sec slope = 4.848 μ in./in. = 0.001 in./17.19 ft
1 solid angle = 4 π steradians = 12.566 steradians = 1 sphere
1 steradian = 0.079580 solid angle

Mass

1 gram = 2.20462×10^{-3} lb (avdp) = 0.03527 oz (avdp) = 15.4324 grains
1 lb (avdp) = 16 oz (avdp) = 7,000 grains = 256 drams (avdp) = 453.5924 grams
1 oz (avdp) = 16 drams (avdp) = 437.5 grains = 28.34953 grams
1 short ton = 2,000 lb (avdp) = 907.185 kg = 20 cwt (short)
1 long ton = 2,240 lb (avdp) = 1,016.0470 kg = 20 cwt (long)

Density

1 lb/cu ft = 5.787×10^{-4} lb/cu in. = 16.018 kg/cu meter = 1.6018×10^{-2} grams/cu cm
1 gram/cu cm = 0.03613 lb/cu in. = 62.43 lb/cu ft

Pressure

1 atmosphere (atm) = 1.0133 bars = 14.696 lb/sq in. = 1.013246×10^6 dynes/sq cm = 1,033.2 grams/sq cm (0°C) = 760 mm Hg (0°C) = 29.921 in. Hg (0°C) = 33.903 ft water (0°C)
1 dyne/sq cm = 1.01971×10^{-3} grams/sq cm = 1.4504×10^{-5} lb/sq in.
1 bar = 1.0×10^6 dynes/sq cm = 0.98692 atm
1 lb wt/sq in. = 70.307 grams/sq cm = 68,947 dynes/sq cm

Temperature

Temperature differences: Δ1° centigrade (C) = Δ1.8° Fahrenheit (F) = Δ1.8° Rankine = Δ1° Kelvin (K) = Δ0.8° Reaumur (R)
0°C = 273°K = 32°F = 492° Rankine = 0°R
100°C = 373°K = 212°F = 672° Rankine = 125°R
0°K = − 273.16°C = − 459.72°F

Force

1 newton = 1×10^5 dynes = 0.22481 lb wt
1 dyne = 2.2481×10^{-6} lb wt = 7.2330×10^{-5} poundal = 0.0010197 gram wt
1 gram wt = 0.07932 poundal = 980.665 dynes = 2.2046×10^{-3} lb wt
1 lb wt = 32.174 poundals = 453.59 gram wt = 4.4482 newtons
1 poundal = 0.031081 lb wt = 14.098 gram wt = 1.3825×10^4 dynes

Units, conversion factors and physical constants (continued)

Work and Energy

1 absolute (abs) joule = 1 newton-meter = 1×10^{7} ergs = 1×10^{7} dyne-cm = 1 watt-sec = 1 volt-coulomb = 0.73756 ft-lb = 2.3889×10^{-4} kg-cal (mean) = 9.4805×10^{-4} Btu (mean) = 23.730 ft-poundal = 2.778×10^{-7} kwhr = 3.725×10^{-7} hp-hr
1 gram-cal (mean) = 4.186 joules (abs)
1 gram-cal (15°C) = 4.1855 joules (abs) = 0.003968 Btu
1 joule (International) = 1.000165 joules (abs)
1 kwhr = 3413.0 Btu (mean) = 2.6552×10^{6} ft-lb = 1.3410 hp-hr
1 liter atm (normal) = 3.7745×10^{-5} hp-hr = 24.206 g-cal (mean) = 101.328 joules (abs)
1 atomic mass unit (amu) = 931.16 Mev = 1.65983×10^{-24} gram mass (energy equiv)
1 electron volt = 1.60207×10^{-19} joule (abs)

Power

1 Btu (mean)/min = 0.023575 hp = 17.580 watts (abs) = 778.0 ft-lb/min
1 erg/sec = 10^{-10} kw = 1.3412×10^{-10} hp = 1×10^{-7} watt = 5.688×10^{-9} Btu (mean)/min = 1.4333×10^{-9} kg-cal (mean)/min = 7.3756×10^{-8} ft-lb/sec
1 hp (mech) = 0.70696 Btu (mean)/sec = 550 ft-lb/sec = 745.70 watts = 10.688 kg-cal (mean)/min
1 hp (elec) = 746.00 watts

Torque

1 dyne-cm = 1.0197×10^{-8} kg-meter = 7.3757×10^{-8} lb-ft = 8.8511×10^{-7} lb-in. = 2.3731×10^{-6} poundal-ft
1 kg-meter = 9.8066×10^{7} dyne-cm
1 lb-ft = 1.3558×10^{7} dyne-cm
1 poundal-ft = 4.2140×10^{5} dyne-cm
1 lb-in. = 1.1298×10^{6} dyne-cm

Physical Constants

h = Planck constant = $(6.6252 \pm 0.0005) \times 10^{-27}$ erg-sec
R_0 = gas constant per mole = $(8.31662 \pm 0.00038) \times 10^{7}$ erg/°K-mole (physical scale) = $(8.20545 + 0.00037) \times 10^{-2}$ liter atm/°K-mole (chem scale)
k = Boltzmann constant = $(1.38042 \pm 0.00010) \times 10^{-16}$ erg/°K
σ = Stefan-Boltzmann constant (for black body) = $(5.6686 \pm 0.0005) \times 10^{-3}$ erg/(sq cm)-(sec)$(°\text{K})^{4}$
C = velocity of light = $(2.99793 \pm 0.00001) \times 10^{10}$ cm/sec = 186,284 miles/sec
$\mathfrak{F}$ = Faraday constant = $(96{,}520 \pm 3)$ abs coulombs/gram equivalent (physical scale)
e = electronic charge = $(1.60207 \pm 0.00007) \times 10^{-19}$ abs coulombs = $(4.80288 \pm 0.00021) \times 10^{-10}$ esu
e/m = specific electronic charge = $(1.75888 \pm 0.00005) \times 10^{11}$ abs coulombs/kg
N_0 = Avogadro's number = $(6.02472 \pm 0.00036) \times 10^{23}$ molecules/gram mole (physical scale) = $(6.02308 \pm 0.00040) \times 10^{23}$ molecules/gram mole (chemical scale)
$\alpha_0 = h^2/4\pi me^2$ = first Bohr radius = $(5.29171 \pm 0.00006) \times 10^{-9}$ cm
$\epsilon_0 = 1/\mu_0 C^2$ = permittivity of free space = $(8.8542 \pm 0.0001) \times 10^{-12}$ farad/meter
μ_0 = permeability of free space = $4\pi \times 10^{-7}$ henry/meter = 12.5664×10^{-7} henry/meter
$Z_0 = (\mu_0/\epsilon_0)^{1/2}$ = impedance of free space = (376.731 ± 0.001) ohm
G = Newton's gravity constant = $(6.670 \pm 0.005) \times 10^{-8}$ cu cm/gram sec

Units, conversion factors and physical constants (continued)

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g = acceleration due to gravity (standard) = 32.174 ft/sec^2 = 980.665 cm/sec^2 = 386.09 in./sec^2 = 21.94 miles/hr-sec

Rest energy of the electron = 0.51098 Mev

Electron mass = $(9.1085 \pm 0.0006) \times 10^{-28}$ gram = 0.51098 Mev = 5.48760×10^{-4} amu

Proton mass = $(1.67243 \pm 0.00010) \times 10^{-24}$ gram = 938.232 Mev = 1.007593 amu

Neutron mass = $(1.67474 \pm 0.00010) \times 10^{-24}$ gram = 939.526 Mev = 1.008982 amu

α particle mass = $(6.6442 \pm 0.0012) \times 10^{-24}$ gram = 3727.377 Mev = 4.00294 amu

H_1 atom mass = $(1.67335 \pm 0.00010) \times 10^{-24}$ gram = 938.743 Mev = 1.008142 amu

SOURCE: Atomic Constants: Dumond and Cohen, *Rev. Mod. Phys.*, **25**:691 (1953).

Electrical Units

Absolute Practical	*Electrostatic (esu)*	*Electromagnetic (emu)*
1 coulomb	= 2.998×10^9 statcoulombs	= 10^{-1} abcoulomb
1 ampere (coulomb/sec)	= 2.998×10^9 statamp	= 10^{-1} abamp
1 volt	= 3.335×10^{-3} statvolt	= 10^8 abvolts
1 farad	= 8.988×10^{11} statfarads	= 10^{-9} abfarad
1 weber/sq meter	= $1/(3 \times 10^6)$	= 10^4 gauss
1 ampere turn/meter	= $12\pi \times 10^7$	= 1.257×10^{-2} oersted
1 ampere turn	= $12\pi \times 10^9$	= 1.257 gilberts
1 henry	= 1.112×10^{-12} stathenry	= 10^9 abhenrys
1 ohm	= 1.112×10^{-12} statohm	= 10^9 abohms

Miscellaneous Constants and Conversions

Density of water at 3.98°C, 760 mm Hg = 0.03613 lb/cu in. = 62.43 lb/cu ft = 1 gram/ml = 0.999973 gram/cu cm

Density of air (stp) = 1.293×10^{-3} gram/cu cm

stp = standard temperature and pressure = 0°C and 760 mm Hg

Water heat of fusion (0°C) = 79.71 g-cal (15°C)/gram

Water heat of vaporization (100°C) = 539.55 g-cal (15°C)/gram

Molar volume (stp) = 359 cu ft/lb mole = (22.4207 ± 0.0006) liters/gram mole (physical scale)

Velocity of sound: in air (stp) = 331.7 meters/sec = 1,088 ft/sec
in water (20°C) = 1,470 meters/sec = 4,823 ft/sec
in pine wood = 3,320 meters/sec = 10,900 ft/sec
in soft steel = 5,000 meters/sec = 16,410 ft/sec

Attenuation of sound in air (stp, 37% rh) = $\delta \cong 6 \times 10^{-14}$ db (kc)^2/ft

Viscosity of water (20°C) = 1.002 centipoises = 0.01002 dyne-sec/sq cm

1 curie activity = 3.7×10^{10} disintegrations/sec = 3.7×10^4 rutherfords

1 roentgen (r) radiation damage = ionization by X or γ rays producing 1 esu of charge in 1 cu cm air (stp)

1 barn cross section = 10^{-24} sq cm

Ratio physical (O^{16} = 16) to chemical scale (O = 16) of atomic weights = 1.000272 ± 0.000005

M = mass of earth = 5.983×10^{24} kg = 6.595×10^{21} tons

Interference fringe = 11.6μ in./fringe (He line: 5,875.6 angstrom units)

$\pi = 3.1415926535+$

$e = 2.7182818284+$

$\text{Log}_e \text{N} = \log_e 10 \times \log_{10} \text{N} = 2.3026 \log_{10} \text{N}$

1 kwhr $\cong 4.2 \times 10^{-5}$ gram U^{235} in fission
$\cong 6.4 \times 10^{-6}$ gram H^3 in H^3 (d,n,) He^4 reaction
$\cong$ average noon insolation in 1 hr on a horizontal plane of 1 sq meter
$\cong$ 0.74 lb (highest modern power station efficiency on 12,500 Btu/lb coal)

Table 1-1. Temperature-conversion table*

The general formulas are

$$^\circ F = (^\circ C \times \tfrac{9}{5}) + 32 \qquad \text{and} \qquad ^\circ C = (^\circ F - 32) \times \tfrac{5}{9}$$

The absolute temperature is defined by the statement that it is directly proportional to the pressure-volume product of an ideal gas. On the centigrade scale the absolute temperature is equal to $273.16 + t^\circ C$. A temperature of 50°C is then equal to $273.16 + 50 = 323.16$ degrees centigrade absolute (°C abs) or Kelvin (°K). On the Fahrenheit scale the absolute temperature is equal to $460 + t^\circ F$ and is termed degrees Rankine (°R).

The numbers in bold-face type refer to the temperature (in either centigrade or Fahrenheit degrees) which it is desired to convert into the other scale. If converting from Fahrenheit degrees to centigrade degrees, the equivalent temperature is in the left column, while if converting from degrees centigrade to degrees Fahrenheit, the equivalent temperature is in the column on the right. Interpolation factors are printed in the last two columns.

GIVEN: 200°C.
REQUIRED: Equivalent temperatures in degrees Fahrenheit, Rankine, and Kelvin.
SOLUTION: 200°C = 392°F
200°C = 200 + 273.16 = 473.16°K
200°C = 392 + 460 = 852°R

C		F	C		F	C		F	C		F
−262	**−440**		−15.0	**5**	41.0	12.8	**55**	131.0	66	**150**	302
−257	**−430**		−14.4	**6**	42.8	13.3	**56**	132.8	71	**160**	320
−251	**−420**		−13.9	**7**	44.6	13.9	**57**	134.6	77	**170**	338
−246	**−410**		−13.3	**8**	46.4	14.4	**58**	136.4	82	**180**	356
−240	**−400**		−12.8	**9**	48.2	15.0	**59**	138.2	88	**190**	374
−234	**−390**		−12.2	**10**	50.0	15.6	**60**	140.0	93	**200**	392
−229	**−380**		−11.7	**11**	51.8	16.1	**61**	141.8	99	**210**	410
−223	**−370**		−11.1	**12**	53.6	16.7	**62**	143.6	104	**220**	428
−218	**−360**		−10.6	**13**	55.4	17.2	**63**	145.4	110	**230**	446
−212	**−350**		−10.0	**14**	57.2	17.8	**64**	147.2	116	**240**	464
−207	**−340**		−9.44	**15**	59.0	18.3	**65**	149.0	121	**250**	482
−201	**−330**		−8.89	**16**	60.8	18.9	**66**	150.8	127	**260**	500
−196	**−320**		−8.33	**17**	62.6	19.4	**67**	152.6	132	**270**	518
−190	**−310**		−7.78	**18**	64.4	20.0	**68**	154.4	138	**280**	536
−184	**−300**		−7.22	**19**	66.2	20.6	**69**	156.2	143	**290**	554
−179	**−290**		−6.67	**20**	68.0	21.1	**70**	158.0	149	**300**	572
−173	**−280**		−6.11	**21**	69.8	21.7	**71**	159.8	154	**310**	590
−169	**−273**	−459.4	−5.56	**22**	71.6	22.2	**72**	161.6	160	**320**	608
−168	**−270**	−454	−5.00	**23**	73.4	22.8	**73**	163.4	166	**330**	626
−162	**−260**	−436	−4.44	**24**	75.2	23.3	**74**	165.2	171	**340**	644
−157	**−250**	−418	−3.89	**25**	77.0	23.9	**75**	167.0	177	**350**	662
−151	**−240**	−400	−3.33	**26**	78.8	24.4	**76**	168.8	182	**360**	680
−146	**−230**	−382	−2.78	**27**	80.6	25.0	**77**	170.6	188	**370**	698
−140	**−220**	−364	−2.22	**28**	82.4	25.6	**78**	172.4	193	**380**	716
−134	**−210**	−346	−1.67	**29**	84.2	26.1	**79**	174.2	199	**390**	734
−129	**−200**	−328	−1.11	**30**	86.0	26.7	**80**	176.0	204	**400**	752
−123	**−190**	−310	−0.56	**31**	87.8	27.2	**81**	177.8	210	**410**	770
−118	**−180**	−292	0	**32**	89.6	27.8	**82**	179.6	216	**420**	788
−112	**−170**	−274	0.56	**33**	91.4	28.3	**83**	181.4	221	**430**	806
−107	**−160**	−256	1.11	**34**	93.2	28.9	**84**	183.2	227	**440**	824
−101	**−150**	−238	1.67	**35**	95.0	29.4	**85**	185.0	232	**450**	842
−95.6	**−140**	−220	2.22	**36**	96.8	30.0	**86**	186.8	238	**460**	860
−90.0	**−130**	−202	2.78	**37**	98.6	30.6	**87**	188.6	243	**470**	878
−84.4	**−120**	−184	3.33	**38**	100.4	31.1	**88**	190.4	249	**480**	896
−78.9	**−110**	−166	3.89	**39**	102.2	31.7	**89**	192.2	254	**490**	914
−73.3	**−100**	−148	4.44	**40**	104.0	32.2	**90**	194.0	260	**500**	932
−67.8	**−90**	−130	5.00	**41**	105.8	32.8	**91**	195.8	266	**510**	950
−62.2	**−80**	−112	5.56	**42**	107.6	33.3	**92**	197.6	271	**520**	968
−56.7	**−70**	−94	6.11	**43**	109.4	33.9	**93**	199.4	277	**530**	986
−51.1	**−60**	−76	6.67	**44**	111.2	34.4	**94**	201.2	282	**540**	1004
−45.6	**−50**	−58	7.22	**45**	113.0	35.0	**95**	203.0	288	**550**	1022
−40.0	**−40**	−40	7.78	**46**	114.8	35.6	**96**	204.8	293	**560**	1040
−34.4	**−30**	−22	8.33	**47**	116.6	36.1	**97**	206.6	299	**570**	1058
−28.9	**−20**	−4	8.89	**48**	118.4	36.7	**98**	208.4	304	**580**	1076
−23.3	**−10**	14	9.44	**49**	120.2	37.2	**99**	210.2	310	**590**	1094
−17.8	**0**	32	10.0	**50**	122.0	38	**100**	212	316	**600**	1112
−17.2	**1**	33.8	10.6	**51**	123.8	43	**110**	230	321	**610**	1130
−16.7	**2**	35.6	11.1	**52**	125.6	49	**120**	248	327	**620**	1148
−16.1	**3**	37.4	11.7	**53**	127.4	54	**130**	266	332	**630**	1166
−15.6	**4**	39.2	12.2	**54**	129.2	60	**140**	284	338	**640**	1184

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* From R. H. Perry, "Engineering Manual," McGraw-Hill Book Company, New York, 1959.

Table 1-1. Temperature-conversion table (continued)

C		F	C		F	C		F	C		F
343	**650**	1202	704	**1300**	2372	1066	**1950**	3542	1371	**2500**	4532
349	**660**	1220	710	**1310**	2390	1071	**1960**	3560	1377	**2510**	4550
354	**670**	1238	716	**1320**	2408	1077	**1970**	3578	1382	**2520**	4568
360	**680**	1256	721	**1330**	2426	1082	**1980**	3596	1388	**2530**	4586
366	**690**	1274	727	**1340**	2444	1088	**1990**	3614	1393	**2540**	4604
371	**700**	1292	732	**1350**	2462	1093	**2000**	3632	1399	**2550**	4622
377	**710**	1310	738	**1360**	2480	1099	**2010**	3650	1404	**2560**	4640
382	**720**	1328	743	**1370**	2498	1104	**2020**	3668	1410	**2570**	4658
388	**730**	1346	749	**1380**	2516	1110	**2030**	3686	1416	**2580**	4676
393	**740**	1364	754	**1390**	2534	1116	**2040**	3704	1421	**2590**	4694
399	**750**	1382	760	**1400**	2552	1121	**2050**	3722	1427	**2600**	4712
404	**760**	1400	766	**1410**	2570	1127	**2060**	3740	1432	**2610**	4730
410	**770**	1418	771	**1420**	2588	1132	**2070**	3758	1438	**2620**	4748
416	**780**	1436	777	**1430**	2606	1138	**2080**	3776	1443	**2630**	4766
421	**790**	1454	782	**1440**	2624	1143	**2090**	3794	1449	**2640**	4784
427	**800**	1472	788	**1450**	2642	1149	**2100**	3812	1454	**2650**	4802
432	**810**	1490	793	**1460**	2660	1154	**2110**	3830	1460	**2660**	4820
438	**820**	1508	799	**1470**	2678	1160	**2120**	3848	1466	**2670**	4838
443	**830**	1526	804	**1480**	2696	1166	**2130**	3866	1471	**2680**	4856
449	**840**	1544	810	**1490**	2714	1171	**2140**	3884	1477	**2690**	4874
454	**850**	1562	816	**1500**	2732	1177	**2150**	3902	1482	**2700**	4892
460	**860**	1580	821	**1510**	2750	1182	**2160**	3920	1488	**2710**	4910
466	**870**	1598	827	**1520**	2768	1188	**2170**	3938	1493	**2720**	4928
471	**880**	1616	832	**1530**	2786	1193	**2180**	3956	1499	**2730**	4946
477	**890**	1634	838	**1540**	2804	1199	**2190**	3974	1504	**2740**	4964
482	**900**	1652	843	**1550**	2822	1204	**2200**	3992	1510	**2750**	4982
488	**910**	1670	849	**1560**	2840	1210	**2210**	4010	1516	**2760**	5000
493	**920**	1688	854	**1570**	2858	1216	**2220**	4028	1521	**2770**	5018
499	**930**	1706	860	**1580**	2876	1221	**2230**	4046	1527	**2780**	5036
504	**940**	1724	866	**1590**	2894	1227	**2240**	4064	1532	**2790**	5054
510	**950**	1742	871	**1600**	2912	1232	**2250**	4082	1538	**2800**	5072
516	**960**	1760	877	**1610**	2930	1238	**2260**	4100	1543	**2810**	5090
521	**970**	1778	882	**1620**	2948	1243	**2270**	4118	1549	**2820**	5108
527	**980**	1796	888	**1630**	2966	1249	**2280**	4136	1554	**2830**	5126
532	**990**	1814	893	**1640**	2984	1254	**2290**	4154	1560	**2840**	5144
538	**1000**	1832	899	**1650**	3002	1260	**2300**	4172	1566	**2850**	5162
543	**1010**	1850	904	**1660**	3020	1266	**2310**	4190	1571	**2860**	5180
549	**1020**	1868	910	**1670**	3038	1271	**2320**	4208	1577	**2870**	5198
554	**1030**	1886	916	**1680**	3056	1277	**2330**	4226	1582	**2880**	5216
560	**1040**	1904	921	**1690**	3074	1282	**2340**	4244	1588	**2890**	5234
566	**1050**	1922	927	**1700**	3092	1288	**2350**	4262	1593	**2900**	5252
571	**1060**	1940	932	**1710**	3110	1293	**2360**	4280	1599	**2910**	5270
577	**1070**	1958	938	**1720**	3128	1299	**2370**	4298	1604	**2920**	5288
582	**1080**	1976	943	**1730**	3146	1304	**2380**	4316	1610	**2930**	5306
588	**1090**	1994	949	**1740**	3164	1310	**2390**	4334	1616	**2940**	5324
593	**1100**	2012	954	**1750**	3182	1316	**2400**	4352	1621	**2950**	5342
599	**1110**	2030	960	**1760**	3200	1321	**2410**	4370	1627	**2960**	5360
604	**1120**	2048	966	**1770**	3218	1327	**2420**	4388	1632	**2970**	5378
610	**1130**	2066	971	**1780**	3236	1332	**2430**	4406	1638	**2980**	5396
616	**1140**	2084	977	**1790**	3254	1338	**2440**	4424	1643	**2990**	5414
621	**1150**	2102	982	**1800**	3272	1343	**2450**	4442	1649	**3000**	5432
627	**1160**	2120	988	**1810**	3290	1349	**2460**	4460			
632	**1170**	2138	993	**1820**	3308	1354	**2470**	4478			
638	**1180**	2156	999	**1830**	3326	1360	**2480**	4496			
643	**1190**	2174	1004	**1840**	3344	1366	**2490**	4514			
649	**1200**	2192	1010	**1850**	3362						
654	**1210**	2210	1016	**1860**	3380						
660	**1220**	2228	1021	**1870**	3398						
666	**1230**	2246	1027	**1880**	3416						
671	**1240**	2264	1032	**1890**	3434						
677	**1250**	2282	1038	**1900**	3452						
682	**1260**	2300	1043	**1910**	3470						
688	**1270**	2318	1049	**1920**	3488						
693	**1280**	2336	1054	**1930**	3506						
699	**1290**	2354	1060	**1940**	3524						

Interpolation Factors

0.56	**1**	1.8	3.33	**6**	10.8
1.11	**2**	3.6	3.89	**7**	12.6
1.67	**3**	5.4	4.44	**8**	14.4
2.22	**4**	7.2	5.00	**9**	16.2
2.78	**5**	9.0	5.56	**10**	18.0

Numerical constants

N	$\log_{10} N$
$\pi = 3.14159265$	0.4971499
$2\pi = 6.28318531$	0.7981799
$\pi/2 = 1.57079633$	0.1961199
$1/\pi = 0.31830989$	$9.5028501 - 10$
$1/2\pi = 0.15915494$	$9.2018201 - 10$
$\pi^2 = 9.86960440$	0.9942997
$1/\pi^2 = 0.10132118$	$9.0057003 - 10$
$\sqrt{\pi} = 1.77245385$	0.2485749
$\sqrt{2\pi} = 2.50662827$	0.3990899
$\sqrt{\pi/2} = 1.25331414$	0.0980599
$1/\sqrt{\pi} = 0.56418958$	$9.7514251 - 10$
$\sqrt{2/\pi} = 0.79788456$	$9.9019401 - 10$
$1/\sqrt{2\pi} = 0.39894228$	$9.6009101 - 10$
$\sqrt{2} = 1.41421356$	0.1505150

$e = 2.71828183$

$M = \log_{10} e = 0.43429448$

$1/M = \log_e 10 = 2.30258509$

$\log_e \pi = 1.14472989$

section
2
FOUR-PLACE TABLES FOR QUICK REFERENCE

Table 2-1. Common logarithms of numbers*

Number	0	1	2	3	4	5	6	7	8	9	Avg diff
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	18
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	17
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	10
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9

$\log \pi = 0.4971$ $\log \pi/2 = 0.1961$ $\log \pi^2 = 0.9943$ $\log \sqrt{\pi} = 0.2486$

$\log e = 0.4343$ $\log (0.4343) = 0.6378 - 1$

These two pages give the common logarithms of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right [or left] in the number is equivalent to adding n [or $-n$] to the logarithm. Thus, $\log 0.017453 = 0.2419 - 2$, which may also be written $\bar{2}.2419$ or $8.2419 - 10$. See p. 1, graphs, p. 2.

$$\log (ab) = \log a + \log b \qquad \log (a^N) = N \log a$$

$$\log \left(\frac{a}{b}\right) = \log a - \log b \qquad \log \left(\sqrt[N]{a}\right) = \frac{1}{N} \log a$$

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* From L. S. Marks, "Mechanical Engineer's Handbook," McGraw-Hill Book Company, New York, 1958.

Table 2-1. Common logarithms of numbers (continued)

Number	0	1	2	3	4	5	6	7	8	9	Avg diff
5.0	0.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
5.1	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8
5.2	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
5.3	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
5.4	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8
5.5	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
5.6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
5.7	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8
5.8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7
5.9	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7
6.0	0.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
6.1	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
6.2	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
6.3	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
6.4	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
6.5	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
6.6	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
6.7	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
6.8	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
6.9	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
7.0	0.8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
7.1	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
7.2	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
7.3	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
7.4	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
7.5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
7.6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
7.7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
7.8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6
7.9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	5
8.0	0.9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
8.1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
8.2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
8.3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
8.4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
8.5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
8.6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
8.7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
8.8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
8.9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5
9.0	0.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
9.1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
9.2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
9.3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5
9.4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5
9.5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
9.6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	4
9.7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4
9.8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
9.9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4

Table 2-2. Natural sines and cosines*

Natural sines at intervals of 0°.1, or 6′

Deg	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg diff
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	0523	87	17
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	0698	86	17
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	0.0872	85	17
5	0.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	1045	84	17
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	1219	83	17
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	1392	82	17
8	1392	1409	1426.	1444	1461	1478	1495	1513	1530	1547	1564	81	17
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	0.1736	80°	17
10°	0.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	1908	79	17
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	2079	78	17
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	2250	77	17
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	2419	76	17
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	0.2588	75	17
15	0.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	2756	74	17
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	2924	73	17
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3090	72	17
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3256	71	17
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	0.3420	70°	16
20°	0.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3584	69	16
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3746	68	16
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3907	67	16
23	3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	4067	66	16
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	0.4226	65	16
25	0.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	4384	64	16
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	4540	63	16
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	4695	62	16
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	4848	61	15
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	0.5000	60°	15
30°	0.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	5150	59	15
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	5299	58	15
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	5446	57	15
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	5592	56	15
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	0.5736	55	14
35	0.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	5878	54	14
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	6018	53	14
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	6157	52	14
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	6293	51	14
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	0.6428	50°	13
40°	0.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	6561	49	13
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	6691	48	13
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	6820	47	13
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	6947	46	13
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	0.7071	45°	12
45°	0.7071												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg	

Natural cosines

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* From L. S. Marks, "Mechanical Engineer's Handbook," McGraw-Hill Book Company, New York, 1958.

Table 2-2. Natural sines and cosines (continued)

Natural sines at intervals of 0°.1, or 6′

Deg	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg diff
											0.7071	45°	
45°	0.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	7193	44	12
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	7314	43	12
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	7431	42	12
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	7547	41	12
49	7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	0.7660	40°	11
50°	0.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	7771	39	11
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	7880	38	11
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	7986	37	11
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	8090	36	10
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	0.8192	35	10
55	0.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	8290	34	10
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	8387	33	10
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	8480	32	9
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	8572	31	9
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	0.8660	30°	9
60°	0.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	8746	29	9
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	8829	28	8
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	8910	27	8
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	8988	26	8
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	0.9063	25	7
65	0.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	9135	24	7
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	9205	23	7
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	9272	22	7
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	9336	21	6
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	0.9397	20°	6
70°	0.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	9455	19	6
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	9511	18	6
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	9563	17	5
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	9613	16	5
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	0.9659	15	5
75	0.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	9703	14	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	9744	13	4
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	9781	12	4
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	9816	11	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	0.9848	10°	3
80°	0.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	9877	9	3
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	9903	8	3
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	9925	7	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	9945	6	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0.9962	5	2
85	0.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	9976	4	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	9986	3	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	9994	2	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0.9998	1	0
89	0.9998	9999	9999	9999	9999	0000	0000	0000	0000	0000	1.0000	0°	0
90°	1.0000												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg	

Natural cosines

Table 2-3. Natural tangents and cotangents*

Natural tangents at intervals of 0°.1, or 6′

Deg	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg diff
											0.0000	90°	
0°	0.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	0175	89	17
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	0349	88	17
2	0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	0524	87	17
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	0699	86	18
4	0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	0.0875	85	18
5	0.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	1051	84	18
6	1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	1228	83	18
7	1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	1405	82	18
8	1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	1584	81	18
9	1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	0.1763	80°	18
10°	0.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	1944	79	18
11	1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	2126	78	18
12	2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	2309	77	18
13	2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	2493	76	18
14	2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	0.2679	75	19
15	0.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	2867	74	19
16	2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3057	73	19
17	3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3249	72	19
18	3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3443	71	19
19	3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	0.3640	70°	20
20°	0.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3839	69	20
21	3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	4040	68	20
22	4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	4245	67	21
23	4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	4452	66	21
24	4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	0.4663	65	21
25	0.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4877	64	21
26	4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	5095	63	22
27	5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	5317	62	22
28	5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	5543	61	23
29	5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	0.5774	60°	23
30°	0.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	6009	59	24
31	6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	6249	58	24
32	6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	6494	57	25
33	6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	6745	56	25
34	6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	0.7002	55	26
35	0.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	7265	54	26
36	7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	7536	53	27
37	7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	7813	52	28
38	7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	8098	51	28
39	8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	0.8391	50°	29
40°	0.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	8693	49	30
41	8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	9004	48	31
42	9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	9325	47	32
43	9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	0.9657	46	33
44	0.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	1.0000	45°	34
45°	1.0000												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg	

Natural cotangents

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* From L. S. Marks, "Mechanical Engineer's Handbook," McGraw-Hill Book Company, New York, 1958.

Table 2-3. Natural tangents and cotangents (continued)

Natural tangents at intervals of 0°.1, or 6′

Deg	°.0 =(0′)	°.1 (6′)	°.2 (12′)	°.3 (18′)	°.4 (24′)	°.5 (30′)	°.6 (36′)	°.7 (42′)	°.8 (48′)	°.9 (54′)			Avg diff
											1.0000	45°	
45°	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	0355	44	35
46	0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	0724	43	37
47	0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	1106	42	38
48	1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	1504	41	40
49	1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	1.1918	40°	41
50°	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	2349	39	43
51	2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	2799	38	45
52	2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	3270	37	47
53	3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	3764	36	49
54	3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	1.4281	35	52
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	4826	34	55
56	4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	5399	33	57
57	5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	6003	32	60
58	6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	6643	31	64
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	1.7321	30°	67
60°	1.732	1.739	1.746	1.753	1.760	1.767	1.775	1.782	1.789	1.797	1.804	29	7
61	1.804	1.811	1.819	1.827	1.834	1.842	1.849	1.857	1.865	1.873	1.881	28	8
62	1.881	1.889	1.897	1.905	1.913	1.921	1.929	1.937	1.946	1.954	1.963	27	8
63	1.963	1.971	1.980	1.988	1.997	2.006	2.014	2.023	2.032	2.041	2.050	26	9
64	2.050	2.059	2.069	2.078	2.087	2.097	2.106	2.116	2.125	2.135	2.145	25	9
65	2.145	2.154	2.164	2.174	2.184	2.194	2.204	2.215	2.225	2.236	2.246	24	10
66	2.246	2.257	2.267	2.278	2.289	2.300	2.311	2.322	2.333	2.344	2.356	23	11
67	2.356	2.367	2.379	2.391	2.402	2.414	2.426	2.438	2.450	2.463	2.475	22	12
68	2.475	2.488	2.500	2.513	2.526	2.539	2.552	2.565	2.578	2.592	2.605	21	13
69	2.605	2.619	2.633	2.646	2.660	2.675	2.689	2.703	2.718	2.733	2.747	20°	14
70°	2.747	2.762	2.778	2.793	2.808	2.824	2.840	2.856	2.872	2.888	2.904	19	16
71	2.904	2.921	2.937	2.954	2.971	2.989	3.006	3.024	3.042	3.060	3.078	18	17
72	3.078	3.096	3.115	3.133	3.152	3.172	3.191	3.211	3.230	3.251	3.271	17	19
73	3.271	3.291	3.312	3.333	3.354	3.376	3.398	3.420	3.442	3.465	3.487	16	22
74	3.487	3.511	3.534	3.558	3.582	3.606	3.630	3.655	3.681	3.706	3.732	15	24
75	3.732	3.758	3.785	3.812	3.839	3.867	3.895	3.923	3.952	3.981	4.011	14	28
76	4.011	4.041	4.071	4.102	4.134	4.165	4.198	4.230	4.264	4.297	4.331	13	32
77	4.331	4.366	4.402	4.437	4.474	4.511	4.548	4.586	4.625	4.665	4.705	12	37
78	4.705	4.745	4.787	4.829	4.872	4.915	4.959	5.005	5.050	5.097	5.145	11	44
79	5.145	5.193	5.242	5.292	5.343	5.396	5.449	5.503	5.558	5.614	5.671	10°	53
80°	5.671	5.730	5.789	5.850	5.912	5.976	6.041	6.107	6.174	6.243	6.314	9	
81	6.314	6.386	6.460	6.535	6.612	6.691	6.772	6.855	6.940	7.026	7.115	8	
82	7.115	7.207	7.300	7.396	7.495	7.596	7.700	7.806	7.916	8.028	8.144	7	
83	8.144	8.264	8.386	8.513	8.643	8.777	8.915	9.058	9.205	9.357	9.514	6	
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20	11.43	5	
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	14.30	4	
86	14.30	14.67	15.06	15.46	15.90	16.35	16.83	17.34	17.89	18.46	19.08	3	
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	28.64	2	
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08	57.29	1	
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0	∞	0°	
90°	∞												
		°.9 =(54′)	°.8 (48′)	°.7 (42′)	°.6 (36′)	°.5 (30′)	°.4 (24′)	°.3 (18′)	°.2 (12′)	°.1 (6′)	°.0 (0′)	Deg	

Natural cotangents

Table 2-4. Logarithms of sines and cosines*

Note: For cosines use right-hand column of degrees and lower line of tenths.

Log sines (read down)

Deg	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°			Avg. diff.
											.....	90	
0	.....	7.2419	7.5429	7.7190	7.8439	7.9408	8.0200	8.0870	8.1450	8.1961	8.2419	89	
1	8.2419	8.2832	8.3210	8.3558	8.3880	8.4179	8.4459	8.4723	8.4971	8.5206	8.5428	88	
2	8.5428	8.5640	8.5842	8.6035	8.6220	8.6397	8.6567	8.6731	8.6889	8.7041	8.7188	87	
3	8.7188	8.7330	8.7468	8.7602	8.7731	8.7857	8.7979	8.8098	8.8213	8.8326	8.8436	86	
4	8.8436	8.8543	8.8647	8.8749	8.8849	8.8946	8.9042	8.9135	8.9226	8.9315	8.9403	85	
5	8.9403	8.9489	8.9573	8.9655	8.9736	8.9816	8.9894	8.9970	9.0046	9.0120	9.0192	84	
6	9.0192	9.0264	9.0334	9.0403	9.0472	9.0539	9.0605	9.0670	9.0734	9.0797	9.0859	83	
7	9.0859	9.0920	9.0981	9.1040	9.1099	9.1157	9.1214	9.1271	9.1326	9.1381	9.1436	82	
8	9.1436	9.1489	9.1542	9.1594	9.1646	9.1697	9.1747	9.1797	9.1847	9.1895	9.1943	81	
9	9.1943	9.1991	9.2038	9.2085	9.2131	9.2176	9.2221	9.2266	9.2310	9.2353	9.2397	80	
10	9.2397	9.2439	9.2482	9.2524	9.2565	9.2606	9.2647	9.2687	9.2727	9.2767	9.2806	79	41
11	9.2806	9.2845	9.2883	9.2921	9.2959	9.2997	9.3034	9.3070	9.3107	9.3143	9.3179	78	37
12	9.3179	9.3214	9.3249	9.3284	9.3319	9.3353	9.3387	9.3421	9.3455	9.3488	9.3521	77	34
13	9.3521	9.3554	9.3586	9.3618	9.3650	9.3682	9.3713	9.3744	9.3775	9.3806	9.3837	76	32
14	9.3837	9.3867	9.3897	9.3927	9.3957	9.3986	9.4015	9.4044	9.4073	9.4102	9.4130	75	29
15	9.4130	9.4158	9.4186	9.4214	9.4242	9.4269	9.4296	9.4323	9.4350	9.4377	9.4403	74	27
16	9.4403	9.4430	9.4456	9.4482	9.4508	9.4533	9.4559	9.4584	9.4609	9.4634	9.4659	73	25
17	9.4659	9.4684	9.4709	9.4733	9.4757	9.4781	9.4805	9.4829	9.4853	9.4876	9.4900	72	24
18	9.4900	9.4923	9.4946	9.4969	9.4992	9.5015	9.5037	9.5060	9.5082	9.5104	9.5126	71	22
19	9.5126	9.5148	9.5170	9.5192	9.5213	9.5235	9.5256	9.5278	9.5299	9.5320	9.5341	70	22
20	9.5341	9.5361	9.5382	9.5402	9.5423	9.5443	9.5463	9.5484	9.5504	9.5523	9.5543	69	20
21	9.5543	9.5563	9.5583	9.5602	9.5621	9.5641	9.5660	9.5679	9.5698	9.5717	9.5736	68	19
22	9.5736	9.5754	9.5773	9.5792	9.5810	9.5828	9.5847	9.5865	9.5883	9.5901	9.5919	67	18
23	9.5919	9.5937	9.5954	9.5972	9.5990	9.6007	9.6024	9.6042	9.6059	9.6076	9.6093	66	17
24	9.6093	9.6110	9.6127	9.6144	9.6161	9.6177	9.6194	9.6210	9.6227	9.6243	9.6260	65	17
25	9.6260	9.6276	9.6292	9.6308	9.6324	9.6340	9.6356	9.6371	9.6387	9.6403	9.6418	64	16
26	9.6418	9.6434	9.6449	9.6465	9.6480	9.6495	9.6510	9.6526	9.6541	9.6556	9.6571	63	15
27	9.6571	9.6585	9.6600	9.6615	9.6629	9.6644	9.6659	9.6673	9.6687	9.6702	9.6716	62	14
28	9.6716	9.6730	9.6744	9.6759	9.6773	9.6787	9.6801	9.6814	9.6828	9.6842	9.6856	61	14
29	9.6856	9.6869	9.6883	9.6896	9.6910	9.6923	9.6937	9.6950	9.6963	9.6977	9.6990	60	14
30	9.6990	9.7003	9.7016	9.7029	9.7042	9.7055	9.7068	9.7080	9.7093	9.7106	9.7118	59	13
31	9.7118	9.7131	9.7144	9.7156	9.7168	9.7181	9.7193	9.7205	9.7218	9.7230	9.7242	58	12
32	9.7242	9.7254	9.7266	9.7278	9.7290	9.7302	9.7314	9.7326	9.7338	9.7349	9.7361	57	12
33	9.7361	9.7373	9.7384	9.7396	9.7407	9.7419	9.7430	9.7442	9.7453	9.7464	9.7476	56	12
34	9.7476	9.7487	9.7498	9.7509	9.7520	9.7531	9.7542	9.7553	9.7564	9.7575	9.7586	55	11
35	9.7586	9.7597	9.7607	9.7618	9.7629	9.7640	9.7650	9.7661	9.7671	9.7682	9.7692	54	11
36	9.7692	9.7703	9.7713	9.7723	9.7734	9.7744	9.7754	9.7764	9.7774	9.7785	9.7795	53	10
37	9.7795	9.7805	9.7815	9.7825	9.7835	9.7845	9.7854	9.7864	9.7874	9.7884	9.7893	52	10
38	9.7893	9.7903	9.7913	9.7922	9.7932	9.7942	9.7951	9.7960	9.7970	9.7979	9.7989	51	10
39	9.7989	9.7998	9.8007	9.8017	9.8026	9.8035	9.8044	9.8053	9.8063	9.8072	9.8081	50	9
40	9.8081	9.8090	9.8099	9.8108	9.8117	9.8125	9.8134	9.8143	9.8152	9.8161	9.8169	49	9
41	9.8169	9.8178	9.8187	9.8195	9.8204	9.8213	9.8221	9.8230	9.8238	9.8247	9.8255	48	9
42	9.8255	9.8264	9.8272	9.8280	9.8289	9.8297	9.8305	9.8313	9.8322	9.8330	9.8338	47	8
43	9.8338	9.8346	9.8354	9.8362	9.8370	9.8378	9.8386	9.8394	9.8402	9.8410	9.8418	46	8
44	9.8418	9.8426	9.8433	9.8441	9.8449	9.8457	9.8464	9.8472	9.8480	9.8487	9.8495	45	8
45	9.8495												
		0.9°	0.8°	0.7°	0.6°	0.5°	0.4°	0.3°	0.2°	0.1°	0.0°	Deg	Avg. diff.

Log cosines (read up)

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* From W. D. Cockrell, "Industrial Electronics Handbook," McGraw-Hill Book Company, New York, 1958.

Table 2-4. Logarithms of sines and cosines (continued)

Note: For cosines use right-hand column of degrees and lower line of tenths.

Log sines (read down)

Deg	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°			Avg. diff.
											9.8495	45	8
45	9.8495	9.8502	9.8510	9.8517	9.8525	9.8532	9.8540	9.8547	9.8555	9.8562	9.8569	44	7
46	9.8569	9.8577	9.8584	9.8591	9.8598	9.8606	9.8613	9.8620	9.8627	9.8634	9.8641	43	7
47	9.8641	9.8648	9.8655	9.8662	9.8669	9.8676	9.8683	9.8690	9.8697	9.8704	9.8711	42	7
48	9.8711	9.8718	9.8724	9.8731	9.8738	9.8745	9.8751	9.8758	9.8765	9.8771	9.8778	41	7
49	9.8778	9.8784	9.8791	9.8797	9.8804	9.8811	9.8817	9.8823	9.8830	9.8836	9.8843	40	6
50	9.8843	9.8849	9.8855	9.8862	9.8868	9.8874	9.8880	9.8887	9.8893	9.8899	9.8905	39	6
51	9.8905	9.8911	9.8917	9.8923	9.8929	9.8935	9.8941	9.8947	9.8953	9.8959	9.8965	38	6
52	9.8965	9.8971	9.8977	9.8983	9.8989	9.8995	9.9000	9.9006	9.9012	9.9018	9.9024	37	6
53	9.9024	9.9029	9.9035	9.9041	9.9046	9.9052	9.9057	9.9063	9.9069	9.9074	9.9080	36	6
54	9.9080	9.9085	9.9091	9.9096	9.9101	9.9107	9.9112	9.9118	9.9123	9.9128	9.9134	35	5
55	9.9134	9.9139	9.9144	9.9149	9.9155	9.9160	9.9165	9.9170	9.9175	9.9181	9.9186	34	5
56	9.9186	9.9191	9.9196	9.9201	9.9206	9.9211	9.9216	9.9221	9.9226	9.9231	9.9236	33	5
57	9.9236	9.9241	9.9246	9.9251	9.9255	9.9260	9.9265	9.9270	9.9275	9.9279	9.9284	32	5
58	9.9284	9.9289	9.9294	9.9298	9.9303	9.9308	9.9312	9.9317	9.9322	9.9326	9.9331	31	5
59	9.9331	9.9335	9.9340	9.9344	9.9349	9.9353	9.9358	9.9362	9.9367	9.9371	9.9375	30	4
60	9.9375	9.9380	9.9384	9.9388	9.9393	9.9397	9.9401	9.9406	9.9410	9.9414	9.9418	29	4
61	9.9418	9.9422	9.9427	9.9431	9.9435	9.9439	9.9443	9.9447	9.9451	9.9455	9.9459	28	4
62	9.9459	9.9463	9.9467	9.9471	9.9475	9.9479	9.9483	9.9487	9.9491	9.9495	9.9499	27	4
63	9.9499	9.9503	9.9506	9.9510	9.9514	9.9518	9.9522	9.9525	9.9529	9.9533	9.9537	26	4
64	9.9537	9.9540	9.9544	9.9548	9.9551	9.9555	9.9558	9.9562	9.9566	9.9569	9.9573	25	3
65	9.9573	9.9576	9.9580	9.9583	9.9587	9.9590	9.9594	9.9597	9.9601	9.9604	9.9607	24	3
66	9.9607	9.9611	9.9614	9.9617	9.9621	9.9624	9.9627	9.9631	9.9634	9.9637	9.9640	23	3
67	9.9640	9.9643	9.9647	9.9650	9.9653	9.9656	9.9659	9.9662	9.9666	9.9669	9.9672	22	3
68	9.9672	9.9675	9.9678	9.9681	9.9684	9.9687	9.9690	9.9693	9.9696	9.9699	9.9702	21	3
69	9.9702	9.9704	9.9707	9.9710	9.9713	9.9716	9.9719	9.9722	9.9724	9.9727	9.9730	20	3
70	9.9730	9.9733	9.9735	9.9738	9.9741	9.9744	9.9746	9.9749	9.9751	9.9754	9.9757	19	3
71	9.9757	9.9759	9.9762	9.9764	9.9767	9.9770	9.9772	9.9775	9.9777	9.9780	9.9782	18	2
72	9.9782	9.9785	9.9787	9.9789	9.9792	9.9794	9.9797	9.9799	9.9801	9.9804	9.9806	17	2
73	9.9806	9.9808	9.9811	9.9813	9.9815	9.9817	9.9820	9.9822	9.9824	9.9826	9.9828	16	2
74	9.9828	9.9831	9.9833	9.9835	9.9837	9.9839	9.9841	9.9843	9.9845	9.9847	9.9849	15	2
75	9.9849	9.9851	9.9853	9.9855	9.9857	9.9859	9.9861	9.9863	9.9865	9.9867	9.9869	14	2
76	9.9869	9.9871	9.9873	9.9875	9.9876	9.9878	9.9880	9.9882	9.9884	9.9885	9.9887	13	2
77	9.9887	9.9889	9.9891	9.9892	9.9894	9.9896	9.9897	9.9899	9.9901	9.9902	9.9904	12	2
78	9.9904	9.9906	9.9907	9.9909	9.9910	9.9912	9.9913	9.9915	9.9916	9.9918	9.9920	11	2
79	9.9920	9.9921	9.9922	9.9924	9.9925	9.9927	9.9928	9.9929	9.9931	9.9932	9.9934	10	1
80	9.9934	9.9935	9.9936	9.9937	9.9939	9.9940	9.9941	9.9943	9.9944	9.9945	9.9946	9	1
81	9.9946	9.9947	9.9949	9.9950	9.9951	9.9952	9.9953	9.9954	9.9955	9.9956	9.9958	8	1
82	9.9958	9.9959	9.9960	9.9961	9.9962	9.9963	9.9964	9.9965	9.9966	9.9967	9.9968	7	1
83	9.9968	9.9968	9.9969	9.9970	9.9971	9.9972	9.9973	9.9974	9.9975	9.9975	9.9976	6	1
84	9.9976	9.9977	9.9978	9.9978	9.9979	9.9980	9.9981	9.9981	9.9982	9.9983	9.9983	5	1
85	9.9983	9.9984	9.9985	9.9985	9.9986	9.9987	9.9987	9.9988	9.9988	9.9989	9.9989	4	1
86	9.9989	9.9990	9.9990	9.9991	9.9991	9.9992	9.9992	9.9993	9.9993	9.9994	9.9994	3	1
87	9.9994	9.9994	9.9995	9.9995	9.9996	9.9996	9.9996	9.9996	9.9997	9.9997	9.9997	2	
88	9.9997	9.9998	9.9998	9.9998	9.9998	9.9999	9.9999	9.9999	9.9999	9.9999	9.9999	1	
89	9.9999	9.9999	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	0	
90	10.0000												
		0.9°	0.8°	0.7°	0.6°	0.5°	0.4°	0.3°	0.2°	0.1°	0.0°	Deg	Avg. diff.

Log cosines (read up)

Table 2-5. Logarithms of tangents and cotangents*

Note: For cotangents use right-hand column of degrees and lower line of tenths.

Log tangents (read down)

Deg	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°			Avg. diff.
											.....	90	
0	.....	7.2419	7.5429	7.7190	7.8439	7.9409	8.0200	8.0870	8.1450	8.1962	8.2419	89	
1	8.2419	8.2833	8.3211	8.3559	8.3881	8.4181	8.4461	8.4725	8.4973	8.5208	8.5431	88	
2	8.5431	8.5643	8.5845	8.6038	8.6223	8.6401	8.6571	8.6736	8.6894	8.7046	8.7194	87	
3	8.7194	8.7337	8.7475	8.7609	8.7739	8.7865	8.7988	8.8107	8.8223	8.8336	8.8446	86	
4	8.8446	8.8554	8.8659	8.8762	8.8862	8.8960	8.9056	8.9150	8.9241	8.9331	8.9420	85	
5	8.9420	8.9506	8.9591	8.9674	8.9756	8.9836	8.9915	8.9992	9.0068	9.0143	9.0216	84	
6	9.0216	9.0289	9.0360	9.0430	9.0499	9.0567	9.0633	9.0699	9.0764	9.0828	9.0891	83	
7	9.0891	9.0954	9.1015	9.1076	9.1135	9.1194	9.1252	9.1310	9.1367	9.1423	9.1478	82	
8	9.1478	9.1533	9.1587	9.1640	9.1693	9.1745	9.1797	9.1848	9.1898	9.1948	9.1997	81	
9	9.1997	9.2046	9.2094	9.2142	9.2189	9.2236	9.2282	9.2328	9.2374	9.2419	9.2463	80	
10	9.2463	9.2507	9.2551	9.2594	9.2537	9.2680	9.2722	9.2764	9.2805	9.2846	9.2887	79	44
11	9.2887	9.2927	9.2967	9.3006	9.3046	9.3085	9.3123	9.3162	9.3200	9.3237	9.3275	78	39
12	9.3275	9.3312	9.3349	9.3385	9.3422	9.3458	9.3493	9.3529	9.3564	9.3599	9.3634	77	35
13	9.3634	9.3668	9.3702	9.3736	9.3770	9.3804	9.3837	9.3870	9.3903	9.3935	9.3968	76	33
14	9.3968	9.4000	9.4032	9.4064	9.4095	9.4127	9.4158	9.4189	9.4220	9.4250	9.4281	75	31
15	9.4281	9.4311	9.4341	9.4371	9.4400	9.4430	9.4459	9.4488	9.4517	9.4546	9.4575	74	29
16	9.4575	9.4603	9.4632	9.4660	9.4688	9.4716	9.4744	9.4771	9.4799	9.4826	9.4853	73	28
17	9.4853	9.4880	9.4907	9.4934	9.4961	9.4987	9.5014	9.5040	9.5066	9.5092	9.5118	72	26
18	9.5118	9.5143	9.5169	9.5195	9.5220	9.5245	9.5270	9.5295	9.5320	9.5345	9.5370	71	25
19	9.5370	9.5394	9.5419	9.5443	9.5467	9.5491	9.5516	9.5539	9.5563	9.5587	9.5611	70	24
20	9.5611	9.5634	9.5658	9.5681	9.5704	9.5727	9.5750	9.5773	9.5796	9.5819	9.5842	69	23
21	9.5842	9.5864	9.5887	9.5909	9.5932	9.5954	9.5976	9.5998	9.6020	9.6042	9.6064	68	22
22	9.6064	9.6086	9.6108	9.6129	9.6151	9.6172	9.6194	9.6215	9.6236	9.6257	9.6279	67	22
23	9.6279	9.6300	9.6321	9.6341	9.6362	9.6383	9.6404	9.6424	9.6445	9.6465	9.6486	66	21
24	9.6486	9.6506	9.6527	9.6547	9.6567	9.6587	9.6607	9.6627	9.6647	9.6667	9.6687	65	20
25	9.6687	9.6706	9.6726	9.6746	9.6765	9.6785	9.6804	9.6824	9.6843	9.6863	9.6882	64	20
26	9.6882	9.6901	9.6920	9.6939	9.6958	9.6977	9.6996	9.7015	9.7034	9.7053	9.7072	63	19
27	9.7072	9.7090	9.7109	9.7128	9.7146	9.7165	9.7183	9.7202	9.7220	9.7238	9.7257	62	19
28	9.7257	9.7275	9.7293	9.7311	9.7330	9.7348	9.7366	9.7384	9.7402	9.7420	9.7438	61	18
29	9.7438	9.7455	9.7473	9.7491	9.7509	9.7526	9.7544	9.7562	9.7579	9.7597	9.7614	60	18
30	9.7614	9.7632	9.7649	9.7667	9.7684	9.7701	9.7719	9.7736	9.7753	9.7771	9.7788	59	17
31	9.7788	9.7805	9.7822	9.7839	9.7856	9.7873	9.7890	9.7907	9.7924	9.7941	9.7958	58	17
32	9.7958	9.7975	9.7992	9.8008	9.8025	9.8042	9.8059	9.8075	9.8092	9.8109	9.8125	57	17
33	9.8125	9.8142	9.8158	9.8175	9.8191	9.8208	9.8224	9.8241	9.8257	9.8274	9.8290	56	16
34	9.8290	9.8306	9.8323	9.8339	9.8355	9.8371	9.8388	9.8404	9.8420	9.8436	9.8452	55	16
35	9.8452	9.8468	9.8484	9.8501	9.8517	9.8533	9.8549	9.8565	9.8581	9.8597	9.8613	54	16
36	9.8613	9.8629	9.8644	9.8660	9.8676	9.8692	9.8708	9.8724	9.8740	9.8755	9.8771	53	16
37	9.8771	9.8787	9.8803	9.8818	9.8834	9.8850	9.8865	9.8881	9.8897	9.8912	9.8928	52	16
38	9.8928	9.8944	9.8959	9.8975	9.8990	9.9006	9.9022	9.9037	9.9053	9.9068	9.9084	51	16
39	9.9084	9.9099	9.9115	9.9130	9.9146	9.9161	9.9176	9.9192	9.9207	9.9223	9.9238	50	15
40	9.9238	9.9254	9.9269	9.9284	9.9300	9.9315	9.9330	9.9346	9.9361	9.9376	9.9392	49	15
41	9.9392	9.9407	9.9422	9.9438	9.9453	9.9468	9.9483	9.9499	9.9514	9.9529	9.9544	48	15
42	9.9544	9.9560	9.9575	9.9590	9.9605	9.9621	9.9636	9.9651	9.9666	9.9681	9.9697	47	15
43	9.9697	9.9712	9.9727	9.9742	9.9757	9.9772	9.9788	9.9803	9.9818	9.9833	9.9848	46	15
44	9.9848	9.9864	9.9879	9.9894	9.9909	9.9924	9.9939	9.9955	9.9970	9.9985	10.0000	45	15
45	10.0000												
		0.9°	0.8°	0.7°	0.6°	0.5°	0.4°	0.3°	0.2°	0.1°	0.0°	Deg	Avg. diff.

Log cotangents (read up)

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* From W. D. Cockrell, "Industrial Electronics Handbook," McGraw-Hill Book Company, New York, 1958.

Table 2-5. Logarithms of tangents and cotangents (continued)

Note: For cotangents use right-hand column of degrees and lower line of tenths.

Log tangents (read down)

Deg	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°			Avg. diff.
											10.0000	45	
45	10.0000	0.0015	0.0030	0.0045	0.0061	0.0076	0.0091	0.0106	0.0121	0.0136	0.0152	44	15
46	0.0152	0.0167	0.0182	0.0197	0.0212	0.0228	0.0243	0.0258	0.0273	0.0288	0.0303	43	15
47	0.0303	0.0319	0.0334	0.0349	0.0364	0.0379	0.0395	0.0410	0.0425	0.0440	0.0456	42	15
48	0.0456	0.0471	0.0486	0.0501	0.0517	0.0532	0.0547	0.0562	0.0578	0.0593	0.0608	41	15
49	0.0608	0.0624	0.0639	0.0654	0.0670	0.0685	0.0700	0.0716	0.0731	0.0746	0.0762	40	15
50	0.0762	0.0777	0.0793	0.0808	0.0824	0.0839	0.0854	0.0870	0.0885	0.0901	0.0916	39	15
51	0.0916	0.0932	0.0947	0.0963	0.0978	0.0994	0.1010	0.1025	0.1041	0.1056	0.1072	38	16
52	0.1072	0.1088	0.1103	0.1119	0.1135	0.1150	0.1166	0.1182	0.1197	0.1213	0.1229	37	16
53	0.1229	0.1245	0.1260	0.1276	0.1292	0.1308	0.1324	0.1340	0.1356	0.1371	0.1387	36	16
54	0.1387	0.1403	0.1419	0.1435	0.1451	0.1467	0.1483	0.1499	0.1516	0.1532	0.1548	35	16
55	0.1548	0.1564	0.1580	0.1596	0.1612	0.1629	0.1645	0.1661	0.1677	0.1694	0.1710	34	16
56	0.1710	0.1726	0.1743	0.1759	0.1776	0.1792	0.1809	0.1825	0.1842	0.1858	0.1875	33	16
57	0.1875	0.1891	0.1908	0.1925	0.1941	0.1958	0.1975	0.1992	0.2008	0.2025	0.2042	32	17
58	0.2042	0.2059	0.2076	0.2093	0.2110	0.2127	0.2144	0.2161	0.2178	0.2195	0.2212	31	17
59	0.2212	0.2229	0.2247	0.2264	0.2281	0.2299	0.2316	0.2333	0.2351	0.2368	0.2386	30	17
60	0.2386	0.2403	0.2421	0.2438	0.2456	0.2474	0.2491	0.2509	0.2527	0.2545	0.2562	29	18
61	0.2562	0.2580	0.2598	0.2616	0.2634	0.2652	0.2670	0.2689	0.2707	0.2725	0.2743	28	18
62	0.2743	0.2762	0.2780	0.2798	0.2817	0.2835	0.2854	0.2872	0.2891	0.2910	0.2928	27	18
63	0.2928	0.2947	0.2966	0.2985	0.3004	0.3023	0.3042	0.3061	0.3080	0.3099	0.3118	26	19
64	0.3118	0.3137	0.3157	0.3176	0.3196	0.3215	0.3235	0.3254	0.3274	0.3294	0.3313	25	20
65	0.3313	0.3333	0.3353	0.3373	0.3393	0.3413	0.3433	0.3453	0.3474	0.3494	0.3514	24	20
66	0.3514	0.3535	0.3555	0.3576	0.3596	0.3617	0.3638	0.3659	0.3679	0.3700	0.3721	23	21
67	0.3721	0.3743	0.3764	0.3785	0.3806	0.3828	0.3849	0.3871	0.3892	0.3914	0.3936	22	21
68	0.3936	0.3958	0.3980	0.4002	0.4024	0.4046	0.4068	0.4091	0.4113	0.4136	0.4158	21	22
69	0.4158	0.4181	0.4204	0.4227	0.4250	0.4273	0.4296	0.4319	0.4342	0.4366	0.4389	20	23
70	0.4389	0.4413	0.4437	0.4461	0.4484	0.4509	0.4533	0.4557	0.4581	0.4606	0.4630	19	24
71	0.4630	0.4655	0.4680	0.4705	0.4730	0.4755	0.4780	0.4805	0.4831	0.4857	0.4882	18	25
72	0.4882	0.4908	0.4934	0.4960	0.4986	0.5013	0.5039	0.5066	0.5092	0.5120	0.5147	17	26
73	0.5147	0.5174	0.5201	0.5229	0.5256	0.5284	0.5312	0.5340	0.5368	0.5397	0.5425	16	28
74	0.5425	0.5454	0.5483	0.5512	0.5541	0.5570	0.5600	0.5629	0.5659	0.5689	0.5719	15	30
75	0.5719	0.5750	0.5780	0.5811	0.5842	0.5873	0.5905	0.5936	0.5968	0.6000	0.6032	14	32
76	0.6032	0.6065	0.6097	0.6130	0.6163	0.6196	0.6230	0.6264	0.6298	0.6332	0.6366	13	34
77	0.6366	0.6401	0.6436	0.6471	0.6507	0.6542	0.6578	0.6615	0.6651	0.6688	0.6725	12	36
78	0.6725	0.6763	0.6800	0.6838	0.6877	0.6915	0.6954	0.6994	0.7033	0.7073	0.7113	11	39
79	0.7113	0.7154	0.7195	0.7236	0.7278	0.7320	0.7363	0.7406	0.7449	0.7493	0.7537	10	43
80	0.7537	0.7581	0.7626	0.7672	0.7718	0.7764	0.7811	0.7858	0.7906	0.7954	0.8003	9	
81	0.8003	0.8052	0.8102	0.8152	0.8203	0.8255	0.8307	0.8360	0.8413	0.8467	0.8522	8	
82	0.8522	0.8577	0.8633	0.8690	0.8748	0.8806	0.8865	0.8924	0.8985	0.9046	0.9109	7	
83	0.9109	0.9172	0.9236	0.9301	0.9366	0.9433	0.9501	0.9570	0.9640	0.9711	0.9784	6	
84	0.9784	0.9857	0.9932	1.0008	1.0085	1.0164	1.0244	1.0326	1.0409	1.0494	1.0580	5	
85	1.0580	1.0669	1.0759	1.0850	1.0944	1.1040	1.1138	1.1238	1.1341	1.1446	1.1554	4	
86	1.1554	1.1664	1.1777	1.1893	1.2012	1.2135	1.2261	1.2391	1.2525	1.2663	1.2806	3	
87	1.2806	1.2953	1.3106	1.3264	1.3428	1.3599	1.3777	1.3961	1.4155	1.4357	1.4569	2	
88	1.4569	1.4792	1.5027	1.5275	1.5539	1.5819	1.6119	1.6441	1.6788	1.7166	1.7581	1	
89	1.7581	1.8038	1.8550	1.9130	1.9800	2.0591	2.1561	2.2810	2.4571	2.7581	∞	0	
90	∞												
		0.9°	0.8°	0.7°	0.6°	0.5°	0.4°	0.3°	0.2°	0.1°	0.0°	Deg	Avg. diff.

Log cotangents (read up)

section

3

BASIC LOGARITHM TABLES

Table 3-1. Five-place common logarithms of numbers†

100–155

No.	L	0	1	2	3	4	5	6	7	8	9
100	00	000	043	087	130	173	217	260	303	346	389
101		432	475	518	561	604	647	689	732	775	817
102		860	903	945	988	*030	*072	*115	*157	*199	*242
103	01	284	326	368	410	452	494	536	578	620	662
104		703	745	787	828	870	912	953	995	*036	*078
105	02	119	160	202	243	284	325	366	408	449	490
106		531	572	612	653	694	735	776	816	857	898
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03	342	383	423	463	503	543	583	623	663	703
109		743	782	822	862	902	941	981	*021	*060	*100
110	04	139	179	218	258	297	336	376	415	454	493
111		532	571	610	650	689	727	766	805	844	883
112		922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05	308	346	385	423	461	500	538	576	614	652
114		690	729	767	805	843	881	918	956	994	*032
115	06	070	108	145	183	221	258	296	333	371	408
116		446	483	521	558	595	633	670	707	744	781
117		819	856	893	930	967	*004	*041	*078	*115	*151
118	07	188	225	262	298	335	372	408	445	482	518
119		555	591	628	664	700	737	773	809	846	882
120		918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08	279	314	350	386	422	458	493	529	565	600
122		636	672	707	743	778	814	849	884	920	955
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09	342	377	412	447	482	517	552	587	621	656
125		691	726	760	795	830	864	899	934	968	*003
126	10	037	072	106	140	175	209	243	278	312	346
127		380	415	449	483	517	551	585	619	653	687
128		721	755	789	823	857	890	924	958	992	*025
129	11	059	093	126	160	193	227	261	294	327	361
130		394	428	461	494	528	561	594	628	661	694
131		727	760	793	826	860	893	926	959	992	*024
132	12	057	090	123	156	189	222	254	287	320	353
133		385	418	450	483	516	548	581	613	646	678
134		710	743	775	808	840	872	905	937	969	*001
135	13	033	066	098	130	162	194	226	258	290	322
136		354	386	418	450	481	513	545	577	609	640
137		672	704	735	767	799	830	862	893	925	956
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14	301	333	364	395	426	457	489	520	551	582
140		613	644	675	706	737	768	799	829	860	891
141		922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15	229	259	290	320	351	381	412	442	473	503
143		534	564	594	625	655	685	715	746	776	806
144		836	866	897	927	957	987	*017	*047	*077	*107
145	16	137	167	197	227	256	286	316	346	376	406
146		435	465	495	524	554	584	613	643	673	702
147		732	761	791	820	850	879	909	938	967	997
148	17	026	056	085	114	143	173	202	231	260	289
149		319	348	377	406	435	464	493	522	551	580
150		609	638	667	696	725	754	782	811	840	869
151		898	926	955	984	*013	*041	*070	*099	*127	*156
152	18	184	213	241	270	299	327	355	384	412	441
153		469	498	526	554	583	611	639	667	696	724
154		752	780	808	837	865	893	921	949	977	*005
155	19	033	061	089	117	145	173	201	229	257	285
No.	L	0	1	2	3	4	5	6	7	8	9

Proportional parts

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Proportional parts

$\log_{10} \pi = 0.49715$
$\log_{10} 2\pi = 0.79818$

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* Indicates change in the first two decimal places.

† From R. H. Perry, "Engineering Manual," McGraw-Hill Book Company, New York, 1959.

Table 3-1. Five-place common logarithms of numbers (continued)

155–210

No.	L	0	1	2	3	4	5	6	7	8	9
155	19	033	061	089	117	145	173	201	229	257	285
156		312	340	368	396	424	451	479	507	535	562
157		590	618	645	673	700	728	756	783	811	838
158		866	893	921	948	976	*003	*030	*058	*085	*112
159	20	140	167	194	222	249	276	303	330	358	385
160		412	439	466	493	520	548	575	602	629	656
161		683	710	737	763	790	817	844	871	898	925
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21	219	245	272	299	325	352	378	405	431	458
164		484	511	537	564	590	617	643	669	696	722
165		748	775	801	827	854	880	906	932	958	985
166	22	011	037	063	089	115	141	168	194	220	246
167		272	298	324	350	376	401	427	453	479	505
168		531	557	583	608	634	660	686	712	737	763
169		789	814	840	866	891	917	943	968	994	*019
170	23	045	070	096	121	147	172	198	223	249	274
171		300	325	350	376	401	426	452	477	502	528
172		553	578	603	629	654	679	704	729	754	776
173		805	830	855	880	905	930	955	980	*005	*030
174	24	055	080	105	130	155	180	204	229	254	279
175		304	329	353	378	403	428	452	477	502	527
176		551	576	601	625	650	674	699	724	748	773
177		797	822	846	871	895	920	944	969	993	*018
178	25	042	066	091	115	139	164	188	212	237	261
179		285	310	334	358	382	406	431	455	479	503
180		527	551	575	600	624	648	672	696	720	744
181		768	792	816	840	864	888	912	935	959	983
182	26	007	031	055	079	102	126	150	174	198	221
183		245	269	293	316	340	364	387	411	435	458
184		482	505	529	553	576	600	623	647	670	694
185		717	741	764	788	811	834	858	881	905	928
186		951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27	184	207	231	254	277	300	323	346	370	393
188		416	439	462	485	508	531	554	577	600	623
189		646	669	692	715	738	761	784	807	830	853
190		875	898	921	944	967	990	*012	*035	*058	*081
191	28	103	126	149	172	194	217	240	262	285	308
192		330	353	375	398	421	443	466	488	511	533
193		556	578	601	623	646	668	691	713	735	758
194		780	803	825	847	870	892	914	937	959	981
195	29	003	026	048	070	092	115	137	159	181	203
196		226	248	270	292	314	336	358	380	403	425
197		447	469	491	513	535	557	579	601	623	645
198		667	688	710	732	754	776	798	820	842	863
199		885	907	929	951	973	994	*016	*038	*060	*081
200	30	103	125	146	168	190	211	233	255	276	298
201		320	341	363	384	406	428	449	471	492	514
202		535	557	578	600	621	643	664	685	707	728
203		750	771	792	814	835	856	878	899	920	942
204		963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31	175	197	218	239	260	281	302	323	345	366
206		387	408	429	450	471	492	513	534	555	576
207		597	618	639	660	681	702	723	744	765	785
208		806	827	848	869	890	911	931	952	973	994
209	32	015	035	056	077	098	118	139	160	181	201
210		222	243	263	284	305	325	346	366	387	408
No.	L	0	1	2	3	4	5	6	7	8	9

Proportional parts

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
1	2.5
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24
1	2.4
2	4.8
3	7.2
4	9.6
5	12.0
6	14.4
7	16.8
8	19.2
9	21.6

	23
1	2.3
2	4.6
3	6.9
4	9.2
5	11.5
6	13.8
7	16.1
8	18.4
9	20.7

	22
1	2.2
2	4.4
3	6.6
4	8.8
5	11.0
6	13.2
7	15.4
8	17.6
9	19.8

Proportional parts

$$\log_{10} \frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10} \frac{1}{2\pi} = 9.20182 - 1$$

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* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

210–265

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
210	32	222	243	263	284	305	325	346	366	387	408	
211		428	449	469	490	511	531	552	572	593	613	
212		634	654	675	695	715	736	756	777	797	818	
213		838	858	879	899	919	940	960	980	*001	*021	
214	33	041	062	082	102	122	143	163	183	203	224	
215		244	264	284	304	325	345	365	385	405	425	
216		445	465	486	506	526	546	566	586	606	626	
217		646	666	686	706	726	746	766	786	806	826	
218		846	866	885	905	925	945	965	985	*005	*025	
219	34	044	064	084	104	124	143	163	183	203	223	
220		242	262	282	301	321	341	361	380	400	420	
221		439	459	479	498	518	537	557	577	596	616	
222		635	655	674	694	713	733	753	772	792	811	
223		830	850	869	889	908	928	947	967	986	*005	
224	35	025	044	064	083	102	122	141	160	180	199	
225		218	238	257	276	295	315	334	353	372	392	
226		411	430	449	468	488	507	526	545	564	583	
227		603	622	641	660	679	698	717	736	755	774	
228		793	813	832	851	870	889	908	927	946	965	
229		984	*003	*021	*040	*059	*078	*097	*116	*135	*154	
230	36	173	192	211	229	248	267	286	305	324	342	
231		361	380	399	418	436	455	474	493	511	530	
232		549	568	586	605	624	642	661	680	698	717	
233		736	754	773	791	810	829	847	866	884	903	
234		922	940	959	977	996	*014	*033	*051	*070	*088	
235	37	107	125	144	162	181	199	218	236	254	273	
236		291	310	328	346	365	383	401	420	438	457	
237		475	493	511	530	548	566	585	603	621	639	
238		658	676	694	712	731	749	767	785	803	822	
239		840	858	876	894	912	931	949	967	985	*003	
240	38	021	039	057	075	093	112	130	148	166	184	
241		202	220	238	256	274	292	310	328	346	364	
242		382	399	417	435	453	471	489	507	525	543	
243		561	579	596	614	632	650	668	686	703	721	
244		739	757	775	792	810	828	846	863	881	899	
245		917	934	952	970	987	*005	*023	*041	*058	*076	
246	39	094	111	129	146	164	182	199	217	235	252	
247		270	287	305	322	340	358	375	393	410	428	
248		445	463	480	498	515	533	550	568	585	602	
249		620	637	655	672	690	707	724	742	759	777	
250		794	811	829	846	863	881	898	915	933	950	
251		967	985	*002	*019	*037	*054	*071	*088	*106	*123	
252	40	140	157	175	192	209	226	243	261	278	295	
253		312	329	346	364	381	398	415	432	449	466	
254		483	500	518	535	552	569	586	603	620	637	
255		654	671	688	705	722	739	756	773	790	807	
256		824	841	858	875	892	909	926	943	960	976	
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145	
258	41	162	179	196	212	229	246	263	280	296	313	
259		330	347	364	380	397	414	430	447	464	481	
260		497	514	531	547	564	581	597	614	631	647	
261		664	681	697	714	731	747	764	780	797	814	
262		830	847	863	880	896	913	929	946	963	979	
263		996	*012	*029	*045	*062	*078	*095	*111	*127	*144	
264	42	160	177	193	210	226	243	259	275	292	308	
265		325	341	357	374	390	406	423	439	456	472	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

Proportional parts

	21
1	2.1
2	4.2
3	6.3
4	8.4
5	10.5
6	12.6
7	14.7
8	16.8
9	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

$\log_{10} \pi = 0.49715$

$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

265–320

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
265	42	325	341	357	374	390	406	423	439	456	472	
266		488	504	521	537	553	570	586	602	619	635	
267		651	667	684	700	716	732	749	765	781	797	
268		813	830	846	862	878	894	911	927	943	959	
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120	
270	43	136	152	169	185	201	217	233	249	265	281	
271		297	313	329	345	361	377	393	409	425	441	
272		457	473	489	505	521	537	553	569	584	600	
273		616	632	648	664	680	696	712	727	743	759	
274		775	791	807	823	838	854	870	886	902	917	
275		933	949	965	981	996	*012	*028	*044	*059	*075	
276	44	091	107	122	138	154	170	185	201	217	232	
277		248	264	279	295	311	326	342	358	373	389	
278		404	420	436	451	467	483	498	514	529	545	
279		560	576	592	607	623	638	654	669	685	700	
280		716	731	747	762	778	793	809	824	840	855	
281		871	886	902	917	932	948	963	979	994	*010	
282	45	025	040	056	071	086	102	117	133	148	163	
283		179	194	209	225	240	255	271	286	301	317	
284		332	347	362	378	393	408	423	439	454	469	
285		484	500	515	530	545	561	576	591	606	621	
286		637	652	667	682	697	712	728	743	758	773	
287		788	803	818	834	849	864	879	894	909	924	
288		939	954	969	984	*000	*015	*030	*045	*060	*075	
289	46	090	105	120	135	150	165	180	195	210	225	
290		240	255	270	285	300	315	330	345	359	374	
291		389	404	419	434	449	464	479	494	509	523	
292		538	553	568	583	598	613	627	642	657	672	
293		687	702	716	731	746	761	776	790	805	820	
294		835	850	864	879	894	909	923	938	953	967	
295		982	997	*012	*026	*041	*056	*070	*085	*100	*115	
296	47	129	144	159	173	188	202	217	232	246	261	
297		276	290	305	319	334	349	363	378	392	407	
298		422	436	451	465	480	494	509	524	538	553	
299		567	582	596	611	625	640	654	669	683	698	
300		712	727	741	756	770	784	799	813	828	842	
301		857	871	886	900	914	929	943	958	972	986	
302	48	001	015	029	044	058	073	087	101	116	130	
303		144	159	173	187	202	216	230	245	259	273	
304		287	302	316	330	344	359	373	387	402	416	
305		430	444	458	473	487	501	515	530	544	558	
306		572	586	601	615	629	643	657	671	686	700	
307		714	728	742	756	770	785	799	813	827	841	
308		855	869	883	897	911	926	940	954	968	982	
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49	136	150	164	178	192	206	220	234	248	262	
311		276	290	304	318	332	346	360	374	388	402	
312		415	429	443	457	471	485	499	513	527	541	
313		554	568	582	596	610	624	638	651	665	679	
314		693	707	721	734	748	762	776	790	803	817	
315		831	845	859	872	886	900	914	927	941	955	
316		969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50	106	120	133	147	161	174	188	202	215	229	
318		243	256	270	284	297	311	325	338	352	365	
319		379	393	406	420	433	447	461	474	488	501	
320		515	529	542	556	569	583	596	610	623	637	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

	16
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6
7	11.2
8	12.8
9	14.4

	15
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5
6	9.0
7	10.5
8	12.0
9	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

$$\log_{10} \frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10} \frac{1}{2\pi} = 9.20182 - 1$$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

320–375

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
320	50	515	529	542	556	569	583	596	610	623	637	
321		651	664	678	691	705	718	732	745	759	772	
322		786	799	813	826	840	853	866	880	893	907	
323		920	934	947	961	974	987	*001	*014	*028	*041	
324	51	055	068	081	095	108	121	135	148	162	175	
325		188	202	215	228	242	255	268	282	295	308	
326		322	335	348	362	375	388	402	415	428	441	
327		455	468	481	495	508	521	534	548	561	574	
328		587	601	614	627	640	654	667	680	693	706	
329		720	733	746	759	772	786	799	812	825	838	
330		851	865	878	891	904	917	930	943	957	970	
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52	114	127	140	153	166	179	192	205	218	231	
333		244	257	271	284	297	310	323	336	349	362	
334		375	388	401	414	427	440	453	466	479	492	
335		504	517	530	543	556	569	582	595	608	621	
336		634	647	660	673	686	699	711	724	737	750	
337		763	776	789	802	815	827	840	853	866	879	
338		892	905	917	930	943	956	969	982	994	*007	
339	53	020	033	046	058	071	084	097	110	122	135	
340		148	161	173	186	199	212	224	237	250	263	
341		275	288	301	314	326	339	352	365	377	390	
342		403	415	428	441	453	466	479	491	504	517	
343		529	542	555	567	580	593	605	618	631	643	
344		656	668	681	694	706	719	732	744	757	769	
345		782	795	807	820	832	845	857	870	883	895	
346		908	920	933	945	958	970	983	995	*008	*020	
347	54	033	045	058	070	083	095	108	120	133	145	
348		158	170	183	195	208	220	233	245	258	270	
349		283	295	307	320	332	345	357	370	382	394	
350		407	419	432	444	456	469	481	494	506	518	
351		531	543	555	568	580	593	605	617	630	642	
352		654	667	679	691	704	716	728	741	753	765	
353		777	790	802	814	827	839	851	864	876	888	
354		900	913	925	937	949	962	974	986	998	*011	
355	55	023	035	047	060	072	084	096	108	121	133	
356		145	157	169	182	194	206	218	230	242	255	
357		267	279	291	303	315	328	340	352	364	376	
358		388	400	413	425	437	449	461	473	485	497	
359		509	522	534	546	558	570	582	594	606	618	
360		630	642	654	666	678	691	703	715	727	739	
361		751	763	775	787	799	811	823	835	847	859	
362		871	883	895	907	919	931	943	955	967	979	
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56	110	122	134	146	158	170	182	194	205	217	
365		229	241	253	265	277	289	301	313	324	336	
366		348	360	372	384	396	407	419	431	443	455	
367		467	478	490	502	514	526	538	549	561	573	
368		585	597	608	620	632	644	656	667	679	691	
369		703	714	726	738	750	761	773	785	797	808	
370		820	832	844	855	867	879	891	902	914	926	
371		937	949	961	972	984	996	*008	*019	*031	*043	
372	57	054	066	078	089	101	113	124	136	148	159	
373		171	183	194	206	217	229	241	252	264	276	
374		287	299	310	322	334	345	357	368	380	392	
375		403	415	426	438	449	461	473	484	496	507	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

	13
1	1.3
2	2.6
3	3.9
4	5.2
5	6.5
6	7.8
7	9.1
8	10.4
9	11.7

	12
1	1.2
2	2.4
3	3.6
4	4.8
5	6.0
6	7.2
7	8.4
8	9.6
9	10.8

$\log_{10} \pi = 0.49715$
$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

375–430

No.	L	0	1	2	3	4	5	6	7	8	9
375	57	403	415	426	438	449	461	473	484	496	507
376		519	530	542	553	565	577	588	600	611	623
377		634	646	657	669	680	692	703	715	726	738
378		749	761	772	784	795	807	818	830	841	852
379		864	875	887	898	910	921	933	944	956	967
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58	093	104	115	127	138	149	161	172	184	195
382		206	218	229	240	252	263	275	286	297	309
383		320	331	343	354	365	377	388	399	411	422
384		433	444	456	467	478	490	501	512	524	535
385		546	557	569	580	591	602	614	625	636	647
386		659	670	681	692	704	715	726	737	749	760
387		771	782	794	805	816	827	838	850	861	872
388		883	894	906	917	928	939	950	961	973	984
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59	106	118	129	140	151	162	173	184	195	207
391		218	229	240	251	262	273	284	295	306	318
392		329	340	351	362	373	384	395	406	417	428
393		439	450	461	472	483	494	506	517	528	539
394		550	561	572	583	594	605	616	627	638	649
395		660	671	682	693	704	715	726	737	748	759
396		770	780	791	802	813	824	835	846	857	868
397		879	890	901	912	923	934	945	956	966	977
398		988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60	097	108	119	130	141	152	163	173	184	195
400		206	217	228	239	249	260	271	282	293	304
401		314	325	336	347	358	369	379	390	401	412
402		423	433	444	455	466	477	487	498	509	520
403		531	541	552	563	574	584	595	606	617	627
404		638	649	660	670	681	692	703	713	724	735
405		746	756	767	778	788	799	810	821	831	842
406		853	863	874	885	895	906	917	927	938	949
407		959	970	981	991	*002	*013	*023	*034	*045	*055
408	61	066	077	087	098	109	119	130	140	151	162
409		172	183	194	204	215	225	236	247	257	268
410		278	289	300	310	321	331	342	352	363	374
411		384	395	405	416	426	437	448	458	469	479
412		490	500	511	521	532	542	553	563	574	584
413		595	606	616	627	637	648	658	669	679	690
414		700	711	721	731	742	752	763	773	784	794
415		805	815	826	836	847	857	868	878	888	899
416		909	920	930	941	951	962	972	982	993	*003
417	62	014	024	034	045	055	066	076	086	097	107
418		118	128	138	149	159	170	180	190	201	211
419		221	232	242	252	263	273	284	294	304	315
420		325	335	346	356	366	377	387	397	408	418
421		428	439	449	459	469	480	490	500	511	521
422		531	542	552	562	572	583	593	603	614	624
423		634	644	655	665	675	685	696	706	716	726
424		737	747	757	767	778	788	798	808	818	829
425		839	849	859	870	880	890	900	910	921	931
426		941	951	961	972	982	992	*002	*012	*022	*033
427	63	043	053	063	073	083	094	104	114	124	134
428		144	155	165	175	185	195	205	215	225	236
429		246	256	266	276	286	296	306	317	327	337
430		347	357	367	377	387	397	407	417	428	438
No.	L	0	1	2	3	4	5	6	7	8	9

Proportional parts

	11
1	1.1
2	2.2
3	3.3
4	4.4
5	5.5
6	6.6
7	7.7
8	8.8
9	9.9

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

$$\log_{10} \frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10} \frac{1}{2\pi} = 9.20182 - 1$$

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* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

430–485

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
430	63	347	357	367	377	387	397	407	417	428	438	
431		448	458	468	478	488	498	508	518	528	538	
432		548	558	568	579	589	599	609	619	629	639	
433		649	659	669	679	689	699	709	719	729	739	
434		749	759	769	779	789	799	809	819	829	839	
435		849	859	869	879	889	899	909	919	929	939	
436		949	959	969	979	988	998	*008	*018	*028	*038	
437	64	048	058	068	078	088	098	108	118	128	137	
438		147	157	167	177	187	197	207	217	227	237	
439		246	256	266	276	286	296	306	316	326	335	
440		345	355	365	375	385	395	404	414	424	434	
441		444	454	464	473	483	493	503	513	523	532	
442		542	552	562	572	582	591	601	611	621	631	
443		640	650	660	670	680	689	699	709	719	729	
444		738	748	758	768	777	787	797	807	816	826	
445		836	846	856	865	875	885	895	904	914	924	
446		933	943	953	963	972	982	992	*002	*011	*021	
447	65	031	040	050	060	070	079	089	099	108	118	
448		128	137	147	157	167	176	186	196	205	215	
449		225	234	244	254	263	273	283	292	302	312	
450		321	331	341	350	360	369	379	389	398	408	
451		418	427	437	447	456	466	475	485	495	504	
452		514	523	533	543	552	562	571	581	591	600	
453		610	619	629	639	648	658	667	677	686	696	
454		706	715	725	734	744	753	763	773	782	792	
455		801	811	820	830	839	849	858	868	877	887	
456		896	906	916	925	935	944	954	963	973	982	
457		992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66	087	096	106	115	124	134	143	153	162	172	
459		181	191	200	210	219	229	238	247	257	266	
460		276	285	295	304	314	323	332	342	351	361	
461		370	380	389	398	408	417	427	436	445	455	
462		464	474	483	492	502	511	521	530	539	549	
463		558	567	577	586	596	605	614	624	633	642	
464		652	661	671	680	689	699	708	717	727	736	
465		745	755	764	773	783	792	801	811	820	829	
466		839	848	857	867	876	885	894	904	913	922	
467		932	941	950	960	969	978	987	997	*006	*015	
468	67	025	034	043	052	062	071	080	090	099	108	
469		117	127	136	145	154	164	173	182	191	201	
470		210	219	228	238	247	256	265	274	284	293	
471		302	311	321	330	339	348	357	367	376	385	
472		394	403	413	422	431	440	449	459	468	477	
473		486	495	504	514	523	532	541	550	560	569	
474		578	587	596	605	614	624	633	642	651	660	
475		669	679	688	697	706	715	724	733	742	752	
476		761	770	779	788	797	806	815	825	834	843	
477		852	861	870	879	888	897	906	916	925	934	
478		943	952	961	970	979	988	997	*006	*015	*024	
479	68	034	043	052	061	070	079	088	097	106	115	
480		124	133	142	151	160	169	178	187	196	205	
481		215	224	233	242	251	260	269	278	287	296	
482		305	314	323	332	341	350	359	368	377	386	
483		395	404	413	422	431	440	449	458	467	476	
484		485	494	502	511	520	529	538	547	556	565	
485		574	583	592	601	610	619	628	637	646	655	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

$\log_{10} \pi = 0.49715$

$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

485–540

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
485	68	574	583	592	601	610	619	628	637	646	655	
486		664	673	682	690	699	708	717	726	735	744	
487		753	762	771	780	789	797	806	815	824	833	
488		842	851	860	869	878	886	895	904	913	922	
489		931	940	949	958	966	975	984	993	*002	*011	
490	69	020	028	037	046	055	064	073	082	090	099	
491		108	117	126	135	144	152	161	170	179	188	
492		197	205	214	223	232	241	249	258	267	276	
493		285	294	302	311	320	329	338	346	355	364	
494		373	381	390	399	408	417	425	434	443	452	
495		461	469	478	487	496	504	513	522	531	539	
496		548	557	566	574	583	592	601	609	618	627	
497		636	644	653	662	671	679	688	697	705	714	
498		723	732	740	749	758	767	775	784	793	801	
499		810	819	827	836	845	854	862	871	880	888	
500		897	906	914	923	932	940	949	958	966	975	
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70	070	079	088	096	105	114	122	131	140	148	
503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	
505		329	338	346	355	364	372	381	389	398	406	
506		415	424	432	441	449	458	467	475	484	492	
507		501	509	518	526	535	544	552	561	569	578	
508		586	595	603	612	621	629	638	646	655	663	
509		672	680	689	697	706	714	723	731	740	749	
510		757	766	774	783	791	800	808	817	825	834	
511		842	851	859	868	876	885	893	902	910	919	
512		927	935	944	952	961	969	978	986	995	*003	
513	71	012	020	029	037	046	054	063	071	079	088	
514		096	105	113	122	130	139	147	155	164	172	
515		181	189	198	206	214	223	231	240	248	257	
516		265	273	282	290	299	307	315	324	332	341	
517		349	357	366	374	383	391	399	408	416	425	
518		433	441	450	458	467	475	483	492	500	508	
519		517	525	533	542	550	559	567	575	584	592	
520		600	609	617	625	634	642	650	659	667	675	
521		684	692	700	709	717	725	734	742	750	759	
522		767	775	784	792	800	809	817	825	834	842	
523		850	858	867	875	883	892	900	908	917	925	
524		933	941	950	958	966	975	983	991	999	*008	
525	72	016	024	032	041	049	057	066	074	082	090	
526		099	107	115	123	132	140	148	156	165	173	
527		181	189	198	206	214	222	230	239	247	255	
528		263	272	280	288	296	305	313	321	329	337	
529		346	354	362	370	378	387	395	403	411	419	
530		428	436	444	452	460	469	477	485	493	501	
531		509	518	526	534	542	550	559	567	575	583	
532		591	599	607	616	624	632	640	648	656	665	
533		673	681	689	697	705	713	722	730	738	746	
534		754	762	770	779	787	795	803	811	819	827	
535		835	844	852	860	868	876	884	892	900	908	
536		916	925	933	941	949	957	965	973	981	989	
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73	078	086	094	102	111	119	127	135	143	151	
539		159	167	175	183	191	199	207	215	223	231	
540		239	247	255	264	272	280	288	296	304	312	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

$$\log_{10} \frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10} \frac{1}{2\pi} = 9.20182 - 1$$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

540–595

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
540	73	239	247	255	264	272	280	288	296	304	312	
541		320	328	336	344	352	360	368	376	384	392	
542		400	408	416	424	432	440	448	456	464	472	
543		480	488	496	504	512	520	528	536	544	552	
544		560	568	576	584	592	600	608	616	624	632	
545		640	648	656	664	672	679	687	695	703	711	
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
548		878	886	894	902	910	918	926	934	941	949	
549		957	965	973	981	989	997	*005	*013	*020	*028	
550	74	036	044	052	060	068	076	084	092	099	107	
551		115	123	131	139	147	155	162	170	178	186	
552		194	202	210	218	225	233	241	249	257	265	
553		273	280	288	296	304	312	320	327	335	343	
554		351	359	367	374	382	390	398	406	414	421	
555		429	437	445	453	461	468	476	484	492	500	
556		507	515	523	531	539	547	554	562	570	578	
557		586	593	601	609	617	624	632	640	648	656	
558		663	671	679	687	695	702	710	718	726	733	
559		741	749	757	764	772	780	788	796	803	811	
560		819	827	834	842	850	858	865	873	881	889	
561		896	904	912	920	927	935	943	950	958	966	
562		974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75	051	059	066	074	082	089	097	105	113	120	
564		128	136	143	151	159	166	174	182	189	197	
565		205	213	220	228	236	243	251	259	266	274	
566		282	289	297	305	312	320	328	335	343	351	
567		358	366	374	381	389	397	404	412	420	427	
568		435	442	450	458	465	473	481	488	496	504	
569		511	519	526	534	542	549	557	565	572	580	
570		587	595	603	610	618	626	633	641	648	656	
571		664	671	679	686	694	702	709	717	724	732	
572		740	747	755	762	770	778	785	793	800	808	
573		815	823	831	838	846	853	861	868	876	884	
574		891	899	906	914	921	929	937	944	952	959	
575		967	974	982	989	997	*005	*012	*020	*027	*035	
576	76	042	050	057	065	072	080	087	095	103	110	
577		118	125	133	140	148	155	163	170	178	185	
578		193	200	208	215	223	230	238	245	253	260	
579		268	275	283	290	298	305	313	320	328	335	
580		343	350	358	365	373	380	388	395	403	410	
581		418	425	433	440	448	455	462	470	477	485	
582		492	500	507	515	522	530	537	545	552	559	
583		567	574	582	589	597	604	612	619	626	634	
584		641	649	656	664	671	678	686	693	701	708	
585		716	723	730	738	745	753	760	768	775	782	
586		790	797	805	812	819	827	834	842	849	856	
587		864	871	879	886	893	901	908	916	923	930	
588		938	945	953	960	967	975	982	989	997	*004	
589	77	012	019	026	034	041	048	056	063	070	078	
590		085	093	100	107	115	122	129	137	144	151	
591		159	166	173	181	188	195	203	210	218	225	
592		232	240	247	254	262	269	276	283	291	298	
593		305	313	320	327	335	342	349	357	364	371	
594		379	386	393	401	408	415	422	430	437	444	
595		452	459	466	474	481	488	495	503	510	517	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

$\log_{10} \pi = 0.49715$
$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

595–650

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
595	77	452	459	466	474	481	488	495	503	510	517	
596		525	532	539	546	554	561	568	576	583	590	
597		597	605	612	619	627	634	641	648	656	663	
598		670	677	685	692	699	706	714	721	728	735	
599		743	750	757	764	772	779	786	793	801	808	
600		815	822	830	837	844	851	859	866	873	880	
601		887	895	902	909	916	924	931	938	945	952	
602		960	967	974	981	989	996	*003	*010	*017	*025	
603	78	032	039	046	053	061	068	075	082	089	097	
604		104	111	118	125	132	140	147	154	161	168	
605		176	183	190	197	204	211	219	226	233	240	
606		247	254	262	269	276	283	290	297	305	312	
607		319	326	333	340	347	355	362	369	376	383	
608		390	398	405	412	419	426	433	440	447	455	
609		462	469	476	483	490	497	505	512	519	526	
610		533	540	547	554	561	569	576	583	590	597	
611		604	611	618	625	633	640	647	654	661	668	
612		675	682	689	696	704	711	718	725	732	739	
613		746	753	760	767	774	781	789	796	803	810	
614		817	824	831	838	845	852	859	866	873	880	
615		888	895	902	909	916	923	930	937	944	951	
616		958	965	972	979	986	993	*000	*007	*014	*021	
617	79	029	036	043	050	057	064	071	078	085	092	
618		099	106	113	120	127	134	141	148	155	162	
619		169	176	183	190	197	204	211	218	225	232	
620		239	246	253	260	267	274	281	288	295	302	
621		309	316	323	330	337	344	351	358	365	372	
622		379	386	393	400	407	414	421	428	435	442	
623		449	456	463	470	477	484	491	498	505	512	
624		518	525	532	539	546	553	560	567	574	581	
625		588	595	602	609	616	623	630	637	644	651	
626		657	664	671	678	685	692	699	706	713	720	
627		727	734	741	748	754	761	768	775	782	789	
628		796	803	810	817	824	831	837	844	851	858	
629		865	872	879	886	893	900	906	913	920	927	
630		934	941	948	955	962	969	975	982	989	996	
631	80	003	010	017	024	030	037	044	051	058	065	
632		072	079	085	092	099	106	113	120	127	134	
633		140	147	154	161	168	175	182	188	195	202	
634		209	216	223	229	236	243	250	257	264	271	
635		277	284	291	298	305	312	318	325	332	339	
636		346	353	359	366	373	380	387	393	400	407	
637		414	421	428	434	441	448	455	462	468	475	
638		482	489	496	502	509	516	523	530	536	543	
639		550	557	564	570	577	584	591	598	604	611	
640		618	625	632	638	645	652	659	665	672	679	
641		686	693	699	706	713	720	726	733	740	747	
642		754	760	767	774	781	787	794	801	808	814	
643		821	828	835	841	848	855	862	868	875	882	
644		889	895	902	909	916	922	929	936	943	949	
645		956	963	969	976	983	990	996	*003	*010	*017	
646	81	023	030	037	043	050	057	064	070	077	084	
647		090	097	104	111	117	124	131	137	144	151	
648		158	164	171	178	184	191	198	204	211	218	
649		224	231	238	245	251	258	265	271	278	285	
650		291	298	305	311	318	325	331	338	345	351	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

$$\log_{10} \frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10} \frac{1}{2\pi} = 9.20182 - 1$$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

650–705

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
650	81	291	298	305	311	318	325	331	338	345	351	
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	618	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	
661	82	020	027	033	040	046	053	060	066	073	079	
662		086	092	099	105	112	119	125	132	138	145	
663		151	158	164	171	178	184	191	197	204	210	
664		217	223	230	236	243	250	256	263	269	276	
665		282	289	295	302	308	315	321	328	334	341	
666		347	354	360	367	374	380	387	393	400	406	
667		413	419	426	432	439	445	452	458	465	471	
668		478	484	491	497	504	510	517	523	530	536	
669		543	549	556	562	569	575	582	588	595	601	
670		607	614	620	627	633	640	646	653	659	666	
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
679		187	193	200	206	213	219	225	232	238	245	
680		251	257	264	270	276	283	289	296	302	308	
681		315	321	327	334	340	347	353	359	366	372	
682		378	385	391	398	404	410	417	423	429	436	
683		442	448	455	461	468	474	480	487	493	499	
684		506	512	518	525	531	537	544	550	556	563	
685		569	575	582	588	594	601	607	613	620	626	
686		632	639	645	651	658	664	670	677	683	689	
687		696	702	708	715	721	727	734	740	746	753	
688		759	765	771	778	784	790	797	803	809	816	
689		822	828	835	841	847	853	860	866	872	879	
690		885	891	898	904	910	916	923	929	935	942	
691		948	954	960	967	973	979	986	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

$\log_{10} \pi = 0.49715$
$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

705–760

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
705	84	819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	059	
709		065	071	077	083	089	095	101	107	114	120	
710		126	132	138	144	150	156	163	169	175	181	
711		187	193	199	205	211	217	224	230	236	242	
712		248	254	260	266	272	278	285	291	297	303	
713		309	315	321	327	333	339	345	352	358	364	
714		370	376	382	388	394	400	406	412	418	425	
715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	510	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	
724		974	980	986	992	998	*004	*010	*016	*022	*028	
725	86	034	040	046	052	058	064	070	076	082	088	
726		094	100	106	112	118	124	130	136	141	147	
727		153	159	165	171	177	183	189	195	201	207	
728		213	219	225	231	237	243	249	255	261	267	
729		273	279	285	291	297	303	308	314	320	326	
730		332	338	344	350	356	362	368	374	380	386	
731		392	398	404	410	416	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	
739		864	870	876	882	888	894	900	906	911	917	
740		923	929	935	941	947	953	958	964	970	976	
741		982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87	040	046	052	058	064	070	075	081	087	093	
743		099	105	111	116	122	128	134	140	146	151	
744		157	163	169	175	181	186	192	198	204	210	
745		216	221	227	233	239	245	251	256	262	268	
746		274	280	286	291	297	303	309	315	320	326	
747		332	338	344	350	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	
751		564	570	576	581	587	593	599	604	610	616	
752		622	628	633	639	645	651	656	662	668	674	
753		680	685	691	697	703	708	714	720	726	731	
754		737	743	749	754	760	766	772	777	783	789	
755		795	800	806	812	818	823	829	835	841	846	
756		852	858	864	869	875	881	887	892	898	904	
757		910	915	921	927	933	938	944	950	955	961	
758		967	973	978	984	990	996	*001	*007	*013	*018	
759	88	024	030	036	041	047	053	059	064	070	076	
760		081	087	093	099	104	110	116	121	127	133	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

$$\log_{10} \frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10} \frac{1}{2\pi} = 9.20182 - 1$$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

760–815

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
760	88	081	087	093	099	104	110	116	121	127	133	
761		138	144	150	156	161	167	173	178	184	190	
762		196	201	207	213	218	224	230	235	241	247	
763		252	258	264	270	275	281	287	292	298	304	
764		309	315	321	326	332	338	343	349	355	360	
765		366	372	378	383	389	395	400	406	412	417	
766		423	429	434	440	446	451	457	463	468	474	
767		480	485	491	497	502	508	514	519	525	530	
768		536	542	547	553	559	564	570	576	581	587	
769		593	598	604	610	615	621	627	632	638	643	
770		649	655	660	666	672	677	683	689	694	700	
771		705	711	717	722	728	734	739	745	750	756	
772		762	767	773	779	784	790	795	801	807	812	
773		818	824	829	835	840	846	852	857	863	868	
774		874	880	885	891	897	902	908	913	919	925	
775		930	936	941	947	953	958	964	969	975	981	
776		986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89	042	048	053	059	064	070	076	081	087	092	
778		098	104	109	115	120	126	131	137	143	148	
779		154	159	165	170	176	182	187	193	198	204	
780		209	215	221	226	232	237	243	248	254	260	
781		265	271	276	282	287	293	298	304	310	315	
782		321	326	332	337	343	348	354	360	365	371	
783		376	382	387	393	398	404	409	415	421	426	
784		432	437	443	448	454	459	465	470	476	481	
785		487	493	498	504	509	515	520	526	531	537	
786		542	548	553	559	564	570	575	581	586	592	
787		597	603	609	614	620	625	631	636	642	647	
788		653	658	664	669	675	680	686	691	697	702	
789		708	713	719	724	730	735	741	746	752	757	
790		763	768	774	779	785	790	796	801	807	812	
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	704	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	
811		902	907	913	918	924	929	934	940	945	950	
812		956	961	966	972	977	982	988	993	998	*004	
813	91	009	014	020	025	030	036	041	046	052	057	
814		062	068	073	078	084	089	094	100	105	110	
815		116	121	126	132	137	142	148	153	158	164	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

$\log_{10} \pi = 0.49715$

$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

815–870

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
815	91	116	121	126	132	137	142	148	153	158	164	
816		169	174	180	185	190	196	201	206	212	217	
817		222	228	233	238	243	249	254	259	265	270	
818		275	281	286	291	297	302	307	312	318	323	
819		328	334	339	344	350	355	360	365	371	376	
820		381	387	392	397	403	408	413	418	424	429	
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	
831		960	965	971	976	981	986	991	997	•002	•007	
832	92	012	018	023	028	033	038	044	049	054	059	
833		065	070	075	080	085	091	096	101	106	111	
834		117	122	127	132	137	143	148	153	158	163	
835		169	174	179	184	189	195	200	205	210	215	
836		221	226	231	236	241	247	252	257	262	267	
837		273	278	283	288	293	298	304	309	314	319	
838		324	330	335	340	345	350	355	361	366	371	
839		376	381	387	392	397	402	407	412	418	423	
840		428	433	438	443	449	454	459	464	469	474	
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	717	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	
851		993	998	•003	•008	•013	•018	•024	•029	•034	•039	
852	93	044	049	054	059	064	069	075	080	085	090	
853		095	100	105	110	115	120	125	131	136	141	
854		146	151	156	161	166	171	176	181	186	192	
855		197	202	207	212	217	222	227	232	237	242	
856		247	252	258	263	268	273	278	283	288	293	
857		298	303	308	313	318	323	328	334	339	344	
858		349	354	359	364	369	374	379	384	389	394	
859		399	404	409	414	420	425	430	435	440	445	
860		450	455	460	465	470	475	480	485	490	495	
861		500	505	510	515	520	526	531	536	541	546	
862		551	556	561	566	571	576	581	586	591	596	
863		601	606	611	616	621	626	631	636	641	646	
864		651	656	661	666	671	677	682	687	692	697	
865		702	707	712	717	722	727	732	737	742	747	
866		752	757	762	767	772	777	782	787	792	797	
867		802	807	812	817	822	827	832	837	842	847	
868		852	857	862	867	872	877	882	887	892	897	
869		902	907	912	917	922	927	932	937	942	947	
870		952	957	962	967	972	977	982	987	992	997	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

$$\log_{10}\frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10}\frac{1}{2\pi} = 9.20182 - 1$$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

870–925

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
870	93	952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	
872		052	057	062	067	072	077	082	087	091	096	
873		101	106	111	116	121	126	131	136	141	146	
874		151	156	161	166	171	176	181	186	191	196	
875		201	206	211	216	221	226	231	236	240	245	
876		250	255	260	265	270	275	280	285	290	295	
877		300	305	310	315	320	325	330	335	340	345	
878		349	354	359	364	369	374	379	384	389	394	
879		399	404	409	414	419	424	429	433	438	443	
880		448	453	458	463	468	473	478	483	488	493	
881		498	503	507	512	517	522	527	532	537	542	
882		547	552	557	562	567	571	576	581	586	591	
883		596	601	606	611	616	621	626	630	635	640	
884		645	650	655	660	665	670	675	680	685	689	
885		694	699	704	709	714	719	724	729	734	738	
886		743	748	753	758	763	768	773	778	783	787	
887		792	797	802	807	812	817	822	827	832	836	
888		841	846	851	856	861	866	871	876	880	885	
889		890	895	900	905	910	915	919	924	929	934	
890		939	944	949	954	959	963	968	973	978	983	
891		988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	95	036	041	046	051	056	061	066	071	075	080	
893		085	090	095	100	105	109	114	119	124	129	
894		134	139	143	148	153	158	163	168	173	177	
895		182	187	192	197	202	207	211	216	221	226	
896		231	236	240	245	250	255	260	265	270	274	
897		279	284	289	294	299	303	308	313	318	323	
898		328	332	337	342	347	352	357	361	366	371	
899		376	381	386	390	395	400	405	410	415	419	
900		424	429	434	439	444	448	453	458	463	468	
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		569	574	578	583	588	593	598	602	607	612	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	866	871	875	880	885	890	895	899	
910		904	909	914	918	923	928	933	938	942	947	
911		952	957	961	966	971	976	980	985	990	995	
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96	047	052	057	061	066	071	076	080	085	090	
914		095	099	104	109	114	118	123	128	133	137	
915		142	147	152	156	161	166	171	175	180	185	
916		190	194	199	204	209	213	218	223	227	232	
917		237	242	246	251	256	261	265	270	275	280	
918		284	289	294	298	303	308	313	317	322	327	
919		332	336	341	346	350	355	360	365	369	374	
920		379	384	388	393	398	402	407	412	417	421	
921		426	431	435	440	445	450	454	459	464	468	
922		473	478	483	487	492	497	501	506	511	515	
923		520	525	530	534	539	544	548	553	558	563	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

$\log_{10} \pi = 0.49715$

$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

925–980

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
925	96	614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	867	872	876	881	886	890	
931		895	900	904	909	914	918	923	928	932	937	
932		942	946	951	956	960	965	970	974	979	984	
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97	035	039	044	049	053	058	063	067	072	077	
935		081	086	090	095	100	104	109	114	118	123	
936		128	132	137	142	146	151	155	160	165	169	
937		174	179	183	188	192	197	202	206	211	216	
938		220	225	230	234	239	243	248	253	257	262	
939		267	271	276	280	285	290	294	299	304	308	
940		313	317	322	327	331	336	341	345	350	354	
941		359	364	368	373	377	382	387	391	396	400	
942		405	410	414	419	424	428	433	437	442	447	
943		451	456	460	465	470	474	479	483	488	493	
944		497	502	506	511	516	520	525	529	534	539	
945		543	548	552	557	562	566	571	575	580	585	
946		589	594	598	603	607	612	617	621	626	630	
947		635	640	644	649	653	658	663	667	672	676	
948		681	685	690	695	699	704	708	713	717	722	
949		727	731	736	740	745	750	754	759	763	768	
950		772	777	782	786	791	795	800	804	809	813	
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	887	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	069	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	205	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	
961		272	277	281	286	290	295	299	304	308	313	
962		318	322	327	331	336	340	345	349	354	358	
963		363	367	372	376	381	385	390	394	399	403	
964		408	412	417	421	426	430	435	439	444	448	
965		453	457	462	466	471	475	480	484	489	493	
966		498	502	507	511	516	520	525	529	534	538	
967		543	547	552	556	561	565	570	574	579	583	
968		588	592	597	601	605	610	614	619	623	628	
969		632	637	641	646	650	655	659	664	668	673	
970		677	682	686	691	695	700	704	709	713	717	
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	785	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

$$\log_{10}\frac{1}{\pi} = 9.50286 - 1$$

$$\log_{10}\frac{1}{2\pi} = 9.20182 - 1$$

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* Indicates change in the first two decimal places.

Table 3-1. Five-place common logarithms of numbers (continued)

980–1000

No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts
980	99	123	127	131	136	140	145	149	154	158	162	
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	5 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
983		255	260	264	269	273	277	282	286	291	295	
984		300	304	308	313	317	322	326	330	335	339	
985		344	348	352	357	361	366	370	374	379	383	
986		388	392	397	401	405	410	414	419	423	427	
987		432	436	441	445	449	454	458	463	467	471	
988		476	480	484	489	493	498	502	506	511	515	
989		520	524	528	533	537	542	546	550	555	559	
990		564	568	572	577	581	585	590	594	599	603	
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	4 1 0.4 2 0.8 3 1.2 4 1.6 5 2.0 6 2.4 7 2.8 8 3.2 9 3.6
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000	00	000	004	009	013	017	022	026	030	035	039	
No.	L	0	1	2	3	4	5	6	7	8	9	Proportional parts

$\log_{10} \pi = 0.49715$
$\log_{10} 2\pi = 0.79818$

* Indicates change in the first two decimal places.

SPECIAL INTERPOLATION IN TABLE 3-2*

In the first three columns on pages 90, 91, and 92, the tabular differences are so large that linear interpolation gives inaccurate results. Since every entry for the logarithm of the cosine of an angle near 90° can be thought of as the logarithm of the sine of an angle near 0°, it is sufficient if we consider only the sines of small angles. In like manner, since the tangent and cotangent are both reciprocals and cofunctions, it is sufficient to consider interpolation for tangents of very small angles, for we have

$$\log \tan A = -\log \cot A = -\log \tan (90° - A) = \log \cot (90° - A)$$

If A and B are sufficiently small angles, both expressed in minutes, it can be shown that the following relations are accurate to five places of decimals:

$$\log \sin A = \log \sin B + \log A - \log B$$
$$\log \tan A = \log \tan B + \log A - \log B$$

In particular, these relations will be reliable to five places of decimals if both A and B are less than 3° and $A - B$ is less than 1′. Since these conditions are true for interpolation problems for the sine and tangent in the first columns on pages 90, 91, and 92, these formulas may be used.

Example 1

Find log cos 89°27.5′. This is the same as finding log sin 0°32.5′. Let $A = 32.5'$, and let $B = 32'$. Then

$$\log \sin 32.5' = \log \sin 32' + \log 32.5 - \log 32$$

whence $\log \cos 89°27.5' = 7.97560 - 10$.

Example 2

Find log tan 1°15.2′. Let $A = 1°15.2' = 75.2'$, and let $B = 1°15' = 75'$. Then

$$\log \tan 1°15.2' = \log \tan 1°15' + \log 75.2 - \log 75 = 8.34002 - 10$$

Note

Ordinary linear interpolation in the first example would have given the result $7.97555 - 10$, with an error of 5 in the last place. Linear interpolation in the last example would have given the result $8.34001 - 10$, with an error of 1 in the last place.

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* From C. R. Wylie, Jr., "Plane Trigonometry," McGraw-Hill Book Company, New York, 1955.

Table 3-2. Common logarithms of trigonometric functions

0°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	———		———		———	0.00 000	**60**
1	6.46 373	30103	6.46 373	30103	13.53 627	0.00 000	59
2	6.76 476	17609	6.76 476	17609	13.23 524	0.00 000	58
3	6.94 085	12494	6.94 085	12494	13.05 915	0.00 000	57
4	7.06 579	9691	7.06 579	9691	12.93 421	0.00 000	56
5	7.16 270	7918	7.16 270	7918	12.83 730	0.00 000	55
6	7.24 188	6694	7.24 188	6694	12.75 812	0.00 000	54
7	7.30 882	5800	7.30 882	5800	12.69 118	0.00 000	53
8	7.36 682	5115	7.36 682	5115	12.63 318	0.00 000	52
9	7.41 797	4576	7.41 797	4576	12.58 203	0.00 000	51
10	7.46 373	4139	7.46 373	4139	12.53 627	0.00 000	**50**
11	7.50 512	3779	7.50 512	3779	12.49 488	0.00 000	49
12	7.54 291	3476	7.54 291	3476	12.45 709	0.00 000	48
13	7.57 767	3218	7.57 767	3219	12.42 233	0.00 000	47
14	7.60 985	2997	7.60 986	2996	12.39 014	0.00 000	46
15	7.63 982	2802	7.63 982	2803	12.36 018	0.00 000	45
16	7.66 784	2633	7.66 785	2633	12.33 215	0.00 000	44
17	7.69 417	2483	7.69 418	2482	12.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	12.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	12.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	12.23 524	9.99 999	**40**
21	7.78 594	2021	7.78 595	2020	12.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	12.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	12.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	12.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	12.13 833	9.99 999	35
26	7.87 870	1639	7.87 871	1639	12.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	12.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	12.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	12.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	12.05 914	9.99 998	**30**
31	7.95 508	1379	7.95 510	1379	12.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	12.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	12.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	12.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	11.99 219	9.99 998	25
36	8.02 002	1190	8.02 004	1190	11.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	11.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	11.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	11.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	11.93 419	9.99 997	**20**
41	8.07 650	1046	8.07 653	1047	11.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	11.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	11.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	11.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	11.88 304	9.99 996	15
46	8.12 647	934	8.12 651	934	11.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	11.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	11.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	11.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	11.83 727	9.99 995	**10**
51	8.17 128	843	8.17 133	843	11.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	11.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	11.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	11.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	11.79 587	9.99 994	5
56	8.21 189	769	8.21 195	769	11.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	11.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	11.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	11.76 538	9.99 994	1
60	8.24 186		8.24 192		11.75 808	9.99 993	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

Since the tabular differences in the first three columns of this page, and on each of the two pages following, are so large and change so rapidly in value that ordinary linear interpolation does not give results accurate to five places of decimals, special methods of interpolation are necessary. A brief account of these special methods is given on page 89.

89°

Table 3-2. Common logarithms of trigonometric functions (continued)

1°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.24 186	717	8.24 192	718	11.75 808	9.99 993	60
1	8.24 903	706	8.24 910	706	11.75 090	9.99 993	59
2	8.25 609	695	8.25 616	696	11.74 384	9.99 993	58
3	8.26 304	684	8.26 312	684	11.73 688	9.99 993	57
4	8.26 988	673	8.26 996	673	11.73 004	9.99 992	56
5	8.27 661	663	8.27 669	663	11.72 331	9.99 992	55
6	8.28 324	653	8.28 332	654	11.71 668	9.99 992	54
7	8.28 977	644	8.28 986	643	11.71 014	9.99 992	53
8	8.29 621	634	8.29 629	634	11.70 371	9.99 992	52
9	8.30 255	624	8.30 263	625	11.69 737	9.99 991	51
10	8.30 879	616	8.30 888	617	11.69 112	9.99 991	50
11	8.31 495	608	8.31 505	607	11.68 495	9.99 991	49
12	8.32 103	599	8.32 112	599	11.67 888	9.99 990	48
13	8.32 702	590	8.32 711	591	11.67 289	9.99 990	47
14	8.33 292	583	8.33 302	584	11.66 698	9.99 990	46
15	8.33 875	575	8.33 886	575	11.66 114	9.99 990	45
16	8.34 450	568	8.34 461	568	11.65 539	9.99 989	44
17	8.35 018	560	8.35 029	561	11.64 971	9.99 989	43
18	8.35 578	553	8.35 590	553	11.64 410	9.99 989	42
19	8.36 131	547	8.36 143	546	11.63 857	9.99 989	41
20	8.36 678	539	8.36 689	540	11.63 311	9.99 988	40
21	8.37 217	533	8.37 229	533	11.62 771	9.99 988	39
22	8.37 750	526	8.37 762	527	11.62 238	9.99 988	38
23	8.83 276	520	8.38 289	520	11.61 711	9.99 987	37
24	8.38 796	514	8.38 809	514	11.61 191	9.99 987	36
25	8.39 310	508	8.39 323	509	11.60 677	9.99 987	35
26	8.39 818	502	8.39 832	502	11.60 168	9.99 986	34
27	8.40 320	496	8.40 334	496	11.59 666	9.99 986	33
28	8.40 816	491	8.40 830	491	11.59 170	9.99 986	32
29	8.41 307	485	8.41 321	486	11.58 679	9.99 985	31
30	8.41 792	480	8.41 807	480	11.58 193	9.99 985	30
31	8.42 272	474	8.42 287	475	11.57 713	9.99 985	29
32	8.42 746	470	8.42 762	470	11.57 238	9.99 984	28
33	8.43 216	464	8.43 232	464	11.56 768	9.99 984	27
34	8.43 680	459	8.43 696	460	11.56 304	9.99 984	26
35	8.44 139	455	8.44 156	455	11.55 844	9.99 983	25
36	8.44 594	450	8.44 611	450	11.55 389	9.99 983	24
37	8.45 044	445	8.45 061	446	11.54 939	9.99 983	23
38	8.45 489	441	8.45 507	441	11.54 493	9.99 982	22
39	8.45 930	436	8.45 948	437	11.54 052	9.99 982	21
40	8.46 366	433	8.46 385	432	11.53 615	9.99 982	20
41	8.46 799	427	8.46 817	428	11.53 183	9.99 981	19
42	8.47 226	424	8.47 245	424	11.52 755	9.99 981	18
43	8.47 650	419	8.47 669	420	11.52 331	9.99 981	17
44	8.48 069	416	8.48 089	416	11.51 911	9.99 980	16
45	8.48 485	411	8.48 505	412	11.51 495	9.99 980	15
46	8.48 896	408	8.48 917	408	11.51 083	9.99 979	14
47	8.49 304	404	8.49 325	404	11.50 675	9.99 979	13
48	8.49 708	400	8.49 729	401	11.50 271	9.99 979	12
49	8.50 108	396	8.50 130	397	11.49 870	9.99 978	11
50	8.50 504	393	8.50 527	393	11.49 473	9.99 978	10
51	8.50 897	390	8.50 920	390	11.49 080	9.99 977	9
52	8.51 287	386	8.51 310	386	11.48 690	9.99 977	8
53	8.51 673	382	8.51 696	383	11.48 304	9.99 977	7
54	8.52 055	379	8.52 079	380	11.47 921	9.99 976	6
55	8.52 434	376	8.52 459	376	11.47 541	9.99 976	5
56	8.52 810	373	8.52 835	373	11.47 165	9.99 975	4
57	8.53 183	369	8.53 208	370	11.46 792	9.99 975	3
58	8.53 552	367	8.53 578	367	11.46 422	9.99 974	2
59	8.53 919	363	8.53 945	363	11.46 055	9.99 974	1
60	8.54 282		8.54 308		11.45 692	9.99 974	0
	L Cos	d	L Cot	c d	L Tan	L Sin	′

88°

If ordinary linear interpolation is not sufficiently accurate, use the special methods described on page 89.

Table 3-2. Common logarithms of trigonometric functions (continued)

2°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.54 282	360	8.54 308	361	11.45 692	9.99 974	**60**
1	8.54 642	357	8.54 669	358	11.45 331	9.99 973	59
2	8.54 999	355	8.55 027	355	11.44 973	9.99 973	58
3	8.55 354	351	8.55 382	352	11.44 618	9.99 972	57
4	8.55 705	349	8.55 734	349	11.44 266	9.99 972	56
5	8.56 054	346	8.56 083	346	11.43 917	9.99 971	55
6	8.56 400	343	8.56 429	344	11.43 571	9.99 971	54
7	8.56 743	341	8.56 773	341	11.43 227	9.99 970	53
8	8.57 084	337	8.57 114	338	11.42 886	9.99 970	52
9	8.57 421	336	8.57 452	336	11.42 548	9.99 969	51
10	8.57 757	332	8.57 788	333	11.42 212	9.99 969	**50**
11	8.58 089	330	8.58 121	330	11.41 879	9.99 968	49
12	8.58 419	328	8.58 451	328	11.41 549	9.99 968	48
13	8.58 747	325	8.58 779	326	11.41 221	9.99 967	47
14	8.59 072	323	8.59 105	323	11.40 895	9.99 967	46
15	8.59 395	320	8.59 428	321	11.40 572	9.99 967	45
16	8.59 715	318	8.59 749	319	11.40 251	9.99 966	44
17	8.60 033	316	8.60 068	316	11.39 932	9.99 966	43
18	8.60 349	313	8.60 384	314	11.39 616	9.99 965	42
19	8.60 662	311	8.60 698	311	11.39 302	9.99 964	41
20	8.60 973	309	8.61 009	310	11.38 991	9.99 964	**40**
21	8.61 282	307	8.61 319	307	11.38 681	9.99 963	39
22	8.61 589	305	8.61 626	305	11.38 374	9.99 963	38
23	8.61 894	302	8.61 931	303	11.38 069	9.99 962	37
24	8.62 196	301	8.62 234	301	11.37 766	9.99 962	36
25	8.62 497	298	8.62 535	299	11.37 465	9.99 961	35
26	8.62 795	296	8.62 834	297	11.37 166	9.99 961	34
27	8.63 091	294	8.63 131	295	11.36 869	9.99 960	33
28	8.63 385	293	8.63 426	292	11.36 574	9.99 960	32
29	8.63 678	290	8.63 718	291	11.36 282	9.99 959	31
30	8.63 968	288	8.64 009	289	11.35 991	9.99 959	**30**
31	8.64 256	287	8.64 298	287	11.35 702	9.99 958	29
32	8.64 543	284	8.64 585	285	11.35 415	9.99 958	28
33	8.64 827	283	8.64 870	284	11.35 130	9.99 957	27
34	8.65 110	281	8.65 154	281	11.34 846	9.99 956	26
35	8.65 391	279	8.65 435	280	11.34 565	9.99 956	25
36	8.65 670	277	8.65 715	278	11.34 285	9.99 955	24
37	8.65 947	276	8.65 993	276	11.34 007	9.99 955	23
38	8.66 223	274	8.66 269	274	11.33 731	9.99 954	22
39	8.66 497	272	8.66 543	273	11.33 457	9.99 954	21
40	8.66 769	270	8.66 816	271	11.33 184	9.99 953	**20**
41	8.67 039	269	8.67 087	269	11.32 913	9.99 952	19
42	8.67 308	267	8.67 356	268	11.32 644	9.99 952	18
43	8.67 575	266	8.67 624	266	11.32 376	9.99 951	17
44	8.67 841	263	8.67 890	264	11.32 110	9.99 951	16
45	8.68 104	263	8.68 154	263	11.31 846	9.99 950	15
46	8.68 367	260	8.68 417	261	11.31 583	9.99 949	14
47	8.68 627	259	8.68 678	260	11.31 322	9.99 949	13
48	8.68 886	258	8.68 938	258	11.31 062	9.99 948	12
49	8.69 144	256	8.69 196	257	11.30 804	9.99 948	11
50	8.69 400	254	8.69 453	255	11.30 547	9.99 947	**10**
51	8.69 654	253	8.69 708	254	11.30 292	9.99 946	9
52	8.69 907	252	8.69 962	252	11.30 038	9.99 946	8
53	8.70 159	250	8.70 214	251	11.29 786	9.99 945	7
54	8.70 409	249	8.70 465	249	11.29 535	9.99 944	6
55	8.70 658	247	8.70 714	248	11.29 286	9.99 944	5
56	8.70 905	246	8.70 962	246	11.29 038	9.99 943	4
57	8.71 151	244	8.71 208	245	11.28 792	9.99 942	3
58	8.71 395	243	8.71 453	244	11.28 547	9.99 942	2
59	8.71 638	242	8.71 697	243	11.28 303	9.99 941	1
60	8.71 880		8.71 940		11.28 060	9.99 940	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

If ordinary linear interpolation is not sufficiently accurate, use the special methods described on page 89.

87°

Table 3-2. Common logarithms of trigonometric functions (continued)

3°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.71 880	240	8.71 940	241	11.28 060	9.99 940	**60**
1	8.72 120	239	8.72 181	239	11.27 819	9.99 940	59
2	8.72 359	238	8.72 420	239	11.27 580	9.99 939	58
3	8.72 597	237	8.72 659	237	11.27 341	9.99 938	57
4	8.72 834	235	8.72 896	236	11.27 104	9.99 938	56
5	8.73 069	234	8.73 132	234	11.26 868	9.99 937	55
6	8.73 303	232	8.73 366	234	11.26 634	9.99 936	54
7	8.73 535	232	8.73 600	232	11.26 400	9.99 936	53
8	8.73 767	230	8.73 832	231	11.26 168	9.99 935	52
9	8.73 997	229	8.74 063	229	11.25 937	9.99 934	51
10	8.74 226	228	8.74 292	229	11.25 708	9.99 934	**50**
11	8.74 454	226	8.74 521	227	11.25 479	9.99 933	49
12	8.74 680	226	8.74 748	226	11.25 252	9.99 932	48
13	8.74 906	224	8.74 974	225	11.25 026	9.99 932	47
14	8.75 130	223	8.75 199	224	11.24 801	9.99 931	46
15	8.75 353	222	8.75 423	222	11.24 577	9.99 930	45
16	8.75 575	220	8.75 645	222	11.24 355	9.99 929	44
17	8.75 795	220	8.75 867	220	11.24 133	9.99 929	43
18	8.76 015	219	8.76 087	219	11.23 913	9.99 928	42
19	8.76 234	217	8.76 306	219	11.23 694	9.99 927	41
20	8.76 451	216	8.76 525	217	11.23 475	9.99 926	**40**
21	8.76 667	216	8.76 742	216	11.23 258	9.99 926	39
22	8.76 883	214	8.76 958	215	11.23 042	9.99 925	38
23	8.77 097	213	8.77 173	214	11.22 827	9.99 924	37
24	8.77 310	212	8.77 387	213	11.22 613	9.99 923	36
25	8.77 522	211	8.77 600	211	11.22 400	9.99 923	35
26	8.77 733	210	8.77 811	211	11.22 189	9.99 922	34
27	8.77 943	209	8.78 022	210	11.21 978	9.99 921	33
28	8.78 152	208	8.78 232	209	11.21 768	9.99 920	32
29	8.78 360	208	8.78 441	208	11.21 559	9.99 920	31
30	8.78 568	206	8.78 649	206	11.21 351	9.99 919	**30**
31	8.78 774	205	8.78 855	206	11.21 145	9.99 918	29
32	8.78 979	204	8.79 061	205	11.20 939	9.99 917	28
33	8.79 183	203	8.79 266	204	11.20 734	9.99 917	27
34	8.79 386	202	8.79 470	203	11.20 530	9.99 916	26
35	8.79 588	201	8.79 673	202	11.20 327	9.99 915	25
36	8.79 789	201	8.79 875	201	11.20 125	9.99 914	24
37	8.79 990	199	8.80 076	201	11.19 924	9.99 913	23
38	8.80 189	199	8.80 277	199	11.19 723	9.99 913	22
39	8.80 388	197	8.80 476	198	11.19 524	9.99 912	21
40	8.80 585	197	8.80 674	198	11.19 326	9.99 911	**20**
41	8.80 782	196	8.80 872	196	11.19 128	9.99 910	19
42	8.80 978	195	8.81 068	196	11.18 932	9.99 909	18
43	8.81 173	194	8.81 264	195	11.18 736	9.99 909	17
44	8.81 367	193	8.81 459	194	11.18 541	9.99 908	16
45	8.81 560	192	8.81 653	193	11.18 347	9.99 907	15
46	8.81 752	192	8.81 846	192	11.18 154	9.99 906	14
47	8.81 944	190	8.82 038	192	11.17 962	9.99 905	13
48	8.82 134	190	8.82 230	190	11.17 770	9.99 904	12
49	8.82 324	189	8.82 420	190	11.17 580	9.99 904	11
50	8.82 513	188	8.82 610	189	11.17 390	9.99 903	**10**
51	8.82 701	187	8.82 799	188	11.17 201	9.99 902	9
52	8.82 888	187	8.82 987	188	11.17 013	9.99 901	8
53	8.83 075	186	8.83 175	186	11.16 825	9.99 900	7
54	8.83 261	185	8.83 361	186	11.16 639	9.99 899	6
55	8.83 446	184	8.83 547	185	11.16 453	9.99 898	5
56	8.83 630	183	8.83 732	184	11.16 268	9.99 898	4
57	8.83 813	183	8.83 916	184	11.16 084	9.99 897	3
58	8.83 996	181	8.84 100	182	11.15 900	9.99 896	2
59	8.84 177	181	8.84 282	182	11.15 718	9.99 895	1
60	8.84 358		8.84 464		11.15 536	9.99 894	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

Prop. Pts.

	239	**237**	**235**	**234**
2	47.8	47.4	47.0	46.8
3	71.7	71.1	70.5	70.2
4	95.6	94.8	94.0	93.6
5	119.5	118.5	117.5	117.0
6	143.4	142.2	141.0	140.4
7	167.3	165.9	164.5	163.8
8	191.2	189.6	188.0	187.2
9	215.1	213.3	211.5	210.6

	232	**229**	**227**	**226**
2	46.4	45.8	45.4	45.2
3	69.6	68.7	68.1	67.8
4	92.8	91.6	90.8	90.4
5	116.0	114.5	113.5	113.0
6	139.2	137.4	136.2	135.6
7	162.4	160.3	158.9	158.2
8	185.6	183.2	181.6	180.8
9	208.8	206.1	204.3	203.4

	224	**222**	**220**	**219**
2	44.8	44.4	44.0	43.8
3	67.2	66.6	66.0	65.7
4	89.6	88.8	88.0	87.6
5	112.0	111.0	110.0	109.5
6	134.4	133.2	132.0	131.4
7	156.8	155.4	154.0	153.3
8	179.2	177.6	176.0	175.2
9	201.6	199.8	198.0	197.1

	217	**215**	**213**	**211**
2	43.4	43.0	42.6	42.2
3	65.1	64.5	63.9	63.3
4	86.8	86.0	85.2	84.4
5	108.5	107.5	106.5	105.5
6	130.2	129.0	127.8	126.6
7	151.9	150.5	149.1	147.7
8	173.6	172.0	170.4	168.8
9	195.3	193.5	191.7	189.9

	208	**206**	**203**	**201**
2	41.6	41.2	40.6	40.2
3	62.4	61.8	60.9	60.3
4	83.2	82.4	81.2	80.4
5	104.0	103.0	101.5	100.5
6	124.8	123.6	121.8	120.6
7	145.6	144.2	142.1	140.7
8	166.4	164.8	162.4	160.8
9	187.2	185.4	182.7	180.9

	199	**197**	**195**	**193**
2	39.8	39.4	39.0	38.6
3	59.7	59.1	58.5	57.9
4	79.6	78.8	78.0	77.2
5	99.5	98.5	97.5	96.5
6	119.4	118.2	117.0	115.8
7	139.3	137.9	136.5	135.1
8	159.2	157.6	156.0	154.4
9	179.1	177.3	175.5	173.7

	192	**190**	**188**	**186**
2	38.4	38.0	37.6	37.2
3	57.6	57.0	56.4	55.8
4	76.8	76.0	75.2	74.4
5	96.0	95.0	94.0	93.0
6	115.2	114.0	112.8	111.6
7	134.4	133.0	131.6	130.2
8	153.6	152.0	150.4	148.8
9	172.8	171.0	169.2	167.4

	184	**183**	**182**	**181**
2	36.8	36.6	36.4	36.2
3	55.2	54.9	54.6	54.3
4	73.6	73.2	72.8	72.4
5	92.0	91.5	91.0	90.5
6	110.4	109.8	109.2	108.6
7	128.8	128.1	127.4	126.7
8	147.2	146.4	145.6	144.8
9	165.6	164.7	163.8	162.9

Prop. Pts.

86°

Table 3-2. Common logarithms of trigonometric functions (continued)

4°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.84 358	181	8.84 464	182	11.15 536	9.99 894	**60**
1	8.84 539	179	8.84 646	180	11.15 354	9.99 893	59
2	8.84 718	179	8.84 826	180	11.15 174	9.99 892	58
3	8.84 897	178	8.85 006	179	11.14 994	9.99 891	57
4	8.85 075	177	8.85 185	178	11.14 815	9.99 891	56
5	8.85 252	177	8.85 363	177	11.14 637	9.99 890	55
6	8.85 429	176	8.85 540	177	11.14 460	9.99 889	54
7	8.85 605	175	8.85 717	176	11.14 283	9.99 888	53
8	8.85 780	175	8.85 893	176	11.14 107	9.99 887	52
9	8.85 955	173	8.86 069	174	11.13 931	9.99 886	51
10	8.86 128	173	8.86 243	174	11.13 757	9.99 885	**50**
11	8.86 301	173	8.86 417	174	11.13 583	9.99 884	49
12	8.86 474	171	8.86 591	172	11.13 409	9.99 883	48
13	8.86 645	171	8.86 763	172	11.13 237	9.99 882	47
14	8.86 816	171	8.86 935	171	11.13 065	9.99 881	46
15	8.86 987	169	8.87 106	171	11.12 894	9.99 880	45
16	8.87 156	169	8.87 277	170	11.12 723	9.99 879	44
17	8.87 325	169	8.87 447	169	11.12 553	9.99 879	43
18	8.87 494	167	8.87 616	169	11.12 384	9.99 878	42
19	8.87 661	168	8.87 785	168	11.12 215	9.99 877	41
20	8.87 829	166	8.87 953	167	11.12 047	9.99 876	**40**
21	8.87 995	166	8.88 120	167	11.11 880	9.99 875	39
22	8.88 161	165	8.88 287	166	11.11 713	9.99 874	38
23	8.88 326	164	8.88 453	165	11.11 547	9.99 873	37
24	8.88 490	164	8.88 618	165	11.11 382	9.99 872	36
25	8.88 654	163	8.88 783	165	11.11 217	9.99 871	35
26	8.88 817	163	8.88 948	163	11.11 052	9.99 870	34
27	8.88 980	162	8.89 111	163	11.10 889	9.99 869	33
28	8.89 142	162	8.89 274	163	11.10 726	9.99 868	32
29	8.89 304	160	8.89 437	161	11.10 563	9.99 867	31
30	8.89 464	161	8.89 598	162	11.10 402	9.99 866	**30**
31	8.89 625	159	8.89 760	160	11.10 240	9.99 865	29
32	8.89 784	159	8.89 920	160	11.10 080	9.99 864	28
33	8.89 943	159	8.90 080	160	11.09 920	9.99 863	27
34	8.90 102	158	8.90 240	159	11.09 760	9.99 862	26
35	8.90 260	157	8.90 399	158	11.09 601	9.99 861	25
36	8.90 417	157	8.90 557	158	11.09 443	9.99 860	24
37	8.90 574	156	8.90 715	157	11.09 285	9.99 859	23
38	8.90 730	155	8.90 872	157	11.09 128	9.99 858	22
39	8.90 885	155	8.91 029	156	11.08 971	9.99 857	21
40	8.91 040	155	8.91 185	155	11.08 815	9.99 856	**20**
41	8.91 195	154	8.91 340	155	11.08 660	9.99 855	19
42	8.91 349	153	8.91 495	155	11.08 505	9.99 854	18
43	8.91 502	153	8.91 650	153	11.08 350	9.99 853	17
44	8.91 655	152	8.91 803	154	11.08 197	9.99 852	16
45	8.91 807	152	8.91 957	153	11.08 043	9.99 851	15
46	8.91 959	151	8.92 110	152	11.07 890	9.99 850	14
47	8.92 110	151	8.92 262	152	11.07 738	9.99 848	13
48	8.92 261	150	8.92 414	151	11.07 586	9.99 847	12
49	8.92 411	150	8.92 565	151	11.07 435	9.99 846	11
50	8.92 561	149	8.92 716	150	11.07 284	9.99 845	**10**
51	8.92 710	149	8.92 866	150	11.07 134	9.99 844	9
52	8.92 859	148	8.93 016	149	11.06 984	9.99 843	8
53	8.93 007	147	8.93 165	148	11.06 835	9.99 842	7
54	8.93 154	147	8.93 313	149	11.06 687	9.99 841	6
55	8.93 301	147	8.93 462	147	11.06 538	9.99 840	5
56	8.93 448	146	8.93 609	147	11.06 391	9.99 839	4
57	8.93 594	146	8.93 756	147	11.06 244	9.99 838	3
58	8.93 740	145	8.93 903	146	11.06 097	9.99 837	2
59	8.93 885	145	8.94 049	146	11.05 951	9.99 836	1
60	8.94 030		8.94 195		11.05 805	9.99 834	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

Prop. Pts.

	182	181	180	179
2	36.4	36.2	36.0	35.8
3	54.6	54.3	54.0	53.7
4	72.8	72.4	72.0	71.6
5	91.0	90.5	90.0	89.5
6	109.2	108.6	108.0	107.4
7	127.4	126.7	126.0	125.3
8	145.6	144.8	144.0	143.2
9	163.8	162.9	162.0	161.1

	178	177	176	175
2	35.6	35.4	35.2	35.0
3	53.4	53.1	52.8	52.5
4	71.2	70.8	70.4	70.0
5	89.0	88.5	88.0	87.5
6	106.8	106.2	105.6	105.0
7	124.6	123.9	123.2	122.5
8	142.4	141.6	140.8	140.0
9	160.2	159.3	158.4	157.5

	174	173	172	171
2	34.8	34.6	34.4	34.2
3	52.2	51.9	51.6	51.3
4	69.6	69.2	68.8	68.4
5	87.0	86.5	86.0	85.5
6	104.4	103.8	103.2	102.6
7	121.8	121.1	120.4	119.7
8	139.2	138.4	137.6	136.8
9	156.6	155.7	154.8	153.9

	170	169	168	167
2	34.0	33.8	33.6	33.4
3	51.0	50.7	50.4	50.1
4	68.0	67.6	67.2	66.8
5	85.0	84.5	84.0	83.5
6	102.0	101.4	100.8	100.2
7	119.0	118.3	117.6	116.9
8	136.0	135.2	134.4	133.6
9	153.0	152.1	151.2	150.3

	166	165	164	163
2	33.2	33.0	32.8	32.6
3	49.8	49.5	49.2	48.9
4	66.4	66.0	65.6	65.2
5	83.0	82.5	82.0	81.5
6	99.6	99.0	98.4	97.8
7	116.2	115.5	114.8	114.1
8	132.8	132.0	131.2	130.4
9	149.4	148.5	147.6	146.7

	162	161	160	159
2	32.4	32.2	32.0	31.8
3	48.6	48.3	48.0	47.7
4	64.8	64.4	64.0	63.6
5	81.0	80.5	80.0	79.5
6	97.2	96.6	96.0	95.4
7	113.4	112.7	112.0	111.3
8	129.6	128.8	128.0	127.2
9	145.8	144.9	144.0	143.1

	158	157	156	155
2	31.6	31.4	31.2	31.0
2	47.4	47.1	46.8	46.5
4	63.2	62.8	62.4	62.0
5	79.0	78.5	78.0	77.5
6	94.8	94.2	93.6	93.0
7	110.6	109.9	109.2	108.5
8	126.4	125.6	124.8	124.0
9	142.2	141.3	140.4	139.5

	154	153	152
2	30.8	30.6	30.4
3	46.2	45.9	45.6
4	61.6	61.2	60.8
5	77.0	76.5	76.0
6	92.4	91.8	91.2
7	107.8	107.1	106.4
8	123.2	122.4	121.6
9	138.6	137.7	136.8

Prop. Pts.

85°

Table 3-2. Common logarithms of trigonometric functions (continued)

5°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	8.94 030	144	8.94 195	145	11.05 805	9.99 834	**60**
1	8.94 174	143	8.94 340	145	11.05 660	9.99 833	59
2	8.94 317	144	8.94 485	145	11.05 515	9.99 832	58
3	8.94 461	142	8.94 630	143	11.05 370	9.99 831	57
4	8.94 603	143	8.94 773	144	11.05 227	9.99 830	56
5	8.94 746	141	8.94 917	143	11.05 083	9.99 829	55
6	8.94 887	142	8.95 060	142	11.04 940	9.99 828	54
7	8.95 029	141	8.95 202	142	11.04 798	9.99 827	53
8	8.95 170	140	8.95 344	142	11.04 656	9.99 825	52
9	8.95 310	140	8.95 486	141	11.04 514	9.99 824	51
10	8.95 450	139	8.95 627	140	11.04 373	9.99 823	**50**
11	8.95 589	139	8.95 767	141	11.04 233	9.99 822	49
12	8.95 728	139	8.95 908	139	11.04 092	9.99 821	48
13	8.95 867	138	8.96 047	140	11.03 953	9.99 820	47
14	8.96 005	138	8.96 187	138	11.03 813	9.99 819	46
15	8.96 143	137	8.96 325	139	11.03 675	9.99 817	45
16	8.96 280	137	8.96 464	138	11.03 536	9.99 816	44
17	8.96 417	136	8.96 602	137	11.03 398	9.99 815	43
18	8.96 553	136	8.96 739	138	11.03 261	9.99 814	42
19	8.96 689	136	8.96 877	136	11.03 123	9.99 813	41
20	8.96 825	135	8.97 013	137	11.02 987	9.99 812	**40**
21	8.96 960	135	8.97 150	135	11.02 850	9.99 810	39
22	8.97 095	134	8.97 285	136	11.02 715	9.99 809	38
23	8.97 229	134	8.97 421	135	11.02 579	9.99 808	37
24	8.97 363	133	8.97 556	135	11.02 444	9.99 807	36
25	8.97 496	133	8.97 691	134	11.02 309	9.99 806	35
26	8.97 629	133	8.97 825	134	11.02 175	9.99 804	34
27	8.97 762	132	8.97 959	133	11.02 041	9.99 803	33
28	8.97 894	132	8.98 092	133	11.01 908	9.99 802	32
29	8.98 026	131	8.98 225	133	11.01 775	9.99 801	31
30	8.98 157	131	8.98 358	132	11.01 642	9.99 800	**30**
31	8.98 288	131	8.98 490	132	11.01 510	9.99 798	29
32	8.98 419	130	8.98 622	131	11.01 378	9.99 797	28
33	8.98 549	130	8.98 753	131	11.01 247	9.99 796	27
34	8.98 679	129	8.98 884	131	11.01 116	9.99 795	26
35	8.98 808	129	8.99 015	130	11.00 985	9.99 793	25
36	8.98 937	129	8.99 145	130	11.00 855	9.99 792	24
37	8.99 066	128	8.99 275	130	11.00 725	9.99 791	23
38	8.99 194	128	8.99 405	129	11.00 595	9.99 790	22
39	8.99 322	128	8.99 534	128	11.00 466	9.99 788	21
40	8.99 450	127	8.99 662	129	11.00 338	9.99 787	**20**
41	8.99 577	127	8.99 791	128	11.00 209	9.99 786	19
42	8.99 704	126	8.99 919	127	11.00 081	9.99 785	18
43	8.99 830	126	9.00 046	128	10.99 954	9.99 783	17
44	8.99 956	126	9.00 174	127	10.99 826	9.99 782	16
45	9.00 082	125	9.00 301	126	10.99 699	9.99 781	15
46	9.00 207	125	9.00 427	126	10.99 573	9.99 780	14
47	9.00 332	124	9.00 553	126	10.99 447	9.99 778	13
48	9.00 456	125	9.00 679	126	10.99 321	9.99 777	12
49	9.00 581	123	9.00 805	125	10.99 195	9.99 776	11
50	9.00 704	124	9.00 930	125	10.99 070	9.99 775	**10**
51	9.00 828	123	9.01 055	124	10.98 945	9.99 773	9
52	9.00 951	123	9.01 179	124	10.98 821	9.99 772	8
53	9.01 074	122	9.01 303	124	10.98 697	9.99 771	7
54	9.01 196	122	9.01 427	123	10.98 573	9.99 769	6
55	9.01 318	122	9.01 550	123	10.98 450	9.99 768	5
56	9.01 440	121	9.01 673	123	10.98 327	9.99 767	4
57	9.01 561	121	9.01 796	122	10.98 204	9.99 765	3
58	9.01 682	121	9.01 918	122	10.98 082	9.99 764	2
59	9.01 803	120	9.02 040	122	10.97 960	9.99 763	1
60	9.01 923		9.02 162		10.97 838	9.99 761	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

Prop. Pts.

	151	**150**	**149**	**148**
2	30.2	30.0	29.8	29.6
3	45.3	45.0	44.7	44.4
4	60.4	60.0	59.6	59.2
5	75.5	75.0	74.5	74.0
6	90.6	90.0	89.4	88.8
7	105.7	105.0	104.3	103.6
8	120.8	120.0	119.2	118.4
9	135.9	135.0	134.1	133.2

	147	**146**	**145**	**144**
2	29.4	29.2	29.0	28.8
3	44.1	43.8	43.5	43.2
4	58.8	58.4	58.0	57.6
5	73.5	73.0	72.5	72.0
6	88.2	87.6	87.0	86.4
7	102.9	102.2	101.5	100.8
8	117.6	116.8	116.0	115.2
9	132.3	131.4	130.5	129.6

	143	**142**	**141**	**140**
2	28.6	28.4	28.2	28.0
3	42.9	42.6	42.3	42.0
4	57.2	56.8	56.4	56.0
5	71.5	71.0	70.5	70.0
6	85.8	85.2	84.6	84.0
7	100.1	99.4	98.7	98.0
8	114.4	113.6	112.8	112.0
9	128.7	127.8	126.9	126.0

	139	**138**	**137**	**136**
2	27.8	27.6	27.4	27.2
3	41.7	41.4	41.1	40.8
4	55.6	55.2	54.8	54.4
5	69.5	69.0	68.5	68.0
6	83.4	82.8	82.2	81.6
7	97.3	96.6	95.9	95.2
8	111.2	110.4	109.6	108.8
9	125.1	124.2	123.3	122.4

	135	**134**	**133**	**132**
2	27.0	26.8	26.6	26.4
3	40.5	40.2	39.9	39.6
4	54.0	53.6	53.2	52.8
5	67.5	67.0	66.5	66.0
6	81.0	80.4	79.8	79.2
7	94.5	93.8	93.1	92.4
8	108.0	107.2	106.4	105.6
9	121.5	120.6	119.7	118.8

	131	**130**	**129**	**128**
2	26.2	26.0	25.8	25.6
3	39.3	39.0	38.7	38.4
4	52.4	52.0	51.6	51.2
5	65.5	65.0	64.5	64.0
6	78.6	78.0	77.4	76.8
7	91.7	91.0	90.3	89.6
8	104.8	104.0	103.2	102.4
9	117.9	117.0	116.1	115.2

	127	**126**	**125**	**124**
2	25.4	25.2	25.0	24.8
3	38.1	37.8	37.5	37.2
4	50.8	50.4	50.0	49.6
5	63.5	63.0	62.5	62.0
6	76.2	75.6	75.0	74.4
7	88.9	88.2	87.5	86.8
8	101.6	100.8	100.0	99.2
9	114.3	113.4	112.5	111.6

	123	**122**	**121**	**120**
2	24.6	24.4	24.2	24.0
3	36.9	36.6	36.3	36.0
4	49.2	48.8	48.4	48.0
5	61.5	61.0	60.5	60.0
6	73.8	73.2	72.6	72.0
7	86.1	85.4	84.7	84.0
8	98.4	97.6	96.8	96.0
9	110.7	109.8	108.9	108.0

Prop. Pts.

84°

Table 3-2. Common logarithms of trigonometric functions (continued)

6°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.01 923	120	9.02 162	121	10.97 838	9.99 761	**60**
1	9.02 043	120	9.02 283	121	10.97 717	9.99 760	59
2	9.02 163	120	9.02 404	121	10.97 596	9.99 759	58
3	9.02 283	119	9.02 525	120	10.97 475	9.99 757	57
4	9.02 402	118	9.02 645	121	10.97 355	9.99 756	56
5	9.02 520	119	9.02 766	119	10.97 234	9.99 755	55
6	9.02 639	118	9.02 885	120	10.97 115	9.99 753	54
7	9.02 757	117	9.03 005	119	10.96 995	9.99 752	53
8	9.02 874	118	9.03 124	118	10.96 876	9.99 751	52
9	9.02 992	117	9.03 242	119	10.96 758	9.99 749	51
10	9.03 109	117	9.03 361	118	10.96 639	9.99 748	**50**
11	9.03 226	116	9.03 479	118	10.96 521	9.99 747	49
12	9.03 342	116	9.03 597	117	10.96 403	9.99 745	48
13	9.03 458	116	9.03 714	118	10.96 286	9.99 744	47
14	9.03 574	116	9.03 832	116	10.96 168	9.99 742	46
15	9.03 690	115	9.03 948	117	10.96 052	9.99 741	45
16	9.03 805	115	9.04 065	116	10.95 935	9.99 740	44
17	9.03 920	114	9.04 181	116	10.95 819	9.99 738	43
18	9.04 034	115	9.04 297	116	10.95 703	9.99 737	42
19	9.04 149	113	9.04 413	115	10.95 587	9.99 736	41
20	9.04 262	114	9.04 528	115	10.95 472	9.99 734	**40**
21	9.04 376	114	9.04 643	115	10.95 357	9.99 733	39
22	9.04 490	113	9.04 758	115	10.95 242	9.99 731	38
23	9.04 603	112	9.04 873	114	10.95 127	9.99 730	37
24	9.04 715	113	9.04 987	114	10.95 013	9.99 728	36
25	9.04 828	112	9.05 101	113	10 94 899	9.99 727	35
26	9.04 940	112	9.05 214	114	10.94 786	9.99 726	34
27	9.05 052	112	9.05 328	113	10.94 672	9.99 724	33
28	9.05 164	111	9.05 441	112	10.94 559	9.99 723	32
29	9.05 275	111	9.05 553	113	10.94 447	9.99 721	31
30	9.05 386	111	9.05 666	112	10.94 334	9.99 720	**30**
31	9.05 497	110	9.05 778	112	10.94 222	9.99 718	29
32	9.05 607	110	9.05 890	112	10.94 110	9.99 717	28
33	9.05 717	110	9.06 002	111	10.93 998	9.99 716	27
34	9.05 827	110	9.06 113	111	10.93 887	9.99 714	26
35	9.05 937	109	9.06 224	111	10.93 776	9.99 713	25
36	9.06 046	109	9.06 335	110	10.93 665	9.99 711	24
37	9.06 155	109	9.06 445	111	10.93 555	9.99 710	23
38	9.06 264	108	9.06 556	110	10.93 444	9.99 708	22
39	9.06 372	109	9.06 666	109	10.93 334	9.99 707	21
40	9.06 481	108	9.06 775	110	10.93 225	9.99 705	**20**
41	9.06 589	107	9.06 885	109	10.93 115	9.99 704	19
42	9.06 696	108	9.06 994	109	10.93 006	9.99 702	18
43	9.06 804	107	9.07 103	108	10.92 897	9.99 701	17
44	9.06 911	107	9.07 211	109	10.92 789	9.99 699	16
45	9.07 018	106	9.07 320	108	10.92 680	9.99 698	15
46	9.07 124	107	9.07 428	108	10.92 572	9.99 696	14
47	9.07 231	106	9.07 536	107	10.92 464	9.99 695	13
48	9.07 337	105	9.07 643	108	10.92 357	9.99 693	12
49	9.07 442	106	9.07 751	107	10.92 249	9.99 692	11
50	9.07 548	105	9.07 858	106	10.92 142	9.99 690	**10**
51	9.07 653	105	9.07 964	107	10.92 036	9.99 689	9
52	9.07 758	105	9.08 071	106	10.91 929	9.99 687	8
53	9.07 863	105	9.08 177	106	10.91 823	9.99 686	7
54	9.07 968	104	9.08 283	106	10.91 717	9.99 684	6
55	9.08 072	104	9.08 389	106	10.91 611	9.99 683	5
56	9.08 176	104	9.08 495	105	10.91 505	9.99 681	4
57	9.08 280	103	9.08 600	105	10.91 400	9.99 680	3
58	9.08 383	103	9.08 705	105	10.91 295	9.99 678	2
59	9.08 486	103	9.08 810	104	10.91 190	9.99 677	1
60	9.08 589		9.08 914		10.91 086	9.99 675	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

83°

Prop. Pts.

	121	120	119
1	12.1	12.0	11.9
2	24.2	24.0	23.8
3	36.3	36.0	35.7
4	48.4	48.0	47.6
5	60.5	60.0	59.5
6	72.6	72.0	71.4
7	84.7	84.0	83.3
8	96.8	96.0	95.2
9	108.9	108.0	107.1

	118	117	116
1	11.8	11.7	11.6
2	23.6	23.4	23.2
3	35.4	35.1	34.8
4	47.2	46.8	46.4
5	59.0	58.5	58.0
6	70.8	70.2	69.6
7	82.6	81.9	81.2
8	94.4	93.6	92.8
9	106.2	105.3	104.4

	115	114	113
1	11.5	11.4	11.3
2	23.0	22.8	22.6
3	34.5	34.2	33.9
4	46.0	45.6	45.2
5	57.5	57.0	56.5
6	69.0	68.4	67.8
7	80.5	79.8	79.1
8	92.0	91.2	90.4
9	103.5	102.6	101.7

	112	111	110
1	11.2	11.1	11.0
2	22.4	22.2	22.0
3	33.6	33.3	33.0
4	44.8	44.4	44.0
5	56.0	55.5	55.0
6	67.2	66.6	66.0
7	78.4	77.7	77.0
8	89.6	88.8	88.0
9	100.8	99.9	99.0

	109	108	107	106
1	10.9	10.8	10.7	10.6
2	21.8	21.6	21.4	21.2
3	32.7	32.4	32.1	31.8
4	43.6	43.2	42.8	42.4
5	54.5	54.0	53.5	53.0
6	65.4	64.8	64.2	63.6
7	76.3	75.6	74.9	74.2
8	87.2	86.4	85.6	84.8
9	98.1	97.2	96.3	95.4

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

7°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.08 589	103	9.08 914	105	10.91 086	9.99 675	**60**
1	9.08 692	103	9.09 019	104	10.90 981	9.99 674	59
2	9.08 795	102	9.09 123	104	10.90 877	9.99 672	58
3	9.08 897	102	9.09 227	103	10.90 773	9.99 670	57
4	9.08 999	102	9.09 330	104	10.90 670	9.99 669	56
5	9.09 101	101	9.09 434	103	10.90 566	9.99 667	55
6	9.09 202	102	9.09 537	103	10.90 463	9.99 666	54
7	9.09 304	101	9.09 640	102	10.90 360	9.99 664	53
8	9.09 405	101	9.09 742	103	10.90 258	9.99 663	52
9	9.09 506	100	9.09 845	102	10.90 155	9.99 661	51
10	9.09 606	101	9.09 947	102	10.90 053	9.99 659	**50**
11	9.09 707	100	9.10 049	101	10.89 951	9.99 658	49
12	9.09 807	100	9.10 150	102	10.89 850	9.99 656	48
13	9.09 907	99	9.10 252	101	10.89 748	9.99 655	47
14	9.10 006	100	9.10 353	101	10.89 647	9.99 653	46
15	9.10 106	99	9.10 454	101	10.89 546	9.99 651	45
16	9.10 205	99	9.10 555	101	10.89 445	9.99 650	44
17	9.10 304	98	9.10 656	100	10.89 344	9.99 648	43
18	9.10 402	99	9.10 756	100	10.89 244	9.99 647	42
19	9.10 501	98	9.10 856	100	10.89 144	9.99 645	41
20	9.10 599	98	9.10 956	100	10.89 044	9.99 643	**40**
21	9.10 697	98	9.11 056	99	10.88 944	9.99 642	39
22	9.10 795	98	9.11 155	99	10.88 845	9.99 640	38
23	9.10 893	97	9.11 254	99	10.88 746	9.99 638	37
24	9.10 990	97	9.11 353	99	10.88 647	9.99 637	36
25	9.11 087	97	9.11 452	99	10.88 548	9.99 635	35
26	9.11 184	97	9.11 551	98	10.88 449	9.99 633	34
27	9.11 281	96	9.11 649	98	10.88 351	9.99 632	33
28	9.11 377	97	9.11 747	98	10.88 253	9.99 630	32
29	9.11 474	96	9.11 845	98	10.88 155	9.99 629	31
30	9.11 570	96	9.11 943	97	10.88 057	9.99 627	**30**
31	9.11 666	95	9.12 040	98	10.87 960	9.99 625	29
32	9.11 761	96	9.12 138	97	10.87 862	9.99 624	28
33	9.11 857	95	9.12 235	97	10.87 765	9.99 622	27
34	9.11 952	95	9.12 332	96	10.87 668	9.99 620	26
35	9.12 047	95	9.12 428	97	10.87 572	9.99 618	25
36	9.12 142	94	9.12 525	96	10.87 475	9.99 617	24
37	9.12 236	95	9.12 621	96	10.87 379	9.99 615	23
38	9.12 331	94	9.12 717	96	10.87 283	9.99 613	22
39	9.12 425	94	9.12 813	96	10.87 187	9.99 612	21
40	9.12 519	93	9.12 909	95	10.87 091	9.99 610	**20**
41	9.12 612	94	9.13 004	95	10.86 996	9.99 608	19
42	9.12 706	93	9.13 099	95	10.86 901	9.99 607	18
43	9.12 799	93	9.13 194	95	10.86 806	9.99 605	17
44	9.12 892	93	9.13 289	95	10.86 711	9.99 603	16
45	9.12 985	93	9.13 384	94	10.86 616	9.99 601	15
46	9.13 078	93	9.13 478	95	10.86 522	9.99 600	14
47	9 13 171	92	9.13 573	94	10.86 427	9.99 598	13
48	9.13 263	92	9.13 667	94	10.86 333	9.99 596	12
49	9.13 355	92	9.13 761	93	10.86 239	9.99 595	11
50	9.13 447	92	9.13 854	94	10.86 146	9.99 593	**10**
51	9.13 539	91	9.13 948	93	10.86 052	9.99 591	9
52	9.13 630	92	9.14 041	93	10.85 959	9.99 589	8
53	9.13 722	91	9.14 134	93	10.85 866	9.99 588	7
54	9.13 813	91	9.14 227	93	10.85 773	9.99 586	6
55	9.13 904	90	9.14 320	92	10.85 680	9.99 584	5
56	9.13 994	91	9.14 412	92	10.85 588	9.99 582	4
57	9.14 085	90	9.14 504	93	10.85 496	9.99 581	3
58	9.14 175	91	9.14 597	91	10.85 403	9.99 579	2
59	9.14 266	90	9.14 688	92	10.85 312	9.99 577	1
60	9.14 356		9.14 780		10.85 220	9.99 575	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

Prop. Pts.

	105	**104**	**103**
1	10.5	10.4	10.3
2	21.0	20.8	20.6
3	31.5	31.2	30.9
4	42.0	41.6	41.2
5	52.5	52.0	51.5
6	63.0	62.4	61.8
7	73.5	72.8	72.1
8	84.0	83.2	82.4
9	94.5	93.6	92.7

	102	**101**	**99**
1	10.2	10.1	9.9
2	20.4	20.2	19.8
3	30.6	30.3	29.7
4	40.8	40.4	39.6
5	51.0	50.5	49.5
6	61.2	60.6	59.4
7	71.4	70.7	69.3
8	81.6	80.8	79.2
9	91.8	90.9	89.1

	98	**97**	**96**
1	9.8	9.7	9.6
2	19.6	19.4	19.2
3	29.4	29.1	28.8
4	39.2	38.8	38.4
5	49.0	48.5	48.0
6	58.8	58.2	57.6
7	68.6	67.9	67.2
8	78.4	77.6	76.8
9	88.2	87.3	86.4

	95	**94**	**93**
1	9.5	9.4	9.3
2	19.0	18.8	18.6
3	28.5	28.2	27.9
4	38.0	37.6	37.2
5	47.5	47.0	46.5
6	57.0	56.4	55.8
7	66.5	65.8	65.1
8	76.0	75.2	74.4
9	85.5	84.6	83.7

	92	**91**	**90**
1	9.2	9.1	9.0
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

Prop. Pts.

82°

Table 3-2. Common logarithms of trigonometric functions (continued)

8°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.14 356	89	9.14 780	92	10.85 220	9.99 575	**60**
1	9.14 445	90	9.14 872	91	10.85 128	9.99 574	59
2	9.14 535	89	9.14 963	91	10.85 037	9.99 572	58
3	9.14 624	90	9.15 054	91	10.84 946	9.99 570	57
4	9.14 714	89	9.15 145	91	10.84 855	9.99 568	56
5	9.14 803	88	9.15 236	91	10.84 764	9.99 566	55
6	9.14 891	89	9.15 327	90	10.84 673	9.99 565	54
7	9.14 980	89	9.15 417	91	10.84 583	9.99 563	53
8	9.15 069	88	9.15 508	90	10.84 492	9.99 561	52
9	9.15 157	88	9.15 598	90	10.84 402	9.99 559	51
10	9.15 245	88	9.15 688	89	10.84 312	9.99 557	**50**
11	9.15 333	88	9.15 777	90	10.84 223	9.99 556	49
12	9.15 421	87	9.15 867	89	10.84 133	9.99 554	48
13	9.15 508	88	9.15 956	90	10.84 044	9.99 552	47
14	9.15 596	87	9.16 046	89	10.83 954	9.99 550	46
15	9.15 683	87	9.16 135	89	10.83 865	9.99 548	45
16	9.15 770	87	9.16 224	88	10.83 776	9.99 546	44
17	9.15 857	87	9.16 312	89	10.83 688	9.99 545	43
18	9.15 944	86	9.16 401	88	10.83 599	9.99 543	42
19	9.16 030	86	9.16 489	88	10.83 511	9.99 541	41
20	9.16 116	87	9.16 577	88	10.83 423	9.99 539	**40**
21	9.16 203	86	9.16 665	88	10.83 335	9.99 537	39
22	9.16 289	85	9.16 753	88	10.83 247	9.99 535	38
23	9.16 374	86	9.16 841	87	10.83 159	9.99 533	37
24	9.16 460	85	9.16 928	88	10.83 072	9.99 532	36
25	9.16 545	86	9.17 016	87	10.82 984	9.99 530	35
26	9.16 631	85	9.17 103	87	10.82 897	9.99 528	34
27	9.16 716	85	9.17 190	87	10.82 810	9.99 526	33
28	9.16 801	85	9.17 277	86	10.82 723	9.99 524	32
29	9.16 886	84	9.17 363	87	10.82 637	9.99 522	31
30	9.16 970	85	9.17 450	86	10.82 550	9.99 520	**30**
31	9.17 055	84	9.17 536	86	10.82 464	9.99 518	29
32	9.17 139	84	9.17 622	86	10.82 378	9.99 517	28
33	9.17 223	84	9.17 708	86	10.82 292	9.99 515	27
34	9.17 307	84	9.17 794	86	10.82 206	9.99 513	26
35	9.17 391	83	9.17 880	85	10.82 120	9.99 511	25
36	9.17 474	84	9.17 965	86	10.82 035	9.99 509	24
37	9.17 558	83	9.18 051	85	10.81 949	9.99 507	23
38	9.17 641	83	9.18 136	85	10.81 864	9.99 505	22
39	9.17 724	83	9.18 221	85	10.81 779	9.99 503	21
40	9.17 807	83	9.18 306	85	10.81 694	9.99 501	**20**
41	9.17 890	83	9.18 391	84	10.81 609	9.99 499	19
42	9.17 983	82	9.18 475	85	10.81 525	9.99 497	18
43	9.18 055	82	9.18 560	84	10.81 440	9.99 495	17
44	9.18 137	83	9.18 644	84	10.81 356	9.99 494	16
45	9.18 220	82	9.18 728	84	10.81 272	9.99 492	15
46	9.18 302	81	9.18 812	84	10.81 188	9.99 490	14
47	9.18 383	82	9.18 896	83	10.81 104	9.99 488	13
48	9.18 465	82	9.18 979	84	10.81 021	9.99 486	12
49	9.18 547	81	9.19 063	83	10.80 937	9.99 484	11
50	9.18 628	81	9.19 146	83	10.80 854	9.99 482	**10**
51	9.18 709	81	9.19 229	83	10.80 771	9.99 480	9
52	9.18 790	81	9.19 312	83	10.80 688	9.99 478	8
53	9.18 871	81	9.19 395	83	10.80 605	9.99 476	7
54	9.18 952	81	9.19 478	83	10.80 522	9.99 474	6
55	9.19 033	80	9.19 561	82	10.80 439	9.99 472	5
56	9.19 113	80	9.19 643	82	10.80 357	9.99 470	4
57	9.19 193	80	9.19 725	82	10.80 275	9.99 468	3
58	9.19 273	80	9.19 807	82	10.80 193	9.99 466	2
59	9.19 353	80	9.19 889	82	10.80 111	9.99 464	1
60	9.19 433		9.19 971		10.80 029	9.99 462	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

Prop. Pts.

	92	91	90
1	9.2	9.1	9.0
2	18.4	18.2	18.0
3	27.6	27.3	27.0
4	36.8	36.4	36.0
5	46.0	45.5	45.0
6	55.2	54.6	54.0
7	64.4	63.7	63.0
8	73.6	72.8	72.0
9	82.8	81.9	81.0

	89	88	87
1	8.9	8.8	8.7
2	17.8	17.6	17.4
3	26.7	26.4	26.1
4	35.6	35.2	34.8
5	44.5	44.0	43.5
6	53.4	52.8	52.2
7	62.3	61.6	60.9
8	71.2	70.4	69.6
9	80.1	79.2	78.3

	86	85	84
1	8.6	8.5	8.4
2	17.2	17.0	16.8
3	25.8	25.5	25.2
4	34.4	34.0	33.6
5	43.0	42.5	42.0
6	51.6	51.0	50.4
7	60.2	59.5	58.8
8	68.8	68.0	67.2
9	77.4	76.5	75.6

	83	82	81	80
1	8.3	8.2	8.1	8.0
2	16.6	16.4	16.2	16.0
3	24.9	24.6	24.3	24.0
4	33.2	32.8	32.4	32.0
5	41.5	41.0	40.5	40.0
6	49.8	49.2	48.6	48.0
7	58.1	57.4	56.7	56.0
8	66.4	65.6	64.8	64.0
9	74.7	73.8	72.9	72.0

Prop. Pts.

81°

Table 3-2. Common logarithms of trigonometric functions (continued)

9°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0	9.19 433	80	9.19 971	82	10.80 029	9.99 462	**60**
1	9.19 513	79	9.20 053	81	10.79 947	9.99 460	59
2	9.19 592	80	9.20 134	82	10.79 866	9.99 458	58
3	9.19 672	79	9.20 216	81	10.79 784	9.99 456	57
4	9.19 751	79	9.20 297	81	10.79 703	9.99 454	56
5	9.19 830	79	9.20 378	81	10.79 622	9.99 452	55
6	9.19 909	79	9.20 459	81	10.79 541	9.99 450	54
7	9.19 988	79	9.20 540	81	10.79 460	9.99 448	53
8	9.20 067	78	9.20 621	80	10.79 379	9.99 446	52
9	9.20 145	78	9.20 701	81	10.79 299	9.99 444	51
10	9.20 223	79	9.20 782	80	10.79 218	9.99 442	**50**
11	9.20 302	78	9.20 862	80	10.79 138	9.99 440	49
12	9.20 380	78	9.20 942	80	10.79 058	9.99 438	48
13	9.20 458	77	9.21 022	80	10.78 978	9.99 436	47
14	9.20 535	78	9.21 102	80	10.78 898	9.99 434	46
15	9.20 613	78	9.21 182	79	10.78 818	9.99 432	45
16	9.20 691	77	9.21 261	80	10.78 739	9.99 429	44
17	9.20 768	77	9.21 341	79	10.78 659	9.99 427	43
18	9.20 845	77	9.21 420	79	10.78 580	9.99 425	42
19	9.20 922	77	9.21 499	79	10.78 501	9.99 423	41
20	9.20 999	77	9.21 578	79	10.78 422	9.99 421	**40**
21	9.21 076	77	9.21 657	79	10.78 343	9.99 419	39
22	9.21 153	76	9.21 736	78	10.78 264	9.99 417	38
23	9.21 229	77	9.21 814	79	10.78 186	9.99 415	37
24	9.21 306	76	9.21 893	78	10.78 107	9.99 413	36
25	9.21 382	76	9.21 971	78	10.78 029	9.99 411	35
26	9.21 458	76	9.22 049	78	10.77 951	9.99 409	34
27	9.21 534	76	9.22 127	78	10.77 873	9.99 407	33
28	9.21 610	75	9.22 205	78	10.77 795	9.99 404	32
29	9.21 685	76	9.22 283	78	10.77 717	9.99 402	31
30	9.21 761	75	9.22 361	77	10.77 639	9.99 400	**30**
31	9.21 836	76	9.22 438	78	10.77 562	9.99 398	29
32	9.21 912	75	9.22 516	77	10.77 484	9.99 396	28
33	9.21 987	75	9.22 593	77	10.77 407	9.99 394	27
34	9.22 062	75	9.22 670	77	10.77 330	9.99 392	26
35	9.22 137	74	9.22 747	77	10.77 253	9.99 390	25
36	9.22 211	75	9.22 824	77	10.77 176	9.99 388	24
37	9.22 286	75	9.22 901	76	10.77 099	9.99 385	23
38	9.22 361	74	9.22 977	77	10.77 023	9.99 383	22
39	9.22 435	74	9.23 054	76	10.76 946	9.99 381	21
40	9.22 509	74	9.23 130	76	10.76 870	9.99 379	**20**
41	9.22 583	74	9.23 206	77	10.76 794	9.99 377	19
42	9.22 657	74	9.23 283	76	10.76 717	9.99 375	18
43	9.22 731	74	9.23 359	76	10.76 641	9.99 372	17
44	9.22 805	73	9.23 435	75	10.76 565	9.99 370	16
45	9.22 878	74	9.23 510	76	10.76 490	9.99 368	15
46	9.22 952	73	9.23 586	75	10.76 414	9.99 366	14
47	9.23 025	73	9.23 661	76	10.76 339	9.99 364	13
48	9.23 098	73	9.23 737	75	10.76 263	9.99 362	12
49	9.23 171	73	9.23 812	75	10.76 188	9.99 359	11
50	9.23 244	73	9.23 887	75	10.76 113	9.99 357	**10**
51	9.23 317	73	9.23 962	75	10.76 038	9.99 355	9
52	9.23 390	72	9.24 037	75	10.75 963	9.99 353	8
53	9.23 462	73	9.24 112	74	10.75 888	9.99 351	7
54	9.23 535	72	9.24 186	75	10.75 814	9.99 348	6
55	9.23 607	72	9.24 261	74	10.75 739	9.99 346	5
56	9.23 679	73	9.24 335	75	10.75 665	9.99 344	4
57	9.23 752	71	9.24 410	74	10.75 590	9.99 342	3
58	9.23 823	72	9.24 484	74	10.75 516	9.99 340	2
59	9.23 895	72	9.24 558	74	10.75 442	9.99 337	1
60	9.23 967		9.24 632		10.75 368	9.99 335	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

80°

Prop. Pts.

	82	**81**	**80**
1	8.2	8.1	8.0
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

	79	**78**	**77**
1	7.9	7.8	7.7
2	15.8	15.6	15.4
3	23.7	23.4	23.1
4	31.6	31.2	30.8
5	39.5	39.0	38.5
6	47.4	46.8	46.2
7	55.3	54.6	53.9
8	63.2	62.4	61.6
9	71.1	70.2	69.3

	76	**75**	**74**
1	7.6	7.5	7.4
2	15.2	15.0	14.8
3	22.8	22.5	22.2
4	30.4	30.0	29.6
5	38.0	37.5	37.0
6	45.6	45.0	44.4
7	53.2	52.5	51.8
8	60.8	60.0	59.2
9	68.4	67.5	66.6

	73	**72**	**71**
1	7.3	7.2	7.1
2	14.6	14.4	14.2
3	21.9	21.6	21.3
4	29.2	28.8	28.4
5	36.5	36.0	35.5
6	43.8	43.2	42.6
7	51.1	50.4	49.7
8	58.4	57.6	56.8
9	65.7	64.8	63.9

	3	**2**
1	0.3	0.2
2	0.6	0.4
3	0.9	0.6
4	1.2	0.8
5	1.5	1.0
6	1.8	1.2
7	2.1	1.4
8	2.4	1.6
9	2.7	1.8

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

10°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.23 967	72	9.24 632	74	10.75 368	9.99 335	2	**60**
1	9.24 039	71	9.24 706	73	10.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	10.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	10.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	10.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	10.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	10.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	10.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	10.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	10.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	10.74 635	9.99 313	3	**50**
11	9.24 748	70	9.25 437	73	10.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	10.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	10.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	10.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	10.74 273	9.99 301	2	45
16	9.25 098	70	9.25 799	72	10.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	10.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	10.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	10.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	10.73 914	9.99 290	2	**40**
21	9.25 445	69	9.26 158	71	10.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	10.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	10.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	10.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	10.73 557	9.99 278	2	35
26	9.25 790	68	9.26 514	71	10.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	10.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	10.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	10.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	10.73 203	9.99 267	3	**30**
31	9.26 131	68	9.26 867	70	10.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	10.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	10.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	10.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	10.72 852	9.99 255	3	25
36	9.26 470	68	9.27 218	70	10.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	10.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	10.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	10.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	10.72 504	9.99 243	2	**20**
41	9.26 806	67	9.27 566	69	10.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	10.72 365	9.99 238	2	18
43	9.26 940	67	9.27 704	69	10.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	10.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	10.72 158	9.99 231	2	15
46	9.27 140	66	9.27 911	69	10.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	10.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	10.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	10.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	10.71 814	9.99 219	2	**10**
51	9.27 471	66	9.28 254	69	10.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	10.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	10.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	10.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	10.71 473	9.99 207	3	5
56	9.27 799	65	9.28 595	67	10.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	10.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	10.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	10.71 202	9.99 197	2	1
60	9.28 060		9.28 865		10.71 135	9.99 195		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	74	73	72
1	7.4	7.3	7.2
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	70	69
1	7.1	7.0	6.9
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	67	66
1	6.8	6.7	6.6
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	3	2
1	6.5	0.3	0.2
2	13.0	0.6	0.4
3	19.5	0.9	0.6
4	26.0	1.2	0.8
5	32.5	1.5	1.0
6	39.0	1.8	1.2
7	45.5	2.1	1.4
8	52.0	2.4	1.6
9	58.5	2.7	1.8

Prop. Pts.

79°

Table 3-2. Common logarithms of trigonometric functions (continued)

11°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.28 060	65	9.28 865	68	10.71 135	9.99 195	3	**60**
1	9.28 125	65	9.28 933	67	10.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	10.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	10.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	10.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	10.70 799	9.99 182	2	55
6	9.28 448	64	9.29 268	67	10.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	10.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	10.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	10.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	10.70 465	9.99 170	3	**50**
11	9.28 769	64	9.29 601	67	10.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	10.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	10.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	10.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	10.70 134	9.99 157	2	45
16	9.29 087	63	9.29 932	66	10.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	10.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	10.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	10.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	10.69 805	9.99 145	3	**40**
21	9.29 403	63	9.30 261	65	10.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	10.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	10.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	10.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	10.69 478	9.99 132	2	35
26	9.29 716	63	9.30 587	65	10.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	10.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	10.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	10.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	10.69 154	9.99 119	2	**30**
31	9.30 028	62	9.30 911	64	10.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	10.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	10.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	10.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	10.68 832	9.99 106	2	25
36	9.30 336	62	9.31 233	64	10.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	10.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	10.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	10.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	10.68 511	9.99 093	2	**20**
41	9.30 643	61	9.31 552	64	10.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	10.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	10.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	10.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	10.68 194	9.99 080	2	15
46	9.30 947	61	9.31 870	63	10.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	10.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	10.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	10.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	10.67 878	9.99 067	3	**10**
51	9.31 250	60	9.32 185	63	10.67 815	9.99 064	2	9
52	9.31 310	60	9.32 248	63	10.67 752	9.99 062	3	8
53	9.31 370	60	9.32 311	62	10.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	63	10.67 627	9.99 056	2	6
55	9.31 490	59	9.32 436	62	10.67 564	9.99 054	3	5
56	9.31 549	60	9.32 498	63	10.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	10.67 439	9.99 048	2	3
58	9.31 669	59	9.32 623	62	10.67 377	9.99 046	8	2
59	9.31 728	60	9.32 685	62	10.67 315	9.99 043	3	1
60	9.31 788		9.32 747		10.67 253	9.99 040		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

78°

Prop. Pts.

	68	**67**	**66**
1	6.8	6.7	6.6
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	**64**	**63**
1	6.5	6.4	6.3
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7

	62	**61**	**60**
1	6.2	6.1	6.0
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0

	59	**3**	**2**
1	5.9	0.3	0.2
2	11.8	0.6	0.4
3	17.7	0.9	0.6
4	23.6	1.2	0.8
5	29.5	1.5	1.0
6	35.4	1.8	1.2
7	41.3	2.1	1.4
8	47.2	2.4	1.6
9	53.1	2.7	1.8

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

12°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.31 788		9.32 747		10.67 253	9.99 040		**60**
1	9.31 847	59	9.32 810	63	10.67 190	9.99 038	2	59
2	9.31 907	60	9.32 872	62	10.67 128	9.99 035	3	58
3	9.31 966	59	9.32 933	61	10.67 067	9.99 032	3	57
4	9.32 025	59	9.32 995	62	10.67 005	9.99 030	2	56
5	9.32 084	59	9.33 057	62	10.66 943	9.99 027	3	55
6	9.32 143	59	9.33 119	62	10.66 881	9.99 024	3	54
7	9.32 202	59	9.33 180	61	10.66 820	9.99 022	2	53
8	9.32 261	59	9.33 242	62	10.66 758	9.99 019	3	52
9	9.32 319	58	9.33 303	61	10.66 697	9.99 016	3	51
10	9.32 378	59	9.33 365	62	10.66 635	9.99 013	3	**50**
11	9.32 437	59	9.33 426	61	10.66 574	9.99 011	2	49
12	9.32 495	58	9.33 487	61	10.66 513	9.99 008	3	48
13	9.32 553	58	9.33 548	61	10.66 452	9.99 005	3	47
14	9.32 612	59	9.33 609	61	10.66 391	9.99 002	3	46
15	9.32 670	58	9.33 670	61	10.66 330	9.99 000	2	45
16	9.32 728	58	9.33 731	61	10.66 269	9.98 997	3	44
17	9.32 786	58	9.33 792	61	10.66 208	9.98 994	3	43
18	9.32 844	58	9.33 853	61	10.66 147	9.98 991	3	42
19	9.32 902	58	9.33 913	60	10.66 087	9.98 989	2	41
20	9.32 960	58	9.33 974	61	10.66 026	9.98 986	3	**40**
21	9.33 018	58	9.34 034	60	10.65 966	9.98 983	3	39
22	9.33 075	57	9.34 095	61	10.65 905	9.98 980	3	38
23	9.33 133	58	9.34 155	60	10.65 845	9.98 978	2	37
24	9.33 190	57	9.34 215	60	10.65 785	9.98 975	3	36
25	9.33 248	58	9.34 276	61	10.65 724	9.98 972	3	35
26	9.33 305	57	9.34 336	60	10.65 664	9.98 969	3	34
27	9.33 362	57	9.34 396	60	10.65 604	9.98 967	2	33
28	9.33 420	58	9.34 456	60	10.65 544	9.98 964	3	32
29	9.33 477	57	9.34 516	60	10.65 484	9.98 961	3	31
30	9.33 534	57	9.34 576	60	10.65 424	9.98 958	3	**30**
31	9.33 591	57	9.34 635	59	10.65 365	9.98 955	3	29
32	9.33 647	56	9.34 695	60	10.65 305	9.98 953	2	28
33	9.33 704	57	9.34 755	60	10.65 245	9.98 950	3	27
34	9.33 761	57	9.34 814	59	10.65 186	9.98 947	3	26
35	9.33 818	57	9.34 874	60	10.65 126	9.98 944	3	25
36	9.33 874	56	9.34 933	59	10.65 067	9.98 941	3	24
37	9.33 931	57	9.34 992	59	10.65 008	9.98 938	3	23
38	9.33 987	56	9.35 051	59	10.64 949	9.98 936	2	22
39	9.34 043	56	9.35 111	60	10.64 889	9.98 933	3	21
40	9.34 100	57	9.35 170	59	10.64 830	9.98 930	3	**20**
41	9.34 156	56	9.35 229	59	10.64 771	9.98 927	3	19
42	9.34 212	56	9.35 288	59	10.64 712	9.98 924	3	18
43	9.34 268	56	9.35 347	59	10.64 653	9.98 921	3	17
44	9.34 324	56	9.35 405	58	10.64 595	9.98 919	2	16
45	9.34 380	56	9.35 464	59	10.64 536	9.98 916	3	15
46	9.34 436	56	9.35 523	59	10.64 477	9.98 913	3	14
47	9.34 491	55	9.35 581	58	10.64 419	9.98 910	3	13
48	9.34 547	56	9.35 640	59	10.64 360	9.98 907	3	12
49	9.34 602	55	9.35 698	58	10.64 302	9.98 904	3	11
50	9.34 658	56	9.35 757	59	10.64 243	9.98 901	3	**10**
51	9.34 713	55	9.35 815	58	10.64 185	9.98 898	3	9
52	9.34 769	56	9.35 873	58	10.64 127	9.98 896	2	8
53	9.34 824	55	9.35 931	58	10.64 069	9.98 893	3	7
54	9.34 879	55	9.35 989	58	10.64 011	9.98 890	3	6
55	9.34 934	55	9.36 047	58	10.63 953	9.98 887	3	5
56	9.34 989	55	9.36 105	58	10.63 895	9.98 884	3	4
57	9.35 044	55	9.36 163	58	10.63 837	9.98 881	3	3
58	9.35 099	55	9.36 221	58	10.63 779	9.98 878	3	2
59	9.35 154	55	9.36 279	58	10.63 721	9.98 875	3	1
60	9.35 209	55	9.36 336	57	10.63 664	9.98 872	3	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

77°

Prop. Pts.

	63	**62**	**61**
1	6.3	6.2	6.1
2	12.6	12.4	12.2
3	18.9	18.6	18.3
4	25.2	24.8	24.4
5	31.5	31.0	30.5
6	37.8	37.2	36.6
7	44.1	43.4	42.7
8	50.4	49.6	48.8
9	56.7	55.8	54.9

	60	**59**	**58**
1	6.0	5.9	5.8
2	12.0	11.8	11.6
3	18.0	17.7	17.4
4	24.0	23.6	23.2
5	30.0	29.5	29.0
6	36.0	35.4	34.8
7	42.0	41.3	40.6
8	48.0	47.2	46.4
9	54.0	53.1	52.2

	57	**56**	**55**
1	5.7	5.6	5.5
2	11.4	11.2	11.0
3	17.1	16.8	16.5
4	22.8	22.4	22.0
5	28.5	28.0	27.5
6	34.2	33.6	33.0
7	39.9	39.2	38.5
8	45.6	44.8	44.0
9	51.3	50.4	49.5

	3	**2**
1	0.3	0.2
2	0.6	0.4
3	0.9	0.6
4	1.2	0.8
5	1.5	1.0
6	1.8	1.2
7	2.1	1.4
8	2.4	1.6
9	2.7	1.8

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

13°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.35 209		9.36 336		10.63 664	9.98 872		60
1	9.35 263	54	9.36 394	58	10.63 606	9.98 869	3	59
2	9.35 318	55	9.36 452	58	10.63 548	9.98 867	2	58
3	9.35 373	55	9.36 509	57	10.63 491	9.98 864	3	57
4	9.35 427	54	9.36 566	57	10.63 434	9.98 861	3	56
5	9.35 481	54	9.36 624	58	10.63 376	9.98 858	3	55
6	9.35 536	55	9.36 681	57	10.63 319	9.98 855	3	54
7	9.35 590	54	9.36 738	57	10.63 262	9.98 852	3	53
8	9.35 644	54	9.36 795	57	10.63 205	9.98 849	3	52
9	9.35 698	54	9.36 852	57	10.63 148	9.98 846	3	51
10	9.35 752	54	9.36 909	57	10.63 091	9.98 843	3	50
11	9.35 806	54	9.36 966	57	10.63 034	9.98 840	3	49
12	9.35 860	54	9.37 023	57	10.62 977	9.98 837	3	48
13	9.35 914	54	9.37 080	57	10.62 920	9.98 834	3	47
14	9.35 968	54	9.37 137	57	10.62 863	9.98 831	3	46
15	9.36 022	54	9.37 193	56	10.62 807	9.98 828	3	45
16	9.36 075	53	9.37 250	57	10.62 750	9.98 825	3	44
17	9.36 129	54	9.37 306	56	10.62 694	9.98 822	3	43
18	9.36 182	53	9.37 363	57	10.62 637	9.98 819	3	42
19	9.36 236	54	9.37 419	56	10.62 581	9.98 816	3	41
20	9.36 289	53	9.37 476	57	10.62 524	9.98 813	3	40
21	9.36 342	53	9.37 532	56	10.62 468	9.98 810	3	39
22	9.36 395	53	9.37 588	56	10.62 412	9.98 807	3	38
23	9.36 449	54	9.37 644	56	10.62 356	9.98 804	3	37
24	9.36 502	53	9.37 700	56	10.62 300	9.98 801	3	36
25	9.36 555	53	9.37 756	56	10.62 244	9.98 798	3	35
26	9.36 608	53	9.37 812	56	10.62 188	9.98 795	3	34
27	9.36 660	52	9.37 868	56	10.62 132	9.98 792	3	33
28	9.36 713	53	9.37 924	56	10.62 076	9.98 789	3	32
29	9.36 766	53	9.37 980	56	10.62 020	9.98 786	3	31
30	9.36 819	53	9.38 035	55	10.61 965	9.98 783	3	30
31	9.36 871	52	9.38 091	56	10.61 909	9.98 780	3	29
32	9.36 924	53	9.38 147	56	10.61 853	9.98 777	3	28
33	9.36 976	52	9.38 202	55	10.61 798	9.98 774	3	27
34	9.37 028	52	9.38 257	55	10.61 743	9.98 771	3	26
35	9.37 081	53	9.38 313	56	10.61 687	9.98 768	3	25
36	9.37 133	52	9.38 368	55	10.61 632	9.98 765	3	24
37	9.37 185	52	9.38 423	55	10.61 577	9.98 762	3	23
38	9.37 237	52	9.38 479	56	10.61 521	9.98 759	3	22
39	9.37 289	52	9.38 534	55	10.61 466	9.98 756	3	21
40	9.37 341	52	9.38 589	55	10.61 411	9.98 753	3	20
41	9.37 393	52	9.38 644	55	10.61 356	9.98 750	3	19
42	9.37 445	52	9.38 699	55	10.61 301	9.98 746	4	18
43	9.37 497	52	9.38 754	55	10.61 246	9.98 743	3	17
44	9.37 549	52	9.38 808	54	10.61 192	9.98 740	3	16
45	9.37 600	51	9.38 863	55	10.61 137	9.98 737	3	15
46	9.37 652	52	9.38 918	55	10.61 082	9.98 734	3	14
47	9.37 703	51	9.38 972	54	10.61 028	9.98 731	3	13
48	9.37 755	52	9.39 027	55	10.60 973	9.98 728	3	12
49	9.37 806	51	9.39 082	55	10.60 918	9.98 725	3	11
50	9.37 858	52	9.39 136	54	10.60 864	9.98 722	3	10
51	9.37 909	51	9.39 190	54	10.60 810	9.98 719	3	9
52	9.37 960	51	9.39 245	55	10.60 755	9.98 715	4	8
53	9.38 011	51	9.39 299	54	10.60 701	9.98 712	3	7
54	9.38 062	51	9.39 353	54	10.60 647	9.98 709	3	6
55	9.38 113	51	9.39 407	54	10.60 593	9.98 706	3	5
56	9.38 164	51	9.39 461	54	10.60 539	9.98 703	3	4
57	9.38 215	51	9.39 515	54	10.60 485	9.98 700	3	3
58	9.38 266	51	9.39 569	54	10.60 431	9.98 697	3	2
59	9.38 317	51	9.39 623	54	10.60 377	9.98 694	3	1
60	9.38 368	51	9.39 677	54	10.60 323	9.98 690	4	0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	58	57	56
1	5.8	5.7	5.6
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	54	53
1	5.5	5.4	5.3
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
1	5.2	5.1
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	3	2
1	0.4	0.3	0.2
2	0.8	0.6	0.4
3	1.2	0.9	0.6
4	1.6	1.2	0.8
5	2.0	1.5	1.0
6	2.4	1.8	1.2
7	2.8	2.1	1.4
8	3.2	2.4	1.6
9	3.6	2.7	1.8

Prop. Pts.

76°

Table 3-2. Common logarithms of trigonometric functions (continued)

14°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Pts.
0	9.38 368		9.39 677		10.60 323	9.98 690		60	
1	9.38 418	50	9.39 731	54	10.60 269	9.98 687	3	59	
2	9.38 469	51	9.39 785	54	10.60 215	9.98 684	3	58	
3	9.38 519	50	9.39 838	53	10.60 162	9.98 681	3	57	
4	9.38 570	51	9.39 892	54	10.60 108	9.98 678	3	56	
5	9.38 620	50	9.39 945	53	10.60 055	9.98 675	3	55	
6	9.38 670	50	9.39 999	54	10.60 001	9.98 671	4	54	
7	9.38 721	51	9.40 052	53	10.59 948	9.98 668	3	53	
8	9.38 771	50	9.40 106	54	10.59 894	9.98 665	3	52	
9	9.38 821	50	9.40 159	53	10.59 841	9.98 662	3	51	
10	9.38 871	50	9.40 212	53	10.59 788	9.98 659	3	50	
11	9.38 921	50	9.40 266	54	10.59 734	9.98 656	3	49	
12	9.38 971	50	9.40 319	53	10.59 681	9.98 652	4	48	
13	9.39 021	50	9.40 372	53	10.59 628	9.98 649	3	47	
14	9.39 071	50	9.40 425	53	10.59 575	9.98 646	3	46	
15	9.39 121	50	9.40 478	53	10.59 522	9.98 643	3	45	
16	9.39 170	49	9.40 531	53	10.59 469	9.98 640	3	44	
17	9.39 220	50	9.40 584	53	10.59 416	9.98 636	4	43	
18	9.39 270	50	9.40 636	52	10.59 364	9.98 633	3	42	
19	9.39 319	49	9.40 689	53	10.59 311	9.98 630	3	41	
20	9.39 369	50	9.40 742	53	10.59 258	9.98 627	3	40	
21	9.39 418	49	9.40 795	53	10.59 205	9.98 623	4	39	
22	9.39 467	49	9.40 847	52	10.59 153	9.98 620	3	38	
23	9.39 517	50	9.40 900	53	10.59 100	9.98 617	3	37	
24	9.39 566	49	9.40 952	52	10.59 048	9.98 614	3	36	
25	9.39 615	49	9.41 005	53	10.58 995	9.98 610	4	35	
26	9.39 664	49	9.41 057	52	10.58 943	9.98 607	3	34	
27	9.39 713	49	9.41 109	52	10.58 891	9.98 604	3	33	
28	9.39 762	49	9.41 161	52	10.58 839	9.98 601	3	32	
29	9.39 811	49	9.41 214	53	10.58 786	9.98 597	4	31	
30	9.39 860	49	9.41 266	52	10.58 734	9.98 594	3	30	
31	9.39 909	49	9.41 318	52	10.58 682	9.98 591	3	29	
32	9.93 958	49	9.41 370	52	10.58 630	9.98 588	3	28	
33	9.40 006	48	9.41 422	52	10.58 578	9.98 584	4	27	
34	9.40 055	49	9.41 474	52	10.58 526	9.98 581	3	26	
35	9.40 103	48	9.41 526	52	10.58 474	9.98 578	3	25	
36	9.40 152	49	9.41 578	52	10.58 422	9.98 574	4	24	
37	9.40 200	48	9.41 629	51	10.58 371	9.98 571	3	23	
38	9.40 249	49	9.41 681	52	10.58 319	9.98 568	3	22	
39	9.40 297	48	9.41 733	52	10.58 267	9.98 565	3	21	
40	9.40 346	49	9.41 784	51	10.58 216	9.98 561	4	20	
41	9.40 394	48	9.41 836	52	10.58 164	9.98 558	3	19	
42	9.40 442	48	9.41 887	51	10.58 113	9.98 555	3	18	
43	9.40 490	48	9.41 939	52	10.58 061	9.98 551	4	17	
44	9.40 538	48	9.41 990	51	10.58 010	9.98 548	3	16	
45	9.40 586	48	9.42 041	51	10.57 959	9.98 545	3	15	
46	9.40 634	48	9.42 093	52	10.57 907	9.98 541	4	14	
47	9.40 682	48	9.42 144	51	10.57 856	9.98 538	3	13	
48	9.40 730	48	9.42 195	51	10.57 805	9.98 535	3	12	
49	9.40 778	48	9.42 246	51	10.57 754	9.98 531	4	11	
50	9.40 825	47	9.42 297	51	10.57 703	9.98 528	3	10	
51	9.40 873	48	9.42 348	51	10.57 652	9.98 525	3	9	
52	9.40 921	48	9.42 399	51	10.57 601	9.98 521	4	8	
53	9.40 968	47	9.42 450	51	10.57 550	9.98 518	3	7	
54	9.41 016	48	9.42 501	51	10.57 499	9.98 515	3	6	
55	9.41 063	47	9.42 552	51	10.57 448	9.98 511	4	5	
56	9.41 111	48	9.42 603	51	10.57 397	9.98 508	3	4	
57	9.41 158	47	9.42 653	50	10.57 347	9.98 505	3	3	
58	9.41 205	47	9.42 704	51	10.57 296	9.98 501	4	2	
59	9.41 252	47	9.42 755	51	10.57 245	9.98 498	3	1	
60	9.41 300	48	9.42 805	50	10.57 195	9.98 494	4	0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	Prop. Pts.

Prop. Pts.

	54	53	52
1	5.4	5.3	5.2
2	10.8	10.6	10.4
3	16.2	15.9	15.6
4	21.6	21.2	20.8
5	27.0	26.5	26.0
6	32.4	31.8	31.2
7	37.8	37.1	36.4
8	43.2	42.4	41.6
9	48.6	47.7	46.8

	51	50	49
1	5.1	5.0	4.9
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47
1	4.8	4.7
2	9.6	9.4
3	14.4	14.1
4	19.2	18.8
5	24.0	23.5
6	28.8	28.2
7	33.6	32.9
8	38.4	37.6
9	43.2	42.3

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

75°

Table 3-2. Common logarithms of trigonometric functions (continued)

15°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Pts.
0	9.41 300	47	9.42 805	51	10.57 195	9.98 494	3	60	
1	9.41 347	47	9.42 856	50	10.57 144	9.98 491	3	59	
2	9.41 394	47	9.42 906	51	10.57 094	9.98 488	4	58	
3	9.41 441	47	9.42 957	50	10.57 043	9.98 484	3	57	
4	9.41 488	47	9.43 007	50	10.56 993	9.98 481	4	56	
5	9.41 535	47	9.43 057	51	10.56 943	9.98 477	3	55	
6	9.41 582	46	9.43 108	50	10.56 892	9.98 474	3	54	
7	9.41 628	47	9.43 158	50	10.56 842	9.98 471	4	53	
8	9.41 675	47	9.43 208	50	10.56 792	9.98 467	3	52	
9	9.41 722	46	9.43 258	50	10.56 742	9.98 464	4	51	
10	9.41 768	47	9.43 308	50	10.56 692	9.98 460	3	**50**	
11	9.41 815	46	9.43 358	50	10.56 642	9.98 457	4	49	
12	9.41 861	47	9.43 408	50	10.56 592	9.98 453	3	48	
13	9.41 908	46	9.43 458	50	10.56 542	9.98 450	3	47	
14	9.41 954	47	9.43 508	50	10.56 492	9.98 447	4	46	
15	9.42 001	46	9.43 558	49	10.56 442	9.98 443	3	45	
16	9.42 047	46	9.43 607	50	10.56 393	9.98 440	4	44	
17	9.42 093	47	9.43 657	50	10.56 343	9.98 436	3	43	
18	9.42 140	46	9.43 707	49	10.56 293	9.98 433	4	42	
19	9.42 186	46	9.43 756	50	10.56 244	9.98 429	3	41	
20	9.42 232	46	9.43 806	49	10.56 194	9.98 426	4	**40**	
21	9.42 278	46	9.43 855	50	10.56 145	9.98 422	3	39	
22	9.42 324	46	9.43 905	49	10.56 095	9.98 419	4	38	
23	9.42 370	46	9.43 954	50	10.56 046	9.98 415	3	37	
24	9.42 416	45	9.44 004	49	10.55 996	9.98 412	3	36	
25	9.42 461	46	9.44 053	49	10.55 947	9.98 409	4	35	
26	9.42 507	46	9.44 102	49	10.55 898	9.98 405	3	34	
27	9.42 553	46	9.44 151	50	10.55 849	9.98 402	4	33	
28	9.42 599	45	9.44 201	49	10.55 799	9.98 398	3	32	
29	9.42 644	46	9.44 250	49	10.55 750	9.98 395	4	31	
30	9.42 690	45	9.44 299	49	10.55 701	9.98 391	3	**30**	
31	9.42 735	46	9.44 348	49	10.55 652	9.98 388	4	29	
32	9.42 781	45	9.44 397	49	10.55 603	9.98 384	3	28	
33	9.42 826	46	9.44 446	49	10.55 554	9.98 381	4	27	
34	9.42 872	45	9.44 495	49	10.55 505	9.98 377	4	26	
35	9.42 917	45	9.44 544	48	10.55 456	9.98 373	3	25	
36	9.42 962	46	9.44 592	49	10.55 408	9.98 370	4	24	
37	9.43 008	45	9.44 641	49	10.55 359	9.98 366	3	23	
38	9.43 053	45	9.44 690	48	10.55 310	9.98 363	4	22	
39	9.43 098	45	9.44 738	49	10.55 262	9.98 359	3	21	
40	9.43 143	45	9.44 787	49	10.55 213	9.98 356	4	**20**	
41	9.43 188	45	9.44 836	48	10.55 164	9.98 352	3	19	
42	9.43 233	45	9.44 884	49	10.55 116	9.98 349	4	18	
43	9.43 278	45	9.44 933	48	10.55 067	9.98 345	3	17	
44	9.43 323	44	9.44 981	48	10.55 019	9.98 342	4	16	
45	9.43 367	45	9.45 029	49	10.54 971	9.98 338	4	15	
46	9.43 412	45	9.45 078	48	10.54 922	9.98 334	3	14	
47	9.43 457	45	9.45 126	48	10.54 874	9.98 331	4	13	
48	9.43 502	44	9.45 174	48	10.54 826	9.98 327	3	12	
49	9.43 546	45	9.45 222	49	10.54 778	9.98 324	4	11	
50	9.43 591	44	9.45 271	48	10.54 729	9.98 320	3	**10**	
51	9.43 635	45	9.45 319	48	10.54 681	9.98 317	4	9	
52	9.43 680	44	9.45 367	48	10.54 633	9.98 313	4	8	
53	9.43 724	45	9.45 415	48	10.54 585	9.98 309	3	7	
54	9.43 769	44	9.45 463	48	10.54 537	9.98 306	4	6	
55	9.43 813	44	9.45 511	48	10.54 489	9.98 302	3	5	
56	9.43 857	44	9.45 559	47	10.54 441	9.98 299	4	4	
57	9.43 901	45	9.45 606	48	10.54 394	9.98 295	4	3	
58	9.43 946	44	9.45 654	48	10.54 346	9.98 291	3	2	
59	9.43 990	44	9.45 702	48	10.54 298	9.98 288	4	1	
60	9.44 034		9.45 750		10.54 250	9.98 284		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	Prop. Pts.

	51	50	49
1	5.1	5.0	4.9
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47	46
1	4.8	4.7	4.6
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44
1	4.5	4.4
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

74°

Table 3-2. Common logarithms of trigonometric functions (continued)

16°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.44 034	44	9.45 750	47	10.54 250	9.98 284	3	**60**
1	9.44 078	44	9.45 797	48	10.54 203	9.98 281	4	59
2	9.44 122	44	9.45 845	47	10.54 155	9.98 277	4	58
3	9.44 166	44	9.45 892	48	10.54 108	9.98 273	3	57
4	9.44 210	43	9.45 940	47	10.54 060	9.98 270	4	56
5	9.44 253	44	9.45 987	48	10.54 013	9.98 266	4	55
6	9.44 297	44	9.46 035	47	10.53 965	9.98 262	3	54
7	9.44 341	44	9.46 082	48	10.53 918	9.98 259	4	53
8	9.44 385	43	9.46 130	47	10.53 870	9.98 255	4	52
9	9.44 428	44	9.46 177	47	10.53 823	9.98 251	3	51
10	9.44 472	44	9.46 224	47	10.53 776	9.98 248	4	**50**
11	9.44 516	43	9.46 271	48	10.53 729	9.98 244	4	49
12	9.44 559	43	9.46 319	47	10.53 681	9.98 240	3	48
13	9.44 602	44	9.46 366	47	10.53 634	9.98 237	4	47
14	9.44 646	43	9.46 413	47	10.53 587	9.98 233	4	46
15	9.44 689	44	9.46 460	47	10.53 540	9.98 229	3	45
16	9.44 733	43	9.46 507	47	10.53 493	9.98 226	4	44
17	9.44 776	43	9.46 554	47	10.53 446	9.98 222	4	43
18	9.44 819	43	9.46 601	47	10.53 399	9.98 218	3	42
19	9.44 862	43	9.46 648	46	10.53 352	9.98 215	4	41
20	9.44 905	43	9.46 694	47	10.53 306	9.98 211	4	**40**
21	9.44 948	44	9.46 741	47	10.53 259	9.98 207	3	39
22	9.44 992	43	9.46 788	47	10.53 212	9.98 204	4	38
23	9.45 035	42	9.46 835	46	10.53 165	9.98 200	4	37
24	9.45 077	43	9.46 881	47	10.53 119	9.98 196	4	36
25	9.45 120	43	9.46 928	47	10.53 072	9.98 192	3	35
26	9.45 163	43	9.46 975	46	10.53 025	9.98 189	4	34
27	9.45 206	43	9.47 021	47	10.52 979	9.98 185	4	33
28	9.45 249	43	9.47 068	46	10.52 932	9.98 181	4	32
29	9.45 292	42	9.47 114	46	10.52 886	9.98 177	3	31
30	9.45 334	43	9.47 160	47	10.52 840	9.98 174	4	**30**
31	9.45 377	42	9.47 207	46	10.52 793	9.98 170	4	29
32	9.45 419	43	9.47 253	46	10.52 747	9.98 166	4	28
33	9.45 462	42	9.47 299	47	10.52 701	9.98 162	3	27
34	9.45 504	43	9.47 346	46	10.52 654	9.98 159	4	26
35	9.45 547	42	9.47 392	46	10.52 608	9.98 155	4	25
36	9.45 589	43	9.47 438	46	10.52 562	9.98 151	4	24
37	9.45 632	42	9.47 484	46	10.52 516	9.98 147	3	23
38	9.45 674	42	9.47 530	46	10.52 470	9.98 144	4	22
39	9.45 716	42	9.47 576	46	10.52 424	9.98 140	4	21
40	9.45 758	43	9.47 622	46	10.52 378	9.98 136	4	**20**
41	9.45 801	42	9.47 668	46	10.52 332	9.98 132	3	19
42	9.45 843	42	9.47 714	46	10.52 286	9.98 129	4	18
43	9.45 885	42	9.47 760	46	10.52 240	9.98 125	4	17
44	9.45 927	42	9.47 806	46	10.52 194	9.98 121	4	16
45	9.45 969	42	9.47 852	45	10.52 148	9.98 117	4	15
46	9.46 011	42	9.47 897	46	10.52 103	9.98 113	3	14
47	9.46 053	42	9.47 943	46	10.52 057	9.98 110	4	13
48	9.46 095	41	9.47 989	46	10.52 011	9.98 106	4	12
49	9.46 136	42	9.48 035	45	10.51 965	9.98 102	4	11
50	9.46 178	42	9.48 080	46	10.51 920	9.98 098	4	**10**
51	9.46 220	42	9.48 126	45	10.51 874	9.98 094	4	9
52	9.46 262	41	9.48 171	46	10.51 829	9.98 090	3	8
53	9.46 303	42	9.48 217	45	10.51 783	9.98 087	4	7
54	9.46 345	41	9.48 262	45	10.51 738	9.98 083	4	6
55	9.46 386	42	9.48 307	46	10.51 693	9.98 079	4	5
56	9.46 428	41	9.48 353	45	10.51 647	9.98 075	4	4
57	9.46 469	42	9.48 398	45	10.51 602	9.98 071	4	3
58	9.46 511	41	9.48 443	46	10.51 557	9.98 067	4	2
59	9.46 552	42	9.48 489	45	10.51 511	9.98 063	3	1
60	9.46 594		9.48 534		10.51 466	9.98 060		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

73°

Prop. Pts.

	48	47	46
1	4.8	4.7	4.6
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44	43
1	4.5	4.4	4.3
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
1	4.2	4.1
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	3
1	0.4	0.3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

17°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.46 594	41	9.48 534	45	10.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	10.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	10.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	10.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	10.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	10.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	10.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	10.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	10.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	10.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	10.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	10.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	10.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	10.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	10.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	10.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	10.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	10.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	10.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	10.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	10.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	10.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	10.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	10.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	10.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	10.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	10.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	10.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	10.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	10.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	10.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	10.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	10.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	10.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	10.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	10.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	10.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	10.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	10.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	10.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	10.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	10.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	10.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	10.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	10.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	10.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	10.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	10.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	10.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	10.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	10.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	10.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	10.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	10.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	10.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	10.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	10.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	10.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	10.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	10.48 865	9.97 825	4	1
60	9.48 998		9.51 178		10.48 822	9.97 821		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	45	44	43
1	4.5	4.4	4.3
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
1	4.2	4.1
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	40	39
1	4.0	3.9
2	8.0	7.8
3	12.0	11.7
4	16.0	15.6
5	20.0	19.5
6	24.0	23.4
7	28.0	27.3
8	32.0	31.2
9	36.0	35.1

	5	4	3
1	0.5	0.4	0.3
2	1.0	0.8	0.6
3	1.5	1.2	0.9
4	2.0	1.6	1.2
5	2.5	2.0	1.5
6	3.0	2.4	1.8
7	3.5	2.8	2.1
8	4.0	3.2	2.4
9	4.5	3.6	2.7

Prop. Pts.

72°

Table 3-2. Common logarithms of trigonometric functions (continued)

18°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.48 998	39	9.51 178	43	10.48 822	9.97 821	4	**60**
1	9.49 037	39	9.51 221	43	10.48 779	9.97 817	5	59
2	9.49 076	39	9.51 264	42	10.48 736	9.97 812	4	58
3	9.49 115	38	9.51 306	43	10.48 694	9.97 808	4	57
4	9.49 153	39	9.51 349	43	10.48 651	9.97 804	4	56
5	9.49 192	39	9.51 392	43	10.48 608	9.97 800	4	55
6	9.49 231	38	9.51 435	43	10.48 565	9.97 796	4	54
7	9.49 269	39	9.51 478	42	10.48 522	9.97 792	4	53
8	9.49 308	39	9.51 520	43	10.48 480	9.97 788	4	52
9	9.49 347	38	9.51 563	43	10.48 437	9.97 784	5	51
10	9.49 385	39	9.51 606	42	10.48 394	9.97 779	4	**50**
11	9.49 424	38	9.51 648	43	10.48 352	9.97 775	4	49
12	9.49 462	38	9.51 691	43	10.48 309	9.97 771	4	48
13	9.49 500	39	9.51 734	42	10.48 266	9.97 767	4	47
14	9.49 539	38	9.51 776	43	10.48 224	9.97 763	4	46
15	9.49 577	38	9.51 819	42	10.48 181	9.97 759	5	45
16	9.49 615	39	9.51 861	42	10.48 139	9.97 754	4	44
17	9.49 654	38	9.51 903	43	10.48 097	9.97 750	4	43
18	9.49 692	38	9.51 946	42	10.48 054	9.97 746	4	42
19	9.49 730	38	9.51 988	43	10.48 012	9.97 742	4	41
20	9.49 768	38	9.52 031	42	10.47 969	9.97 738	4	**40**
21	9.49 806	38	9.52 073	42	10.47 927	9.97 734	5	39
22	9.49 844	38	9.52 115	42	10.47 885	9.97 729	4	38
23	9.49 882	38	9.52 157	43	10.47 843	9.97 725	4	37
24	9.49 920	38	9.52 200	42	10.47 800	9.97 721	4	36
25	9.49 958	38	9.52 242	42	10.47 758	9.97 717	4	35
26	9.49 996	38	9.52 284	42	10.47 716	9.97 713	5	34
27	9.50 034	38	9.52 326	42	10.47 674	9.97 708	4	33
28	9.50 072	38	9.52 368	42	10.47 632	9.97 704	4	32
29	9.50 110	38	9.52 410	42	10.47 590	9.97 700	4	31
30	9.50 148	37	9.52 452	42	10.47 548	9.97 696	5	**30**
31	9.50 185	38	9.52 494	42	10.47 506	9.97 691	4	29
32	9.50 223	38	9.52 536	42	10.47 464	9.97 687	4	28
33	9.50 261	37	9.52 578	42	10.47 422	9.97 683	4	27
34	9.50 298	38	9.52 620	41	10.47 380	9.97 679	5	26
35	9.50 336	38	9.52 661	42	10.47 339	9.97 674	4	25
36	9.50 374	37	9.52 703	42	10.47 297	9.97 670	4	24
37	9.50 411	38	9.52 745	42	10.47 255	9.97 666	4	23
38	9.50 449	37	9.52 787	42	10.47 213	9.97 662	5	22
39	9.50 486	37	9.52 829	41	10.47 171	9.97 657	4	21
40	9.50 523	38	9.52 870	42	10.47 130	9.97 653	4	**20**
41	9.50 561	37	9.52 912	41	10.47 088	9.97 649	4	19
42	9.50 598	37	9.52 953	42	10.47 047	9.97 645	5	18
43	9.50 635	38	9.52 995	42	10.47 005	9.97 640	4	17
44	9.50 673	37	9.53 037	41	10.46 963	9.97 636	4	16
45	9.50 710	37	9.53 078	42	10.46 922	9.97 632	4	15
46	9.50 747	37	9.53 120	41	10.46 880	9.97 628	5	14
47	9.50 784	37	9.53 161	41	10.46 839	9.97 623	4	13
48	9.50 821	37	9.53 202	42	10.46 798	9.97 619	4	12
49	9.50 858	38	9.53 244	41	10.46 756	9.97 615	5	11
50	9.50 896	37	9.53 285	42	10.46 715	9.97 610	4	**10**
51	9.50 933	37	9.53 327	41	10.46 673	9.97 606	4	9
52	9.50 970	37	9.53 368	41	10.46 632	9.97 602	5	8
53	9.51 007	36	9.53 409	41	10.46 591	9.97 597	4	7
54	9.51 043	37	9.53 450	42	10.46 550	9.97 593	4	6
55	9.51 080	37	9.53 492	41	10.46 508	9.97 589	5	5
56	9.51 117	37	9.53 533	41	10.46 467	9.97 584	4	4
57	9.51 154	37	9.53 574	41	10.46 426	9.97 580	4	3
58	9.51 191	36	9.53 615	41	10.46 385	9.97 576	5	2
59	9.51 227	37	9.53 656	41	10.46 344	9.97 571	4	1
60	9.51 264		9.53 697		10.46 303	9.97 567		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

71°

Prop. Pts.

	43	**42**	**41**
1	4.3	4.2	4.1
2	8.6	8.4	8.2
3	12.9	12.6	12.3
4	17.2	16.8	16.4
5	21.5	21.0	20.5
6	25.8	25.2	24.6
7	30.1	29.4	28.7
8	34.4	33.6	32.8
9	38.7	37.8	36.9

	39	**38**	**37**
1	3.9	3.8	3.7
2	7.8	7.6	7.4
3	11.7	11.4	11.1
4	15.6	15.2	14.8
5	19.5	19.0	18.5
6	23.4	22.8	22.2
7	27.3	26.6	25.9
8	31.2	30.4	29.6
9	35.1	34.2	33.3

	36	**5**	**4**
1	3.6	0.5	0.4
2	7.2	1.0	0.8
3	10.8	1.5	1.2
4	14.4	2.0	1.6
5	18.0	2.5	2.0
6	21.6	3.0	2.4
7	25.2	3.5	2.8
8	28.8	4.0	3.2
9	32.4	4.5	3.6

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

19°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.51 264	37	9.53 697	41	10.46 303	9.97 567	4	60
1	9.51 301	37	9.53 738	41	10.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	10.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	10.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	10.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	10.46 098	9.97 545	4	55
6	9.51 484	36	9.53 943	41	10.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	10.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	10.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	10.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	10.45 894	9.97 523	4	50
11	9.51 666	36	9.54 147	40	10.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	10.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	10.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	10.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	10.45 691	9.97 501	4	45
16	9.51 847	36	9.54 350	40	10.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	10.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	10.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	10.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	10.45 488	9.97 479	4	40
21	9.52 027	36	9.54 552	41	10.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	10.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	10.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	10.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	10.45 286	9.97 457	4	35
26	9.52 207	35	9.54 754	40	10.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	10.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	10.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	10.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	10.45 085	9.97 435	5	30
31	9.52 385	36	9.54 955	40	10.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	10.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	10.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	10.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	10.44 885	9.97 412	4	25
36	9.52 563	35	9.55 155	40	10.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	10.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	10.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	10.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	10.44 685	9.97 390	5	20
41	9.52 740	35	9.55 355	40	10.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	10.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	10.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	10.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	10.44 486	9.97 367	4	15
46	9.52 916	35	9.55 554	39	10.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	10.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	10.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	10.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	10.44 288	9.97 344	4	10
51	9.53 092	34	9.55 752	39	10.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	10.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	39	10.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	10.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	10.44 090	9.97 322	5	5
56	9.53 266	35	9.55 949	40	10.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	10.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	10.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	10.43 933	9.97 303	4	1
60	9.53 405		9.56 107		10.43 893	9.97 299		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	37	36	35
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	5	4
1	3.4	0.5	0.4
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

Prop. Pts.

70°

Table 3-2. Common logarithms of trigonometric functions (continued)

20°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.53 405	35	9.56 107	39	10.43 893	9.97 299	5	**60**
1	9.53 440	35	9.56 146	39	10.43 854	9.97 294	5	59
2	9.53 475	34	9.56 185	39	10.43 815	9.97 289	4	58
3	9.53 509	35	9.56 224	40	10.43 776	9.97 285	5	57
4	9.53 544	34	9.56 264	39	10.43 736	9.97 280	4	56
5	9.53 578	35	9.56 303	39	10.43 697	9.97 276	5	55
6	9.53 613	34	9.56 342	39	10.43 658	9.97 271	5	54
7	9.53 647	35	9.56 381	39	10.43 619	9.97 266	4	53
8	9.53 682	34	9.56 420	39	10.43 580	9.97 262	5	52
9	9.53 716	35	9.56 459	39	10.43 541	9.97 257	5	51
10	9.53 751	34	9.56 498	39	10.43 502	9.97 252	4	**50**
11	9.53 785	34	9.56 537	39	10.43 463	9.97 248	5	49
12	9.53 819	35	9.56 576	39	10.43 424	9.97 243	5	48
13	9.53 854	34	9.56 615	39	10.43 385	9.97 238	4	47
14	9.53 888	34	9.56 654	39	10.43 346	9.97 234	5	46
15	9.53 922	35	9.56 693	39	10.43 307	9.97 229	5	45
16	9.53 957	34	9.56 732	39	10.43 268	9.97 224	4	44
17	9.53 991	34	9.56 771	39	10.43 229	9.97 220	5	43
18	9.54 025	34	9.56 810	39	10.43 190	9.97 215	5	42
19	9.54 059	34	9.56 849	38	10.43 151	9.97 210	4	41
20	9.54 093	34	9.56 887	39	10.43 113	9.97 206	5	**40**
21	9.54 127	34	9.56 926	39	10.43 074	9.97 201	5	39
22	9.54 161	34	9.56 965	39	10.43 035	9.97 196	4	38
23	9.54 195	34	9.57 004	38	10.42 996	9.97 192	5	37
24	9.54 229	34	9.57 042	39	10.42 958	9.97 187	5	36
25	9.54 263	34	9.57 081	39	10.42 919	9.97 182	4	35
26	9.54 297	34	9.57 120	38	10.42 880	9.97 178	5	34
27	9.54 331	34	9.57 158	39	10.42 842	9.97 173	5	33
28	9.54 365	34	9.57 197	38	10.42 803	9.97 168	5	32
29	9.54 399	34	9.57 235	39	10.42 765	9.97 163	4	31
30	9.54 433	33	9.57 274	38	10.42 726	9.97 159	5	**30**
31	9.54 466	34	9.57 312	39	10.42 688	9.97 154	5	29
32	9.54 500	34	9.57 351	38	10.42 649	9.97 149	4	28
33	9.54 534	33	9.57 389	39	10.42 611	9.97 145	5	27
34	9.54 567	34	9.57 428	38	10.42 572	9.97 140	5	26
35	9.54 601	34	9.57 466	38	10.42 534	9.97 135	5	25
36	9.54 635	33	9.57 504	39	10.42 496	9.97 130	4	24
37	9.54 668	34	9.57 543	38	10.42 457	9.97 126	5	23
38	9.54 702	33	9.57 581	38	10.42 419	9.97 121	5	22
39	9.54 735	34	9.57 619	39	10.42 381	9.97 116	5	21
40	9.54 769	33	9.57 658	38	10.42 342	9.97 111	4	**20**
41	9.54 802	34	9.57 696	38	10.42 304	9.97 107	5	19
42	9.54 836	33	9.57 734	38	10.42 266	9.97 102	5	18
43	9.54 869	34	9.57 772	38	10.42 228	9.97 097	5	17
44	9.54 903	33	9.57 810	39	10.42 190	9.97 092	5	16
45	9.54 936	33	9.57 849	38	10.42 151	9.97 087	4	15
46	9.54 969	34	9.57 887	38	10.42 113	9.97 083	5	14
47	9.55 003	33	9.57 925	38	10.42 075	9.97 078	5	13
48	9.55 036	33	9.57 963	38	10.42 037	9.97 073	5	12
49	9.55 069	33	9.58 001	38	10.41 999	9.97 068	5	11
50	9.55 102	34	9.58 039	38	10.41 961	9.97 063	4	**10**
51	9.55 136	33	9.58 077	38	10.41 923	9.97 059	5	9
52	9.55 169	33	9.58 115	38	10.41 885	9.97 054	5	8
53	9.55 202	33	9.58 153	38	10.41 847	9.97 049	5	7
54	9.55 235	33	9.58 191	38	10.41 809	9.97 044	5	6
55	9.55 268	33	9.58 229	38	10.41 771	9.97 039	4	5
56	9.55 301	33	9.58 267	37	10.41 733	9.97 035	5	4
57	9.55 334	33	9.58 304	38	10.41 696	9.97 030	5	3
58	9.55 367	33	9.58 342	38	10.41 658	9.97 025	5	2
59	9.55 400	33	9.58 380	38	10.41 620	9.97 020	5	1
60	9.55 433		9.58 418		10.41 582	9.97 015		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

69°

Prop. Pts.

	40	39	38
1	4.0	3.9	3.8
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

	37	35	34
1	3.7	3.5	3.4
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

	33	5	4
1	3.3	0.5	0.4
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

21°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.55 433	33	9.58 418	37	10.41 582	9.97 015	5	**60**
1	9.55 466	33	9.58 455	38	10.41 545	9.97 010	5	59
2	9.55 499	33	9.58 493	38	10.41 507	9.97 005	4	58
3	9.55 532	32	9.58 531	38	10.41 469	9.97 001	5	57
4	9.55 564	33	9.58 569	37	10.41 431	9.96 996	5	56
5	9.55 597	33	9.58 606	38	10.41 394	9.96 991	5	55
6	9.55 630	33	9.58 644	37	10.41 356	9.96 986	5	54
7	9.55 663	32	9.58 681	38	10.41 319	9.96 981	5	53
8	9.55 695	33	9.58 719	38	10.41 281	9.96 976	5	52
9	9.55 728	33	9.58 757	37	10.41 243	9.96 971	5	51
10	9.55 761	32	9.58 794	38	10.41 206	9.96 966	4	**50**
11	9.55 793	33	9.58 832	37	10.41 168	9.96 962	5	49
12	9.55 826	32	9.58 869	38	10.41 131	9.96 957	5	48
13	9.55 858	33	9.58 907	37	10.41 093	9.96 952	5	47
14	9.55 891	32	9.58 944	37	10.41 056	9.96 947	5	46
15	9.55 923	33	9.58 981	38	10.41 019	9.96 942	5	45
16	9.55 956	32	9.59 019	37	10.40 981	9.96 937	5	44
17	9.55 988	33	9.59 056	38	10.40 944	9.96 932	5	43
18	9.56 021	32	9.59 094	37	10.40 906	9.96 927	5	42
19	9.56 053	32	9.59 131	37	10.40 869	9.96 922	5	41
20	9.56 085	33	9.59 168	37	10.40 832	9.96 917	5	**40**
21	9.56 118	32	9.59 205	38	10.40 795	9.96 912	5	39
22	9.56 150	32	9.59 243	37	10.40 757	9.96 907	4	38
23	9.56 182	33	9.59 280	37	10.40 720	9.96 903	5	37
24	9.56 215	32	9.59 317	37	10.40 683	9.96 898	5	36
25	9.56 247	32	9.59 354	37	10.40 646	9.96 893	5	35
26	9.56 279	32	9.59 391	38	10.40 609	9.96 888	5	34
27	9.56 311	32	9.59 429	37	10.40 571	9.96 883	5	33
28	9.56 343	32	9.59 466	37	10.40 534	9.96 878	5	32
29	9.56 375	33	9.59 503	37	10.40 497	9.96 873	5	31
30	9.56 408	32	9.59 540	37	10.40 460	9.96 868	5	**30**
31	9.56 440	32	9.59 577	37	10.40 423	9.96 863	5	29
32	9.56 472	32	9.59 614	37	10.40 386	9.96 858	5	28
33	9.56 504	32	9.59 651	37	10.40 349	9.96 853	5	27
34	9.56 536	32	9.59 688	37	10.40 312	9.96 848	5	26
35	9.56 568	31	9.59 725	37	10.40 275	9.96 843	5	25
36	9.56 599	32	9.59 762	37	10.40 238	9.96 838	5	24
37	9.56 631	32	9.59 799	36	10.40 201	9.96 833	5	23
38	9.56 663	32	9.59 835	37	10.40 165	9.96 828	5	22
39	9.56 695	32	9.59 872	37	10.40 128	9.96 823	5	21
40	9.56 727	32	9.59 909	37	10.40 091	9.96 818	5	**20**
41	9.56 759	31	9.59 946	37	10.40 054	9.96 813	5	19
42	9.56 790	32	9.59 983	36	10.40 017	9.96 808	5	18
43	9.56 822	32	9.60 019	37	10.39 981	9.96 803	5	17
44	9.56 854	32	9.60 056	37	10.39 944	9.96 798	5	16
45	9.56 886	31	9.60 093	37	10.39 907	9.96 793	5	15
46	9.56 917	32	9.60 130	36	10.39 870	9.96 788	5	14
47	9.56 949	31	9.60 166	37	10.39 834	9.96 783	5	13
48	9.56 980	32	9.60 203	37	10.39 797	9.96 778	6	12
49	9.57 012	32	9.60 240	36	10.39 760	9.96 772	5	11
50	9.57 044	31	9.60 276	37	10.39 724	9.96 767	5	**10**
51	9.57 075	32	9.60 313	36	10.39 687	9.96 762	5	9
52	9.57 107	31	9.60 349	37	10.39 651	9.96 757	5	8
53	9.57 138	31	9.60 386	36	10.39 614	9.96 752	5	7
54	9.57 169	32	9.60 422	37	10.39 578	9.96 747	5	6
55	9.57 201	31	9.60 459	36	10.39 541	9.96 742	5	5
56	9.57 232	32	9.60 495	37	10.39 505	9.96 737	5	4
57	9.57 264	31	9.60 532	36	10.39 468	9.96 732	5	3
58	9.57 295	31	9.60 568	37	10.39 432	9.96 727	5	2
59	9.57 326	32	9.60 605	36	10.39 395	9.96 722	5	1
60	9.57 358		9.60 641		10.39 359	9.96 717		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

68°

Prop. Pts.

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	33	32	31
1	3.3	3.2	3.1
2	6.6	6.4	6.2
3	9.9	9.6	9.3
4	13.2	12.8	12.4
5	16.5	16.0	15.5
6	19.8	19.2	18.6
7	23.1	22.4	21.7
8	26.4	25.6	24.8
9	29.7	28.8	27.9

	6	5	4
1	0.6	0.5	0.4
2	1.2	1.0	0.8
3	1.8	1.5	1.2
4	2.4	2.0	1.6
5	3.0	2.5	2.0
6	3.6	3.0	2.4
7	4.2	3.5	2.8
8	4.8	4.0	3.2
9	5.4	4.5	3.6

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

22°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Pts.
0	9.57 358	31	9.60 641	36	10.39 359	9.96 717	6	**60**	
1	9.57 389	31	9.60 677	37	10.39 323	9.96 711	5	59	
2	9.57 420	31	9.60 714	36	10.39 286	9.96 706	5	58	
3	9.57 451	31	9.60 750	36	10.39 250	9.96 701	5	57	
4	9.57 482	32	9.60 786	37	10.39 214	9.96 696	5	56	
5	9.57 514	31	9.60 823	36	10.39 177	9.96 691	5	55	
6	9.57 545	31	9.60 859	36	10.39 141	9.96 686	5	54	
7	9.57 576	31	9.60 895	36	10.39 105	9.96 681	5	53	
8	9.57 607	31	9.60 931	36	10.39 069	9.96 676	6	52	
9	9.57 638	31	9.60 967	37	10.39 033	9.96 670	5	51	
10	9.57 669	31	9.61 004	36	10.38 996	9.96 665	5	**50**	
11	9.57 700	31	9.61 040	36	10.38 960	9.96 660	5	49	
12	9.57 731	31	9.61 076	36	10.38 924	9.96 655	5	48	
13	9.57 762	31	9.61 112	36	10.38 888	9.96 650	5	47	
14	9.57 793	31	9.61 148	36	10.38 852	9.96 645	5	46	
15	9.57 824	31	9.61 184	36	10.38 816	9.96 640	6	45	
16	9.57 855	30	9.61 220	36	10.38 780	9.96 634	5	44	
17	9.57 885	31	9.61 256	36	10.38 744	9.96 629	5	43	
18	9.57 916	31	9.61 292	36	10.38 708	9.96 624	5	42	
19	9.57 947	31	9.61 328	36	10.38 672	9.96 619	5	41	
20	9.57 978	30	9.61 364	36	10.38 636	9.96 614	6	**40**	
21	9.58 008	31	9.61 400	36	10.38 600	9.96 608	5	39	
22	9.58 039	31	9.61 436	36	10.38 564	9.96 603	5	38	
23	9.58 070	31	9.61 472	36	10.38 528	9.96 598	5	37	
24	9.58 101	30	9.61 508	36	10.38 492	9.96 593	5	36	
25	9.58 131	31	9.61 544	35	10.38 456	9.96 588	6	35	
26	9.58 162	30	9.61 579	36	10.38 421	9.96 582	5	34	
27	9.58 192	31	9.61 615	36	10.38 385	9.96 577	5	33	
28	9.58 223	30	9.61 651	36	10.38 349	9.96 572	5	32	
29	9.58 253	31	9.61 687	35	10.38 313	9.96 567	5	31	
30	9.58 284	30	9.61 722	36	10.38 278	9.96 562	6	**30**	
31	9.58 314	31	9.61 758	36	10.38 242	9.96 556	5	29	
32	9.58 345	30	9.61 794	36	10.38 206	9.96 551	5	28	
33	9.58 375	31	9.61 830	35	10.38 170	9.96 546	5	27	
34	9.58 406	30	9.61 865	36	10.38 135	9.96 541	6	26	
35	9.58 436	31	9.61 901	35	10.38 099	9.96 535	5	25	
36	9.58 467	30	9.61 936	36	10.38 064	9.96 530	5	24	
37	9.58 497	30	9.61 972	36	10.38 028	9.96 525	5	23	
38	9.58 527	30	9.62 008	35	10.37 992	9.96 520	6	22	
39	9.58 557	31	9.62 043	36	10.37 957	9.96 514	5	21	
40	9.58 588	30	9.62 079	35	10.37 921	9.96 509	5	**20**	
41	9.58 618	30	9.62 114	36	10.37 886	9.96 504	6	19	
42	9.58 648	30	9.62 150	35	10.37 850	9.96 498	5	18	
43	9.58 678	31	9.62 185	36	10.37 815	9.96 493	5	17	
44	9.58 709	30	9.62 221	35	10.37 779	9.96 488	5	16	
45	9.58 739	30	9.62 256	36	10.37 744	9.96 483	6	15	
46	9.58 769	30	9.62 292	35	10.37 708	9.96 477	5	14	
47	9.58 799	30	9.62 327	35	10.37 673	9.96 472	5	13	
48	9.58 829	30	9.62 362	36	10.37 638	9.96 467	6	12	
49	9.58 859	30	9.62 398	35	10.37 602	9.96 461	5	11	
50	9.58 889	30	9.62 433	35	10.37 567	9.96 456	5	**10**	
51	9.58 919	30	9.62 468	36	10.37 532	9.96 451	6	9	
52	9.58 949	30	9.62 504	35	10.37 496	9.96 445	5	8	
53	9.58 979	30	9.62 539	35	10.37 461	9.96 440	5	7	
54	9.59 009	30	9.62 574	35	10.37 426	9.96 435	6	6	
55	9.59 039	30	9.62 609	36	10.37 391	9.96 429	5	5	
56	9.59 069	29	9.62 645	35	10.37 355	9.96 424	5	4	
57	9.59 098	30	9.62 680	35	10.37 320	9.96 419	6	3	
58	9.59 128	30	9.62 715	35	10.37 285	9.96 413	5	2	
59	9.59 158	30	9.62 750	35	10.37 250	9.96 408	5	1	
60	9.59 188		9.62 785		10.37 215	9.96 403		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	Prop. Pts.

	37	36	35
1	3.7	3.6	3.5
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
1	2.9	0.6	0.5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

67°

Table 3-2. Common logarithms of trigonometric functions (continued)

23°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.59 188	30	9.62 785	35	10.37 215	9.96 403	6	**60**
1	9.59 218	29	9.62 820	35	10.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	10.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	10.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	10.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	10.37 039	9.96 376	6	55
6	9.59 366	30	9.62 996	35	10.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	10.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	10.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	10.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	10.36 865	9.96 349	6	**50**
11	9.59 514	29	9.63 170	35	10.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	10.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	10.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	10.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	10.36 690	9.96 322	6	45
16	9.59 661	29	9.63 345	34	10.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	10.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	10.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	10.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	10.36 516	9.96 294	5	**40**
21	9.59 808	29	9.63 519	34	10.36 481	9.96 289	5	39
22	9.59 839	29	9.63 553	35	10.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	10.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	10.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	10.36 343	9.96 267	5	35
26	9.59 954	29	9.63 692	34	10.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	10.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	10.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	10.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	10.36 170	9.96 240	6	**30**
31	9.60 099	29	9.63 865	34	10.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	10.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	10.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	10.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	10.35 997	9.96 212	5	25
36	9.60 244	29	9.64 037	35	10.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	10.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	10.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	10.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	10.35 825	9.96 185	6	**20**
41	9.60 388	29	9.64 209	34	10.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	10.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	10.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	10.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	10.35 654	9.96 157	6	15
46	9.60 532	29	9.64 381	34	10.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	10.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	10.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	10.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	10.35 483	9.96 129	6	**10**
51	9.60 675	29	9.64 552	34	10.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	10.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	10.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	10.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	10.35 312	9.96 101	6	5
56	9.60 818	28	9.64 722	34	10.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	10.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	10.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	10.35 176	9.96 079	6	1
60	9.60 931		9.64 858		10.35 142	9.96 073		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

66°

Prop. Pts.

	36	35	34
1	3.6	3.5	3.4
2	7.2	7.0	6.8
3	10.8	10.5	10.2
4	14.4	14.0	13.6
5	18.0	17.5	17.0
6	21.6	21.0	20.4
7	25.2	24.5	23.8
8	28.8	28.0	27.2
9	32.4	31.5	30.6

	30	29	28
1	3.0	2.9	2.8
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

24°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.60 931	29	9.64 858	34	10.35 142	9.96 073	6	**60**
1	9.60 960	28	9.64 892	34	10.35 108	9.96 067	5	59
2	9.60 988	28	9.64 926	34	10.35 074	9.96 062	6	58
3	9.61 016	29	9.64 960	34	10.35 040	9.96 056	6	57
4	9.61 045	28	9.64 994	34	10.35 006	9.96 050	5	56
5	9.61 073	28	9.65 028	34	10.34 972	9.96 045	6	55
6	9.61 101	28	9.65 062	34	10.34 938	9.96 039	5	54
7	9.61 129	29	9.65 096	34	10.34 904	9.96 034	6	53
8	9.61 158	28	9.65 130	34	10.34 870	9.96 028	6	52
9	9.61 186	28	9.65 164	33	10.34 836	9.96 022	5	51
10	9.61 214	28	9.65 197	34	10.34 803	9.96 017	6	**50**
11	9.61 242	28	9.65 231	34	10.34 769	9.96 011	6	49
12	9.61 270	28	9.65 265	34	10.34 735	9.96 005	5	48
13	9.61 298	28	9.65 299	34	10.34 701	9.96 000	6	47
14	9.61 326	28	9.65 333	33	10.34 667	9.95 994	6	46
15	9.61 354	28	9.65 366	34	10.34 634	9.95 988	6	45
16	9.61 382	29	9.65 400	34	10.34 600	9.95 982	5	44
17	9.61 411	27	9.65 434	33	10.34 566	9.95 977	6	43
18	9.61 438	28	9.65 467	34	10.34 533	9.95 971	6	42
19	9.61 466	28	9.65 501	34	10.34 499	9.95 965	5	41
20	9.61 494	28	9.65 535	33	10.34 465	9.95 960	6	**40**
21	9.61 522	28	9.65 568	34	10.34 432	9.95 954	6	39
22	9.61 550	28	9.65 602	34	10.34 398	9.95 948	6	38
23	9.61 578	28	9.65 636	33	10.34 364	9.95 942	5	37
24	9.61 606	28	9.65 669	34	10.34 331	9.95 937	6	36
25	9.61 634	28	9.65 703	33	10.34 297	9.95 931	6	35
26	9.61 662	27	9.65 736	34	10.34 264	9.95 925	5	34
27	9.61 689	28	9.65 770	33	10.34 230	9.95 920	6	33
28	9.61 717	28	9.65 803	34	10.34 197	9.95 914	6	32
29	9.61 745	28	9.65 837	33	10.34 163	9.95 908	6	31
30	9.61 773	27	9.65 870	34	10.34 130	9.95 902	5	**30**
31	9.61 800	28	9.65 904	33	10.34 096	9.95 897	6	29
32	9.61 828	28	9.65 937	34	10.34 063	9.95 891	6	28
33	9.61 856	27	9.65 971	33	10.34 029	9.95 885	6	27
34	9.61 883	28	9.66 004	34	10.33 996	9.95 879	6	26
35	9.61 911	28	9.66 038	33	10.33 962	9.95 873	5	25
36	9.61 939	27	9.66 071	33	10.33 929	9.95 868	6	24
37	9.61 966	28	9.66 104	34	10.33 896	9.95 862	6	23
38	9.61 994	27	9.66 138	33	10.33 862	9.95 856	6	22
39	9.62 021	28	9.66 171	33	10.33 829	9.95 850	6	21
40	9.62 049	27	9.66 204	34	10.33 796	9.95 844	5	**20**
41	9.62 076	28	9.66 238	33	10.33 762	9.95 839	6	19
42	9.62 104	27	9.66 271	33	10.33 729	9.95 833	6	18
43	9.62 131	28	9.66 304	33	10.33 696	9.95 827	6	17
44	9.62 159	27	9.66 337	34	10.33 663	9.95 821	6	16
45	9.62 186	28	9.66 371	33	10.33 629	9.95 815	5	15
46	9.62 214	27	9.66 404	33	10.33 596	9.95 810	6	14
47	9.62 241	27	9.66 437	33	10.33 563	9.95 804	6	13
48	9.62 268	28	9.66 470	33	10.33 530	9.95 798	6	12
49	9.62 296	27	9.66 503	34	10.33 497	9.95 792	6	11
50	9.62 323	27	9.66 537	33	10.33 463	9.95 786	6	**10**
51	9.62 350	27	9.66 570	33	10.33 430	9.95 780	5	9
52	9.62 377	28	9.66 603	33	10.33 397	9.95 775	6	8
53	9.62 405	27	9.66 636	33	10.33 364	9.95 769	6	7
54	9.62 432	27	9.66 669	33	10.33 331	9.95 763	6	6
55	9.62 459	27	9.66 702	33	10.33 298	9.95 757	6	5
56	9.62 486	27	9.66 735	33	10.33 265	9.95 751	6	4
57	9.62 513	28	9.66 768	33	10.33 232	9.95 745	6	3
58	9.62 541	27	9.66 801	33	10.33 199	9.95 739	6	2
59	9.62 568	27	9.66 834	33	10.33 166	9.95 733	5	1
60	9.62 595		9.66 867		10.33 133	9.95 728		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	34	33
1	3.4	3.3
2	6.8	6.6
3	10.2	9.9
4	13.6	13.2
5	17.0	16.5
6	20.4	19.8
7	23.8	23.1
8	27.2	26.4
9	30.6	29.7

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

Prop. Pts.

65°

Table 3-2. Common logarithms of trigonometric functions (continued)

25°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.62 595	27	9.66 867	33	10.33 133	9.95 728	6	60
1	9.62 622	27	9.66 900	33	10.33 100	9.95 722	6	59
2	9.62 649	27	9.66 933	33	10.33 067	9.95 716	6	58
3	9.62 676	27	9.66 966	33	10.33 034	9.95 710	6	57
4	9.62 703	27	9.66 999	33	10.33 001	9.95 704	6	56
5	9.62 730	27	9.67 032	33	10.32 968	9.95 698	6	55
6	9.62 757	27	9.67 065	33	10.32 935	9.95 692	6	54
7	9.62 784	27	9.67 098	33	10.32 902	9.95 686	6	53
8	9.62 811	27	9.67 131	32	10.32 869	9.95 680	6	52
9	9.62 838	27	9.67 163	33	10.32 837	9.95 674	6	51
10	9.62 865	27	9.67 196	33	10.32 804	9.95 668	5	50
11	9.62 892	26	9.67 229	33	10.32 771	9.95 663	6	49
12	9.62 918	27	9.67 262	33	10.32 738	9.95 657	6	48
13	9.62 945	27	9.67 295	32	10.32 705	9.95 651	6	47
14	9.62 972	27	9.67 327	33	10.32 673	9.95 645	6	46
15	9.62 999	27	9.67 360	33	10.32 640	9.95 639	6	45
16	9.63 026	26	9.67 393	33	10.32 607	9.95 633	6	44
17	9.63 052	27	9.67 426	32	10.32 574	9.95 627	6	43
18	9.63 079	27	9.67 458	33	10.32 542	9.95 621	6	42
19	9.63 106	27	9.67 491	33	10.32 509	9.95 615	6	41
20	9.63 133	26	9.67 524	32	10.32 476	9.95 609	6	40
21	9.63 159	27	9.67 556	33	10.32 444	9.95 603	6	39
22	9.63 186	27	9.67 589	33	10.32 411	9.95 597	6	38
23	9.63 213	26	9.67 622	32	10.32 378	9.95 591	6	37
24	9.63 239	27	9.67 654	33	10.32 346	9.95 585	6	36
25	9.63 266	26	9.67 687	32	10.32 313	9.95 579	6	35
26	9.63 292	27	9.67 719	33	10.32 281	9.95 573	6	34
27	9.63 319	26	9.67 752	33	10.32 248	9.95 567	6	33
28	9.63 345	27	9.67 785	32	10.32 215	9.95 561	6	32
29	9.63 372	26	9.67 817	33	10.32 183	9.95 555	6	31
30	9.63 398	27	9.67 850	32	10.32 150	9.95 549	6	30
31	9.63 425	26	9.67 882	33	10.32 118	9.95 543	6	29
32	9.63 451	27	9.67 915	32	10.32 085	9.95 537	6	28
33	9.63 478	26	9.67 947	33	10.32 053	9.95 531	6	27
34	9.63 504	27	9.67 980	32	10.32 020	9.95 525	6	26
35	9.63 531	26	9.68 012	32	10.31 988	9.95 519	6	25
36	9.63 557	26	9.68 044	33	10.31 956	9.95 513	6	24
37	9.63 583	27	9.68 077	32	10.31 923	9.95 507	7	23
38	9.63 610	26	9.68 109	33	10.31 891	9.95 500	6	22
39	9.63 636	26	9.68 142	32	10.31 858	9.95 494	6	21
40	9.63 662	27	9.68 174	32	10.31 826	9.95 488	6	20
41	9.63 689	26	9.68 206	33	10.31 794	9.95 482	6	19
42	9.63 715	26	9.68 239	32	10.31 761	9.95 476	6	18
43	9.63 741	26	9.68 271	32	10.31 729	9.95 470	6	17
44	9.63 767	27	9.68 303	33	10.31 697	9.95 464	6	16
45	9.63 794	26	9.68 336	32	10.31 664	9.95 458	6	15
46	9.63 820	26	9.68 368	32	10.31 632	9.95 452	6	14
47	9.63 846	26	9.68 400	32	10.31 600	9.95 446	6	13
48	9.63 872	26	9.68 432	33	10.31 568	9.95 440	6	12
49	9.63 898	26	9.68 465	32	10.31 535	9.95 434	7	11
50	9.63 924	26	9.68 497	32	10.31 503	9.95 427	6	10
51	9.63 950	26	9.68 529	32	10.31 471	9.95 421	6	9
52	9.63 976	26	9.68 561	32	10.31 439	9.95 415	6	8
53	9.64 002	26	9.68 593	33	10.31 407	9.95 409	6	7
54	9.64 028	26	9.68 626	32	10.31 374	9.95 403	6	6
55	9.64 054	26	9.68 658	32	10.31 342	9.95 397	6	5
56	9.64 080	26	9.68 690	32	10.31 310	9.95 391	7	4
57	9.64 106	26	9.68 722	32	10.31 278	9.95 384	6	3
58	9.64 132	26	9.68 754	32	10.31 246	9.95 378	6	2
59	9.64 158	26	9.68 786	32	10.31 214	9.95 372	6	1
60	9.64 184		9.68 818		10.31 182	9.95 366		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

64°

Prop. Pts.

	33	32
1	3.3	3.2
2	6.6	6.4
3	9.9	9.6
4	13.2	12.8
5	16.5	16.0
6	19.8	19.2
7	23.1	22.4
8	26.4	25.6
9	29.7	28.8

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	7	6	5
1	0.7	0.6	0.5
2	1.4	1.2	1.0
3	2.1	1.8	1.5
4	2.8	2.4	2.0
5	3.5	3.0	2.5
6	4.2	3.6	3.0
7	4.9	4.2	3.5
8	5.6	4.8	4.0
9	6.3	5.4	4.5

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

26°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.64 184	26	9.68 818	32	10.31 182	9.95 366	6	**60**
1	9.64 210	26	9.68 850	32	10.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	10.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	10.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	10.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	10.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	10.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	10.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	10.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	10.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	10.30 862	9.95 304	6	**50**
11	9.64 468	26	9.69 170	32	10.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	10.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	10.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	10.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	10.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	10.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	10.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	10.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	10.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	10.30 543	9.95 242	6	**40**
21	9.64 724	25	9.69 488	32	10.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	10.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	10.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	10.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	10.30 385	9.95 211	7	35
26	9.64 851	26	9.69 647	32	10.30 353	9.95 204	6	34
27	9.64 877	25	9.69 679	31	10.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	10.30 290	9.95 192	7	32
29	9.64 927	26	9.69 742	32	10.30 258	9.95 185	6	31
30	9.64 953	25	9.69 774	31	10.30 226	9.95 179	6	**30**
31	9.64 978	25	9.69 805	32	10.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	10.30 163	9.95 167	7	28
33	9.65 029	25	9.69 868	32	10.30 132	9.95 160	6	27
34	9.65 054	25	9.69 900	32	10.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	10.30 068	9.95 148	7	25
36	9.65 104	26	9.69 963	32	10.30 037	9.95 141	6	24
37	9.65 130	25	9.69 995	31	10.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	10.29 974	9.95 129	7	22
39	9.65 180	25	9.70 058	31	10.29 942	9.95 122	6	21
40	9.65 205	25	9.70 089	32	10.29 911	9.95 116	6	**20**
41	9.65 230	25	9.70 121	31	10.29 879	9.95 110	7	19
42	9.65 255	26	9.70 152	32	10.29 848	9.95 103	6	18
43	9.65 281	25	9.70 184	31	10.29 816	9.95 097	7	17
44	9.65 306	25	9.70 215	32	10.29 785	9.95 090	6	16
45	9.65 331	25	9.70 247	31	10.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	10.29 722	9.95 078	7	14
47	9.65 381	25	9.70 309	32	10.29 691	9.95 071	6	13
48	9.65 406	25	9.70 341	31	10.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	10.29 628	9.95 059	7	11
50	9.65 456	25	9.70 404	31	10.29 596	9.95 052	6	**10**
51	9.65 481	25	9.70 435	31	10.29 565	9.95 046	7	9
52	9.65 506	25	9.70 466	32	10.29 534	9.95 039	6	8
53	9.65 531	25	9.70 498	31	10.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	10.29 471	9.95 027	7	6
55	9.65 580	25	9.70 560	32	10.29 440	9.95 020	6	5
56	9.65 605	25	9.70 592	31	10.29 408	9.95 014	7	4
57	9.65 630	25	9.70 623	31	10.29 377	9.95 007	6	3
58	9.65 655	25	9.70 654	31	10.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	10.29 315	9.94 995	7	1
60	9.65 705		9.70 717		10.29 283	9.94 988		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	32	31
1	3.2	3.1
2	6.4	6.2
3	9.6	9.3
4	12.8	12.4
5	16.0	15.5
6	19.2	18.6
7	22.4	21.7
8	25.6	24.8
9	28.8	27.9

	26	25	24
1	2.6	2.5	2.4
2	5.2	5.0	4.8
3	7.8	7.5	7.2
4	10.4	10.0	9.6
5	13.0	12.5	12.0
6	15.6	15.0	14.4
7	18.2	17.5	16.8
8	20.8	20.0	19.2
9	23.4	22.5	21.6

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

Prop. Pts.

63°

Table 3-2. Common logarithms of trigonometric functions (continued)

27°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Pts.
0	9.65 705	24	9.70 717	31	10.29 283	9.94 988	6	**60**	
1	9.65 729	25	9.70 748	31	10.29 252	9.94 982	7	59	
2	9.65 754	25	9.70 779	31	10.29 221	9.94 975	6	58	
3	9.65 779	25	9.70 810	31	10.29 190	9.94 969	7	57	
4	9.65 804	24	9.70 841	32	10.29 159	9.94 962	6	56	
5	9.65 828	25	9.70 873	31	10.29 127	9.94 956	7	55	
6	9.65 853	25	9.70 904	31	10.29 096	9.94 949	6	54	
7	9.65 878	24	9.70 935	31	10.29 065	9.94 943	7	53	
8	9.65 902	25	9.70 966	31	10.29 034	9.94 936	6	52	
9	9.65 927	25	9.70 997	31	10.29 003	9.94 930	7	51	
10	9.65 952	24	9.71 028	31	10.28 972	9.94 923	6	**50**	
11	9.65 976	25	9.71 059	31	10.28 941	9.94 917	6	49	
12	9.66 001	24	9.71 090	31	10.28 910	9.94 911	7	48	
13	9.66 025	25	9.71 121	32	10.28 879	9.94 904	6	47	
14	9.66 050	25	9.71 153	31	10.28 847	9.94 898	7	46	
15	9.66 075	24	9.71 184	31	10.28 816	9.94 891	6	45	
16	9.66 099	25	9.71 215	31	10.28 785	9.94 885	7	44	
17	9.66 124	24	9.71 246	31	10.28 754	9.94 878	7	43	
18	9.66 148	25	9.71 277	31	10.28 723	9.94 871	6	42	
19	9.66 173	24	9.71 308	31	10.28 692	9.94 865	7	41	
20	9.66 197	24	9.71 339	31	10.28 661	9.94 858	6	**40**	
21	9.66 221	25	9.71 370	31	10.28 630	9.94 852	7	39	
22	9.66 246	24	9.71 401	30	10.28 599	9.94 845	6	38	
23	9.66 270	25	9.71 431	31	10.28 569	9.94 839	7	37	
24	9.66 295	24	9.71 462	31	10.28 538	9.94 832	6	36	
25	9.66 319	24	9.71 493	31	10.28 507	9.94 826	7	35	
26	9.66 343	25	9.71 524	31	10.28 476	9.94 819	6	34	
27	9.66 368	24	9.71 555	31	10.28 445	9.94 813	7	33	
28	9.66 392	24	9.71 586	31	10.28 414	9.94 806	7	32	
29	9.66 416	25	9.71 617	31	10.28 383	9.94 799	6	31	
30	9.66 441	24	9.71 648	31	10.28 352	9.94 793	7	**30**	
31	9.66 465	24	9.71 679	30	10.28 321	9.94 786	6	29	
32	9.66 489	24	9.71 709	31	10.28 291	9.94 780	7	28	
33	9.66 513	24	9.71 740	31	10.28 260	9.94 773	6	27	
34	9.66 537	25	9.71 771	31	10.28 229	9.94 767	7	26	
35	9.66 562	24	9.71 802	31	10.28 198	9.94 760	7	25	
36	9.66 586	24	9.71 833	30	10.28 167	9.94 753	6	24	
37	9.66 610	24	9.71 863	31	10.28 137	9.94 747	7	23	
38	9.66 634	24	9.71 894	31	10.28 106	9.94 740	6	22	
39	9.66 658	24	9.71 925	30	10.28 075	9.94 734	7	21	
40	9.66 682	24	9.71 955	31	10.28 045	9.94 727	7	**20**	
41	9.66 706	25	9.71 986	31	10.28 014	9.94 720	6	19	
42	9.66 731	24	9.72 017	31	10.27 983	9.94 714	7	18	
43	9.66 755	24	9.72 048	30	10.27 952	9.94 707	7	17	
44	9.66 779	24	9.72 078	31	10.27 922	9.94 700	6	16	
45	9.66 803	24	9.72 109	31	10.27 891	9.94 694	7	15	
46	9.66 827	24	9.72 140	30	10.27 860	9.94 687	7	14	
47	9.66 851	24	9.72 170	31	10.27 830	9.94 680	6	13	
48	9.66 875	24	9.72 201	30	10.27 799	9.94 674	7	12	
49	9.66 899	23	9.72 231	31	10.27 769	9.94 667	7	11	
50	9.66 922	24	9.72 262	31	10.27 738	9.94 660	6	**10**	
51	9.66 946	24	9.72 293	30	10.27 707	9.94 654	7	9	
52	9.66 970	24	9.72 323	31	10.27 677	9.94 647	7	8	
53	9.66 994	24	9.72 354	30	10.27 646	9.94 640	6	7	
54	9.67 018	24	9.72 384	31	10.27 616	9.94 634	7	6	
55	9.67 042	24	9.72 415	30	10.27 585	9.94 627	7	5	
56	9.67 066	24	9.72 445	31	10.27 555	9.94 620	6	4	
57	9.67 090	23	9.72 476	30	10.27 524	9.94 614	7	3	
58	9.67 113	24	9.72 506	31	10.27 494	9.94 607	7	2	
59	9.67 137	24	9.72 537	30	10.27 463	9.94 600	7	1	
60	9.67 161		9.72 567		10.27 433	9.94 593		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	Prop. Pts.

Prop. Pts.

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	25	24	23
1	2.5	2.4	2.3
2	5.0	4.8	4.6
3	7.5	7.2	6.9
4	10.0	9.6	9.2
5	12.5	12.0	11.5
6	15.0	14.4	13.8
7	17.5	16.8	16.1
8	20.0	19.2	18.4
9	22.5	21.6	20.7

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

62°

Table 3-2. Common logarithms of trigonometric functions (continued)

28°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.67 161	24	9.72 567	31	10.27 433	9.94 593	6	**60**
1	9.67 185	23	9.72 598	30	10.27 402	9.94 587	7	59
2	9.67 208	24	9.72 628	31	10.27 372	9.94 580	7	58
3	9.67 232	24	9.72 659	30	10.27 341	9.94 573	6	57
4	9.67 256	24	9.72 689	31	10.27 311	9.94 567	7	56
5	9.67 280	23	9.72 720	30	10.27 280	9.94 560	7	55
6	9.67 303	24	9.72 750	30	10.27 250	9.94 553	7	54
7	9.67 327	23	9.72 780	31	10.27 220	9.94 546	6	53
8	9.67 350	24	9.72 811	30	10.27 189	9.94 540	7	52
9	9.67 374	24	9.72 841	31	10.27 159	9.94 533	7	51
10	9.67 398	23	9.72 872	30	10.27 128	9.94 526	7	**50**
11	9.67 421	24	9.72 902	30	10.27 098	9.94 519	6	49
12	9.67 445	23	9.72 932	31	10.27 068	9.94 513	7	48
13	9.67 468	24	9.72 963	30	10.27 037	9.94 506	7	47
14	9.67 492	23	9.72 993	30	10.27 007	9.94 499	7	46
15	9.67 515	24	9.73 023	31	10.26 977	9.94 492	7	45
16	9.67 539	23	9.73 054	30	10.26 946	9.94 485	6	44
17	9.67 562	24	9.73 084	30	10.26 916	9.94 479	7	43
18	9.67 586	23	9.73 114	30	10.26 886	9.94 472	7	42
19	9.67 609	24	9.73 144	31	10.26 856	9.94 465	7	41
20	9.67 633	23	9.73 175	30	10.26 825	9.94 458	7	**40**
21	9.67 656	24	9.73 205	30	10.26 795	9.94 451	6	39
22	9.67 680	23	9.73 235	30	10.26 765	9.94 445	7	38
23	9.67 703	23	9.73 265	30	10.26 735	9.94 438	7	37
24	9.67 726	24	9.73 295	31	10.26 705	9.94 431	7	36
25	9.67 750	23	9.73 326	30	10.26 674	9.94 424	7	35
26	9.67 773	23	9.73 356	30	10.26 644	9.94 417	7	34
27	9.67 796	24	9.73 386	30	10.26 614	9.94 410	6	33
28	9.67 820	23	9.73 416	30	10.26 584	9.94 404	7	32
29	9.67 843	23	9.73 446	30	10.26 554	9.94 397	7	31
30	9.67 866	24	9.73 476	31	10.26 524	9.94 390	7	**30**
31	9.67 890	23	9.73 507	30	10.26 493	9.94 383	7	29
32	9.67 913	23	9.73 537	30	10.26 463	9.94 376	7	28
33	9.67 936	23	9.73 567	30	10.26 433	9.94 369	7	27
34	9.67 959	23	9.73 597	30	10.26 403	9.94 362	7	26
35	9.67 982	24	9.73 627	30	10.26 373	9.94 355	6	25
36	9.68 006	23	9.73 657	30	10.26 343	9.94 349	7	24
37	9.68 029	23	9.73 687	30	10.26 313	9.94 342	7	23
38	9.68 052	23	9.73 717	30	10.26 283	9.94 335	7	22
39	9.68 075	23	9.73 747	30	10.26 253	9.94 328	7	21
40	9.68 098	23	9.73 777	30	10.26 223	9.94 321	7	**20**
41	9.68 121	23	9.73 807	30	10.26 193	9.94 314	7	19
42	9.68 144	23	9.73 837	30	10.26 163	9.94 307	7	18
43	9.68 167	23	9.73 867	30	10.26 133	9.94 300	7	17
44	9.68 190	23	9.73 897	30	10.26 103	9.94 293	7	16
45	9.68 213	24	9.73 927	30	10.26 073	9.94 286	7	15
46	9.68 237	23	9.73 957	30	10.26 043	9.94 279	6	14
47	9.68 260	23	9.73 987	30	10.26 013	9.94 273	7	13
48	9.68 283	22	9.74 017	30	10.25 983	9.94 266	7	12
49	9.68 305	23	9.74 047	30	10.25 953	9.94 259	7	11
50	9.68 328	23	9.74 077	30	10.25 923	9.94 252	7	**10**
51	9.68 351	23	9.74 107	30	10.25 893	9.94 245	7	9
52	9.68 374	23	9.74 137	29	10.25 863	9.94 238	7	8
53	9.68 397	23	9.74 166	30	10.25 834	9.94 231	7	7
54	9.68 420	23	9.74 196	30	10.25 804	9.94 224	7	6
55	9.68 443	23	9.74 226	30	10.25 774	9.94 217	7	5
56	9.68 466	23	9.74 256	30	10.25 744	9.94 210	7	4
57	9.68 489	23	9.74 286	30	10.25 714	9.94 203	7	3
58	9.68 512	22	9.74 316	29	10.25 684	9.94 196	7	2
59	9.68 534	23	9.74 345	30	10.25 655	9.94 189	7	1
60	9.68 557		9.74 375		10.25 625	9.94 182		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	31	30	29
1	3.1	3.0	2.9
2	6.2	6.0	5.8
3	9.3	9.0	8.7
4	12.4	12.0	11.6
5	15.5	15.0	14.5
6	18.6	18.0	17.4
7	21.7	21.0	20.3
8	24.8	24.0	23.2
9	27.9	27.0	26.1

	24	23	22
1	2.4	2.3	2.2
2	4.8	4.6	4.4
3	7.2	6.9	6.6
4	9.6	9.2	8.8
5	12.0	11.5	11.0
6	14.4	13.8	13.2
7	16.8	16.1	15.4
8	19.2	18.4	17.6
9	21.6	20.7	19.8

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

Prop. Pts.

61°

Table 3-2. Common logarithms of trigonometric functions (continued)

29°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.68 557	23	9.74 375	30	10.25 625	9.94 182	7	**60**
1	9.68 580	23	9.74 405	30	10.25 595	9.94 175	7	59
2	9.68 603	22	9.74 435	30	10.25 565	9.94 168	7	58
3	9.68 625	23	9.74 465	29	10.25 535	9.94 161	7	57
4	9.68 648	23	9.74 494	30	10.25 506	9.94 154	7	56
5	9.68 671	23	9.74 524	30	10.25 476	9.94 147	7	55
6	9.68 694	22	9.74 554	29	10.25 446	9.94 140	7	54
7	9.68 716	23	9.74 583	30	10.25 417	9.94 133	7	53
8	9.68 739	23	9.74 613	30	10.25 387	9.94 126	7	52
9	9.68 762	22	9.74 643	30	10.25 357	9.94 119	7	51
10	9.68 784	23	9.74 673	29	10.25 327	9.94 112	7	**50**
11	9.68 807	22	9.74 702	30	10.25 298	9.94 105	7	49
12	9.68 829	23	9.74 732	30	10.25 268	9.94 098	8	48
13	9.68 852	23	9.74 762	29	10.25 238	9.94 090	7	47
14	9.68 875	22	9.74 791	30	10.25 209	9.94 083	7	46
15	9.68 897	23	9.74 821	30	10.25 179	9.94 076	7	45
16	9.68 920	22	9.74 851	29	10.25 149	9.94 069	7	44
17	9.68 942	23	9.74 880	30	10.25 120	9.94 062	7	43
18	9.68 965	22	9.74 910	29	10.25 090	9.94 055	7	42
19	9.68 987	23	9.74 939	30	10.25 061	9.94 048	7	41
20	9.69 010	22	9.74 969	29	10.25 031	9.94 041	7	**40**
21	9.69 032	23	9.74 998	30	10.25 002	9.94 034	7	39
22	9.69 055	22	9.75 028	30	10.24 972	9.94 027	7	38
23	9.69 077	23	9.75 058	29	10.24 942	9.94 020	8	37
24	9.69 100	22	9.75 087	30	10.24 913	9.94 012	7	36
25	9.69 122	22	9.75 117	29	10.24 883	9.94 005	7	35
26	9.69 144	23	9.75 146	30	10.24 854	9.93 998	7	34
27	9.69 167	22	9.75 176	29	10.24 824	9.93 991	7	33
28	9.69 189	23	9.75 205	30	10.24 795	9.93 984	7	32
29	9.69 212	22	9.75 235	29	10.24 765	9.93 977	7	31
30	9.69 234	22	9.75 264	30	10.24 736	9.93 970	7	**30**
31	9.69 256	23	9.75 294	29	10.24 706	9.93 963	8	29
32	9.69 279	22	9.75 323	30	10.24 677	9.93 955	7	28
33	9.69 301	22	9.75 353	29	10.24 647	9.93 948	7	27
34	9.69 323	22	9.75 382	29	10.24 618	9.93 941	7	26
35	9.69 345	23	9.75 411	30	10.24 589	9.93 934	7	25
36	9.69 368	22	9.75 441	29	10.24 559	9.93 927	7	24
37	9.69 390	22	9.75 470	30	10.24 530	9.93 920	8	23
38	9.69 412	22	9.75 500	29	10.24 500	9.93 912	7	22
39	9.69 434	22	9.75 529	29	10.24 471	9.93 905	7	21
40	9.69 456	23	9.75 558	30	10.24 442	9.93 898	7	**20**
41	9.69 479	22	9.75 588	29	10.24 412	9.93 891	7	19
42	9.69 501	22	9.75 617	30	10.24 383	9.93 884	8	18
43	9.69 523	22	9.75 647	29	10.24 353	9.93 876	7	17
44	9.69 545	22	9.75 676	29	10.24 324	9.93 869	7	16
45	9.69 567	22	9.75 705	30	10.24 295	9.93 862	7	15
46	9.69 589	22	9.75 735	29	10.24 265	9.93 855	8	14
47	9.69 611	22	9.75 764	29	10.24 236	9.93 847	7	13
48	9.69 633	22	9.75 793	29	10.24 207	9.93 840	7	12
49	9.69 655	22	9.75 822	30	10.24 178	9.93 833	7	11
50	9.69 677	22	9.75 852	29	10.24 148	9.93 826	7	**10**
51	9.69 699	22	9.75 881	29	10.24 119	9.93 819	8	9
52	9.69 721	22	9.75 910	29	10.24 090	9.93 811	7	8
53	9.69 743	22	9.75 939	30	10.24 061	9.93 804	7	7
54	9.69 765	22	9.75 969	29	10.24 031	9.93 797	8	6
55	9.69 787	22	9.75 998	29	10.24 002	9.93 789	7	5
56	9.69 809	22	9.76 027	29	10.23 973	9.93 782	7	4
57	9.69 831	22	9.76 056	30	10.23 944	9.93 775	7	3
58	9.69 853	22	9.76 086	29	10.23 914	9.93 768	8	2
59	9.69 875	22	9.76 115	29	10.23 885	9.93 760	7	1
60	9.69 897		9.76 144		10.23 856	9.93 753		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	30	**29**
1	3.0	2.9
2	6.0	5.8
3	9.0	8.7
4	12.0	11.6
5	15.0	14.5
6	18.0	17.4
7	21.0	20.3
8	24.0	23.2
9	27.0	26.1

	23	**22**
1	2.3	2.2
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

	8	**7**
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

Prop. Pts.

60°

Table 3-2: Common logarithms of trigonometric functions (continued)

30°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.69 897	22	9.76 144	29	10.23 856	9.93 753	7	**60**
1	9.69 919	22	9.76 173	29	10.23 827	9.93 746	8	59
2	9.69 941	22	9.76 202	29	10.23 798	9.93 738	7	58
3	9.69 963	21	9.76 231	30	10.23 769	9.93 731	7	57
4	9.69 984	22	9.76 261	29	10.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	10.23 710	9.93 717	8	55
6	9.70 028	22	9.76 319	29	10.23 681	9.93 709	7	54
7	9.70 050	22	9.76 348	29	10.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	10.23 623	9.93 695	8	52
9	9.70 093	22	9.76 406	29	10.23 594	9.93 687	7	51
10	9.70 115	22	9.76 435	29	10.23 565	9.93 680	7	**50**
11	9.70 137	22	9.76 464	29	10.23 536	9.93 673	8	49
12	9.70 159	21	9.76 493	29	10.23 507	9.93 665	7	48
13	9.70 180	22	9.76 522	29	10.23 478	9.93 658	8	47
14	9.70 202	22	9.76 551	29	10.23 449	9.93 650	7	46
15	9.70 224	21	9.76 580	29	10.23 420	9.93 643	7	45
16	9.70 245	22	9.76 609	30	10.23 391	9.93 636	8	44
17	9.70 267	21	9.76 639	29	10.23 361	9.93 628	7	43
18	9.70 288	22	9.76 668	29	10.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	10.23 303	9.93 614	8	41
20	9.70 332	21	9.76 725	29	10.23 275	9.93 606	7	**40**
21	9.70 353	22	9.76 754	29	10.23 246	9.93 599	8	39
22	9.70 375	21	9.76 783	29	10.23 217	9.93 591	7	38
23	9.70 396	22	9.76 812	29	10.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	10.23 159	9.93 577	8	36
25	9.70 439	22	9.76 870	29	10.23 130	9.93 569	7	35
26	9.70 461	21	9.76 899	29	10.23 101	9.93 562	8	34
27	9.70 482	22	9.76 928	29	10.23 072	9.93 554	7	33
28	9.70 504	21	9.76 957	29	10.23 043	9.93 547	8	32
29	9.70 525	22	9.76 986	29	10.23 014	9.93 539	7	31
30	9.70 547	21	9.77 015	29	10.22 985	9.93 532	7	**30**
31	9.70 568	22	9.77 044	29	10.22 956	9.93 525	8	29
32	9.70 590	21	9.77 073	28	10.22 927	9.93 517	7	28
33	9.70 611	22	9.77 101	29	10.22 899	9.93 510	8	27
34	9.70 633	21	9.77 130	29	10.22 870	9.93 502	7	26
35	9.70 654	21	9.77 159	29	10.22 841	9.93 495	8	25
36	9.70 675	22	9.77 188	29	10.22 812	9.93 487	7	24
37	9.70 796	21	9.77 217	29	10.22 783	9.93 480	8	23
38	9.70 718	21	9.77 246	28	10.22 754	9.93 472	7	22
39	9.70 739	22	9.77 274	29	10.22 726	9.93 465	8	21
40	9.70 761	21	9.77 303	29	10.22 697	9.93 457	7	**20**
41	9.70 782	21	9.77 332	29	10.22 668	9.93 450	8	19
42	9.70 803	21	9.77 361	29	10.22 639	9.93 442	7	18
43	9.70 824	22	9.77 390	28	10.22 610	9.93 435	8	17
44	9.70 846	21	9.77 418	29	10.22 582	9.93 427	7	16
45	9.70 867	21	9.77 447	29	10.22 553	9.93 420	8	15
46	9.70 888	21	9.77 476	29	10.22 524	9.93 412	7	14
47	9.70 909	22	9.77 505	28	10.22 495	9.93 405	8	13
48	9.70 931	21	9.77 533	29	10.22 467	9.93 397	7	12
49	9.70 952	21	9.77 562	29	10.22 438	9.93 390	8	11
50	9.70 973	21	9.77 591	28	10.22 409	9.93 382	7	**10**
51	9.70 994	21	9.77 619	29	10.22 381	9.93 375	8	9
52	9.71 015	21	9.77 648	29	10.22 352	9.93 367	7	8
53	9.71 036	22	9.77 677	29	10.22 323	9.93 360	8	7
54	9.71 058	21	9.77 706	28	10.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	29	10.22 266	9.93 344	7	5
56	9.71 100	21	9.77 763	28	10.22 237	9.93 337	8	4
57	9.71 121	21	9.77 791	29	10.22 209	9.93 329	7	3
58	9.71 142	21	9.77 820	29	10.22 180	9.93 322	8	2
59	9.71 163	21	9.77 849	28	10.22 151	9.93 314	7	1
60	9.71 184		9.77 877		10.22 123	9.93 307		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

59°

Prop. Pts.

	30	29	28
1	3.0	2.9	2.8
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

31°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Pts.
0	9.71 184		9.77 877		10.22 123	9.93 307		60	
1	9.71 205	21	9.77 906	29	10.22 094	9.93 299	8	59	
2	9.71 226	21	9.77 935	29	10.22 065	9.93 291	8	58	
3	9.71 247	21	9.77 963	28	10.22 037	9.93 284	7	57	
4	9.71 268	21	9.77 992	29	10.22 008	9.93 276	8	56	
5	9.71 289	21	9.78 020	28	10.21 980	9.93 269	7	55	
6	9.71 310	21	9.78 049	29	10.21 951	9.93 261	8	54	
7	9.71 331	21	9.78 077	28	10.21 923	9.93 253	8	53	
8	9.71 352	21	9.78 106	29	10.21 894	9.93 246	7	52	
9	9.71 373	21	9.78 135	29	10.21 865	9.93 238	8	51	
10	9.71 393	20	9.78 163	28	10.21 837	9.93 230	8	50	
11	9.71 414	21	9.78 192	29	10.21 808	9.93 223	7	49	
12	9.71 435	21	9.78 220	28	10.21 780	9.93 215	8	48	
13	9.71 456	21	9.78 249	29	10.21 751	9.93 207	8	47	
14	9.71 477	21	9.78 277	28	10.21 723	9.93 200	7	46	
15	9.71 498	21	9.78 306	29	10.21 694	9.93 192	8	45	
16	9.71 519	21	9.78 334	28	10.21 666	9.93 184	8	44	
17	9.71 539	20	9.78 363	29	10.21 637	9.93 177	7	43	
18	9.71 560	21	9.78 391	28	10.21 609	9.93 169	8	42	
19	9.71 581	21	9.78 419	28	10.21 581	9.93 161	8	41	
20	9.71 602	21	9.78 448	29	10.21 552	9.93 154	7	40	
21	9.71 622	20	9.78 476	28	10.21 524	9.93 146	8	39	
22	9.71 643	21	9.78 505	29	10.21 495	9.93 138	8	38	
23	9.71 664	21	9.78 533	28	10.21 467	9.93 131	7	37	
24	9.71 685	21	9.78 562	29	10.21 438	9.93 123	8	36	
25	9.71 705	20	9.78 590	28	10.21 410	9.93 115	8	35	
26	9.71 726	21	9.78 618	28	10.21 382	9.93 108	7	34	
27	9.71 747	21	9.78 647	29	10.21 353	9.93 100	8	33	
28	9.71 767	20	9.78 675	28	10.21 325	9.93 092	8	32	
29	9.71 788	21	9.78 704	29	10.21 296	9.93 084	8	31	
30	9.71 809	21	9.78 732	28	10.21 268	9.93 077	7	30	
31	9.71 829	20	9.78 760	28	10.21 240	9.93 069	8	29	
32	9.71 850	21	9.78 789	29	10.21 211	9.93 061	8	28	
33	9.71 870	20	9.78 817	28	10.21 183	9.93 053	8	27	
34	9.71 891	21	9.78 845	28	10.21 155	9.93 046	7	26	
35	9.71 911	20	9.78 874	29	10.21 126	9.93 038	8	25	
36	9.71 932	21	9.78 902	28	10.21 098	9.93 030	8	24	
37	9.71 952	20	9.78 930	28	10.21 070	9.93 022	8	23	
38	9.71 973	21	9.78 959	29	10.21 041	9.93 014	8	22	
39	9.71 994	21	9.78 987	28	10.21 013	9.93 007	7	21	
40	9.72 014	20	9.79 015	28	10.20 985	9.92 999	8	20	
41	9.72 034	20	9.79 043	28	10.20 957	9.92 991	8	19	
42	9.72 055	21	9.79 072	29	10.20 928	9.92 983	8	18	
43	9.72 075	20	9.79 100	28	10.20 900	9.92 976	7	17	
44	9.72 096	21	9.79 128	28	10.20 872	9.92 968	8	16	
45	9.72 116	20	9.79 156	28	10.20 844	9.92 960	8	15	
46	9.72 137	21	9.79 185	29	10.20 815	9.92 952	8	14	
47	9.72 157	20	9.79 213	28	10.20 787	9.92 944	8	13	
48	9.72 177	20	9.79 241	28	10.20 759	9.92 936	8	12	
49	9.72 198	21	9.79 269	28	10.20 731	9.92 929	7	11	
50	9.72 218	20	9.79 297	28	10.20 703	9.92 921	8	10	
51	9.72 238	20	9.79 326	29	10.20 674	9.92 913	8	9	
52	9.72 259	21	9.79 354	28	10.20 646	9.92 905	8	8	
53	9.72 279	20	9.79 382	28	10.20 618	9.92 897	8	7	
54	9.72 299	20	9.79 410	28	10.20 590	9.92 889	8	6	
55	9.72 320	21	9.79 438	28	10.20 562	9.92 881	8	5	
56	9.72 340	20	9.79 466	28	10.20 534	9.92 874	7	4	
57	9.72 360	20	9.79 495	29	10.20 505	9.92 866	8	3	
58	9.72 381	21	9.79 523	28	10.20 477	9.92 858	8	2	
59	9.72 401	20	9.79 551	28	10.20 449	9.92 850	8	1	
60	9.72 421	20	9.79 579	28	10.20 421	9.92 842	8	0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	Prop. Pts.

	29	28
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	21	20
1	2.1	2.0
2	4.2	4.0
3	6.3	6.0
4	8.4	8.0
5	10.5	10.0
6	12.6	12.0
7	14.7	14.0
8	16.8	16.0
9	18.9	18.0

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

58°

Table 3-2. Common logarithms of trigonometric functions (continued)

32°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.72 421	20	9.79 579	28	10.20 421	9.92 842	8	**60**
1	9.72 441	20	9.79 607	28	10.20 393	9.92 834	8	59
2	9.72 461	21	9.79 635	28	10.20 365	9.92 826	8	58
3	9.72 482	20	9.79 663	28	10.20 337	9.92 818	8	57
4	9.72 502	20	9.79 691	28	10.20 309	9.92 810	7	56
5	9.72 522	20	9.79 719	28	10.20 281	9.92 803	8	55
6	9.72 542	20	9.79 747	29	10.20 253	9.92 795	8	54
7	9.72 562	20	9.79 776	28	10.20 224	9.92 787	8	53
8	9.72 582	20	9.79 804	28	10.20 196	9.92 779	8	52
9	9.72 602	20	9.79 832	28	10.20 168	9.92 771	8	51
10	9.72 622	21	9.79 860	28	10.20 140	9.92 763	8	**50**
11	9.72 643	20	9.79 888	28	10.20 112	9.92 755	8	49
12	9.72 663	20	9.79 916	28	10.20 084	9.92 747	8	48
13	9.72 683	20	9.79 944	28	10.20 056	9.92 739	8	47
14	9.72 703	20	9.79 972	28	10.20 028	9.92 731	8	46
15	9.72 723	20	9.80 000	28	10.20 000	9.92 723	8	45
16	9.72 743	20	9.80 028	28	10.19 972	9.92 715	8	44
17	9.72 763	20	9.80 056	28	10.19 944	9.92 707	8	43
18	9.72 783	20	9.80 084	28	10.19 916	9.92 699	8	42
19	9.72 803	20	9.80 112	28	10.19 888	9.92 691	8	41
20	9.72 823	20	9.80 140	28	10.19 860	9.92 683	8	**40**
21	9.72 843	20	9.80 168	27	10.19 832	9.92 675	8	39
22	9.72 863	20	9.80 195	28	10.19 805	9.92 667	8	38
23	9.72 883	19	9.80 223	28	10.19 777	9.92 659	8	37
24	9.72 902	20	9.80 251	28	10.19 749	9.92 651	8	36
25	9.72 922	20	9.80 279	28	10.19 721	9.92 643	8	35
26	9.72 942	20	9.80 307	28	10.19 693	9.92 635	8	34
27	9.72 962	20	9.80 335	28	10.19 665	9.92 627	8	33
28	9.72 982	20	9.80 363	28	10.19 637	9.92 619	8	32
29	9.73 002	20	9.80 391	28	10.19 609	9.92 611	8	31
30	9.73 022	19	9.80 419	28	10.19 581	9.92 603	8	**30**
31	9.73 041	20	9.80 447	27	10.19 553	9.92 595	8	29
32	9.73 061	20	9.80 474	28	10.19 526	9.92 587	8	28
33	9.73 081	20	9.80 502	28	10.19 498	9.92 579	8	27
34	9.73 101	20	9.80 530	28	10.19 470	9.92 571	8	26
35	9.73 121	19	9.80 558	28	10.19 442	9.92 563	8	25
36	9.73 140	20	9.80 586	28	10.19 414	9.92 555	9	24
37	9.73 160	20	9.80 614	28	10.19 386	9.92 546	8	23
38	9.73 180	20	9.80 642	27	10.19 358	9.92 538	8	22
39	9.73 200	19	9.80 669	28	10.19 331	9.92 530	8	21
40	9.73 219	20	9.80 697	28	10.19 303	9.92 522	8	**20**
41	9.73 239	20	9.80 725	28	10.19 275	9.92 514	8	19
42	9.73 259	19	9.80 753	28	10.19 247	9.92 506	8	18
43	9.73 278	20	9.80 781	27	10.19 219	9.92 498	8	17
44	9.73 298	20	9.80 808	28	10.19 192	9.92 490	8	16
45	9.73 318	19	9.80 836	28	10.19 164	9.92 482	9	15
46	9.73 337	20	9.80 864	28	10.19 136	9.92 473	8	14
47	9.73 357	20	9.80 892	27	10.19 108	9.92 465	8	13
48	9.73 377	19	9.80 919	28	10.19 081	9.92 457	8	12
49	9.73 396	20	9.80 947	28	10.19 053	9.92 449	8	11
50	9.73 416	19	9.80 975	28	10.19 025	9.92 441	8	**10**
51	9.73 435	20	9.81 003	27	10.18 997	9.92 433	8	9
52	9.73 455	19	9.81 030	28	10.18 970	9.92 425	9	8
53	9.73 474	20	9.81 058	28	10.18 942	9.92 416	8	7
54	9.73 494	19	9.81 086	27	10.18 914	9.92 408	8	6
55	9.73 513	20	9.81 113	28	10.18 887	9.92 400	8	5
56	9.73 533	19	9.81 141	28	10.18 859	9.92 392	8	4
57	9.73 552	20	9.81 169	27	10.18 831	9.92 384	8	3
58	9.73 572	19	9.81 196	28	10.18 804	9.92 376	9	2
59	9.73 591	20	9.81 224	28	10.18 776	9.92 367	8	1
60	9.73 611		9.81 252		10.18 748	9.92 359		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	29	28	27
1	2.9	2.8	2.7
2	5.8	5.6	5.4
3	8.7	8.4	8.1
4	11.6	11.2	10.8
5	14.5	14.0	13.5
6	17.4	16.8	16.2
7	20.3	19.6	18.9
8	23.2	22.4	21.6
9	26.1	25.2	24.3

	21	20	19
1	2.1	2.0	1.9
2	4.2	4.0	3.8
3	6.3	6.0	5.7
4	8.4	8.0	7.6
5	10.5	10.0	9.5
6	12.6	12.0	11.4
7	14.7	14.0	13.3
8	16.8	16.0	15.2
9	18.9	18.0	17.1

	9	8	7
1	0.9	0.8	0.7
2	1.8	1.6	1.4
3	2.7	2.4	2.1
4	3.6	3.2	2.8
5	4.5	4.0	3.5
6	5.4	4.8	4.2
7	6.3	5.6	4.9
8	7.2	6.4	5.6
9	8.1	7.2	6.3

Prop. Pts.

57°

Table 3-2. Common logarithms of trigonometric functions (continued)

33°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Pts.
0	9.73 611	19	9.81 252	27	10.18 748	9.92 359	8	**60**	
1	9.73 630	20	9.81 279	28	10.18 721	9.92 351	8	59	
2	9.73 650	19	9.81 307	28	10.18 693	9.92 343	8	58	
3	9.73 669	20	9.81 335	27	10.18 665	9.92 335	9	57	
4	9.73 689	19	9.81 362	28	10.18 638	9.92 326	8	56	
5	9.73 708	19	9.81 390	28	10.18 610	9.92 318	8	55	
6	9.73 727	20	9.81 418	27	10.18 582	9.92 310	8	54	
7	9.73 747	19	9.81 445	28	10.18 555	9.92 302	9	53	
8	9.73 766	19	9.81 473	27	10.18 527	9.92 293	8	52	
9	9.73 785	20	9.81 500	28	10.18 500	9.92 285	8	51	
10	9.73 805	19	9.81 528	28	10.18 472	9.92 277	8	**50**	
11	9.73 824	19	9.81 556	27	10.18 444	9.92 269	9	49	
12	9.73 843	20	9.81 583	28	10.18 417	9.92 260	8	48	
13	9.73 863	19	9.81 611	27	10.18 389	9.92 252	8	47	
14	9.73 882	19	9.81 638	28	10.18 362	9.92 244	9	46	
15	9.73 901	20	9.81 666	27	10.18 334	9.92 235	8	45	
16	9.73 921	19	9.81 693	28	10.18 307	9.92 227	8	44	
17	9.73 940	19	9.81 721	27	10.18 279	9.92 219	8	43	
18	9.73 959	19	9.81 748	28	10.18 252	9.92 211	9	42	
19	9.73 978	19	9.81 776	27	10.18 224	9.92 202	8	41	
20	9.73 997	20	9.81 803	28	10.18 197	9.92 194	8	**40**	
21	9.74 017	19	9.81 831	27	10.18 169	9.92 186	9	39	
22	9.74 036	19	9.81 858	28	10.18 142	9.92 177	8	38	
23	9.74 055	19	9.81 886	27	10.18 114	9.92 169	8	37	
24	9.74 074	19	9.81 913	28	10.18 087	9.92 161	9	36	
25	9.74 093	20	9.81 941	27	10.18 059	9.92 152	8	35	
26	9.74 113	19	9.81 968	28	10.18 032	9.92 144	8	34	
27	9.74 132	19	9.81 996	27	10.18 004	9.92 136	9	33	
28	9.74 151	19	9.82 023	28	10.17 977	9.92 127	8	32	
29	9.74 170	19	9.82 051	27	10.17 949	9.92 119	8	31	
30	9.74 189	19	9.82 078	28	10.17 922	9.92 111	9	**30**	
31	9.74 208	19	9.82 106	27	10.17 894	9.92 102	8	29	
32	9.74 227	19	9.82 133	28	10.17 867	9.92 094	8	28	
33	9.74 246	19	9.82 161	27	10.17 839	9.92 086	9	27	
34	9.74 265	19	9.82 188	27	10.17 812	9.92 077	8	26	
35	9.74 284	19	9.82 215	28	10.17 785	9.92 069	9	25	
36	9.74 303	19	9.82 243	27	10.17 757	9.92 060	8	24	
37	9.74 322	19	9.82 270	28	10.17 730	9.92 052	8	23	
38	9.74 341	19	9.82 298	27	10.17 702	9.92 044	9	22	
39	9.74 360	19	9.82 325	27	10.17 675	9.92 035	8	21	
40	9.74 379	19	9.82 352	28	10.17 648	9.92 027	9	**20**	
41	9.74 398	19	9.82 380	27	10.17 620	9.92 018	8	19	
42	9.74 417	19	9.82 407	28	10.17 593	9.92 010	8	18	
43	9.74 436	19	9.82 435	27	10.17 565	9.92 002	9	17	
44	9.74 455	19	9.82 462	27	10.17 538	9.91 993	8	16	
45	9.74 474	19	9.82 489	28	10.17 511	9.91 985	9	15	
46	9.74 493	19	9.82 517	27	10.17 483	9.91 976	8	14	
47	9.74 512	19	9.82 544	27	10.17 456	9.91 968	9	13	
48	9.74 531	18	9.82 571	28	10.17 429	9.91 959	8	12	
49	9.74 549	19	9.82 599	27	10.17 401	9.91 951	9	11	
50	9.74 568	19	9.82 626	27	10.17 374	9.91 942	8	**10**	
51	9.74 587	19	9.82 653	28	10.17 347	9.91 934	9	9	
52	9.74 606	19	9.82 681	27	10.17 319	9.91 925	8	8	
53	9.74 625	19	9.82 708	27	10.17 292	9.91 917	9	7	
54	9.74 644	18	9.82 735	27	10.17 265	9.91 908	8	6	
55	9.74 662	19	9.82 762	28	10.17 238	9.91 900	9	5	
56	9.74 681	19	9.82 790	27	10.17 210	9.91 891	8	4	
57	9.74 700	19	9.82 817	27	10.17 183	9.91 883	9	3	
58	9.74 719	18	9.82 844	27	10.17 156	9.91 874	8	2	
59	9.74 737	19	9.82 871	28	10.17 129	9.91 866	9	1	
60	9.74 756		9.82 899		10.17 101	9.91 857		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	Prop. Pts.

	28	**27**
1	2.8	2.7
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	20	**19**	**18**
1	2.0	1.9	1.8
2	4.0	3.8	3.6
3	6.0	5.7	5.4
4	8.0	7.6	7.2
5	10.0	9.5	9.0
6	12.0	11.4	10.8
7	14.0	13.3	12.6
8	16.0	15.2	14.4
9	18.0	17.1	16.2

	9	**8**
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

56°

Table 3-2. Common logarithms of trigonometric functions (continued)

34°

'	L Sin	d	L Tan	c d	L Cot	L Cos	d	'
0	9.74 756	19	9.82 899	27	10.17 101	9.91 857	8	**60**
1	9.74 775	19	9.82 926	27	10.17 074	9.91 849	9	59
2	9.74 794	18	9.82 953	27	10.17 047	9.91 840	8	58
3	9.74 812	19	9.82 980	28	10.17 020	9.91 832	9	57
4	9.74 831	19	9.83 008	27	10.16 992	9.91 823	8	56
5	9.74 850	18	9.83 035	27	10.16 965	9.91 815	9	55
6	9.74 868	19	9.83 062	27	10.16 938	9.91 806	8	54
7	9.74 887	19	9.83 089	28	10.16 911	9.91 798	9	53
8	9.74 906	18	9.83 117	27	10.16 883	9.91 789	8	52
9	9.74 924	19	9.83 144	27	10.16 856	9.91 781	9	51
10	9.74 943	18	9.83 171	27	10.16 829	9.91 772	9	**50**
11	9.74 961	19	9.83 198	27	10.16 802	9.91 763	8	49
12	9.74 980	19	9.83 225	27	10.16 775	9.91 755	9	48
13	9.74 999	18	9.83 252	28	10.16 748	9.91 746	8	47
14	9.75 017	19	9.83 280	27	10.16 720	9.91 738	9	46
15	9.75 036	18	9.83 307	27	10.16 693	9.91 729	9	45
16	9.75 054	19	9.83 334	27	10.16 666	9.91 720	8	44
17	9.75 073	18	9.83 361	27	10.16 639	9.91 712	9	43
18	9.75 091	19	9.83 388	27	10.16 612	9.91 703	8	42
19	9.75 110	18	9.83 415	27	10.16 585	9.91 695	9	41
20	9.75 128	19	9.83 442	28	10.16 558	9.91 686	9	**40**
21	9.75 147	18	9.83 470	27	10.16 530	9.91 677	8	39
22	9.75 165	19	9.83 497	27	10.16 503	9.91 669	9	38
23	9.75 184	18	9.83 524	27	10.16 476	9.91 660	9	37
24	9.75 202	19	9.83 551	27	10.16 449	9.91 651	8	36
25	9.75 221	18	9.83 578	27	10.16 422	9.91 643	9	35
26	9.75 239	19	9.83 605	27	10.16 395	9.91 634	9	34
27	9.75 258	18	9.83 632	27	10.16 368	9.91 625	8	33
28	9.75 276	18	9.83 659	27	10.16 341	9.91 617	9	32
29	9.75 294	19	9.83 686	27	10.16 314	9.91 608	9	31
30	9.75 313	18	9.83 713	27	10.16 287	9.91 599	8	**30**
31	9.75 331	19	9.83 740	28	10.16 260	9.91 591	9	29
32	9.75 350	18	9.83 768	27	10.16 232	9.91 582	9	28
33	9.75 368	18	9.83 795	27	10.16 205	9.91 573	8	27
34	9.75 386	19	9.83 822	27	10.16 178	9.91 565	9	26
35	9.75 405	18	9.83 849	27	10.16 151	9.91 556	9	25
36	9.75 423	18	9.83 876	27	10.16 124	9.91 547	9	24
37	9.75 441	18	9.83 903	27	10.16 097	9.91 538	8	23
38	9.75 459	19	9.83 930	27	10.16 070	9.91 530	9	22
39	9.75 478	18	9.83 957	27	10.16 043	9.91 521	9	21
40	9.75 496	18	9.83 984	27	10.16 016	9.91 512	8	**20**
41	9.75 514	19	9.84 011	27	10.15 989	9.91 504	9	19
42	9.75 533	18	9.84 038	27	10.15 962	9.91 495	9	18
43	9.75 551	18	9.84 065	27	10.15 935	9.91 486	9	17
44	9.75 569	18	9.84 092	27	10.15 908	9.91 477	8	16
45	9.75 587	18	9.84 119	27	10.15 881	9.91 469	9	15
46	9.75 605	19	9.84 146	27	10.15 854	9.91 460	9	14
47	9.75 624	18	9.84 173	27	10.15 827	9.91 451	9	13
48	9.75 642	18	9.84 200	27	10.15 800	9.91 442	9	12
49	9.75 660	18	9.84 227	27	10.15 773	9.91 433	8	11
50	9.75 678	18	9.84 254	26	10.15 746	9.91 425	9	**10**
51	9.75 696	18	9.84 280	27	10.15 720	9.91 416	9	9
52	9.75 714	19	9.84 307	27	10.15 693	9.91 407	9	8
53	9.75 733	18	9.84 334	27	10.15 666	9.91 398	9	7
54	9.75 751	18	9.84 361	27	10.15 639	9.91 389	8	6
55	9.75 769	18	9.84 388	27	10.15 612	9.91 381	9	5
56	9.75 787	18	9.84 415	27	10.15 585	9.91 372	9	4
57	9.75 805	18	9.84 442	27	10.15 558	9.91 363	9	3
58	9.75 823	18	9.84 469	27	10.15 531	9.91 354	9	2
59	9.75 841	18	9.84 496	27	10.15 504	9.91 345	9	1
60	9.75 859		9.84 523		10.15 477	9.91 336		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	'

Prop. Pts.

	28	27	26
1	2.8	2.7	2.6
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	19	18
1	1.9	1.8
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

Prop. Pts.

55°

Table 3-2. Common logarithms of trigonometric functions (continued)

35°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.75 859	18	9.84 523	27	10.15 477	9.91 336	8	**60**
1	9.75 877	18	9.84 550	26	10.15 450	9.91 328	9	59
2	9.75 895	18	9.84 576	27	10.15 424	9.91 319	9	58
3	9.75 913	18	9.84 603	27	10.15 397	9.91 310	9	57
4	9.75 931	18	9.84 630	27	10.15 370	9.91 301	9	56
5	9.75 949	18	9.84 657	27	10.15 343	9.91 292	9	55
6	9.75 967	18	9.84 684	27	10.15 316	9.91 283	9	54
7	9.75 985	18	9.84 711	27	10.15 289	9.91 274	8	53
8	9.76 003	18	9.84 738	26	10.15 262	9.91 266	9	52
9	9.76 021	18	9.84 764	27	10.15 236	9.91 257	9	51
10	9.76 039	18	9.84 791	27	10.15 209	9.91 248	9	**50**
11	9.76 057	18	9.84 818	27	10.15 182	9.91 239	9	49
12	9.76 075	18	9.84 845	27	10.15 155	9.91 230	9	48
13	9.76 093	18	9.84 872	27	10.15 128	9.91 221	9	47
14	9.76 111	18	9.84 899	26	10.15 101	9.91 212	9	46
15	9.76 129	17	9.84 925	27	10.15 075	9.91 203	9	45
16	9.76 146	18	9.84 952	27	10.15 048	9.91 194	9	44
17	9.76 164	18	9.84 979	27	10.15 021	9.91 185	9	43
18	9.76 182	18	9.85 006	27	10.14 994	9.91 176	9	42
19	9.76 200	18	9.85 033	26	10.14 967	9.91 167	9	41
20	9.76 218	18	9.85 059	27	10.14 941	9.91 158	9	**40**
21	9.76 236	17	9.85 086	27	10.14 914	9.91 149	8	39
22	9.76 253	18	9.85 113	27	19.14 887	9.91 141	9	38
23	9.76 271	18	9.85 140	26	10.14 860	9.91 132	9	37
24	9.76 289	18	9.85 166	27	10.14 834	9.91 123	9	36
25	9.76 307	17	9.85 193	27	10.14 807	9.91 114	9	35
26	9.76 324	18	9.85 220	27	10.14 780	9.91 105	9	34
27	9.76 342	18	9.85 247	26	10.14 753	9.91 096	9	33
28	9.76 360	18	9.85 273	27	10.14 727	9.91 087	9	32
29	9.76 378	17	9.85 300	27	10.14 700	9.91 078	9	31
30	9.76 395	18	9.85 327	27	10.14 673	9.91 069	9	**30**
31	9.76 413	18	9.85 354	26	10.14 646	9.91 060	9	29
32	9.76 431	17	9.85 380	27	10.14 620	9.91 051	9	28
33	9.76 448	18	9.85 407	27	10.14 593	9.91 042	9	27
34	9.76 466	18	9.85 434	26	10.14 566	9.91 033	10	26
35	9.76 484	17	9.85 460	27	10.14 540	9.91 023	9	25
36	9.76 501	18	9.85 487	27	10.14 513	9.91 014	9	24
37	9.76 519	18	9.85 514	26	10.14 486	9.91 005	9	23
38	9.76 537	17	9.85 540	27	10.14 460	9.90 996	9	22
39	9.76 554	18	9.85 567	27	10.14 433	9.90 987	9	21
40	9.76 572	18	9.85 594	26	10.14 406	9.90 978	9	**20**
41	9.76 590	17	9.85 620	27	10.14 380	9.90 969	9	19
42	9.76 607	18	9.85 647	27	10.14 353	9.90 960	9	18
43	9.76 625	17	9.85 674	26	10.14 326	9.90 951	9	17
44	9.76 642	18	9.85 700	27	10.14 300	9.90 942	9	16
45	9.76 660	17	9.85 727	27	10.14 273	9.90 933	9	15
46	9.76 677	18	9.85 754	26	10.14 246	9.90 924	9	14
47	9.76 695	17	9.85 780	27	10.14 220	9.90 915	9	13
48	9.76 712	18	9.85 807	27	10.14 193	9.90 906	10	12
49	9.76 730	17	9.85 834	26	10.14 166	9.90 896	9	11
50	9.76 747	18	9.85 860	27	10.14 140	9.90 887	9	**10**
51	9.76 765	17	9.85 887	26	10.14 113	9.90 878	9	9
52	9.76 782	18	9.85 913	27	10.14 087	9.90 869	9	8
53	9.76 800	17	9.85 940	27	10.14 060	9.90 860	9	7
54	9.76 817	18	9.85 967	26	10.14 033	9.90 851	9	6
55	9.76 835	17	9.85 993	27	10.14 007	9.90 842	10	5
56	9.76 852	18	9.86 020	26	10.13 980	9.90 832	9	4
57	9.76 870	17	9.86 046	27	10.13 954	9.90 823	9	3
58	9.76 887	17	9.86 073	27	10.13 927	9.90 814	9	2
59	9.76 904	18	9.86 100	26	10.13 900	9.90 805	9	1
60	9.76 922		9.86 126		10.13 874	9.90 796		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

	10	9	8
1	1.0	0.9	0.8
2	2.0	1.8	1.6
3	3.0	2.7	2.4
4	4.0	3.6	3.2
5	5.0	4.5	4.0
6	6.0	5.4	4.8
7	7.0	6.3	5.6
8	8.0	7.2	6.4
9	9.0	8.1	7.2

Prop. Pts.

54°

Table 3-2. Common logarithms of trigonometric functions (continued)

36°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.76 922	17	9.86 126	27	10.13 874	9.90 796	9	60
1	9.76 939	18	9.86 153	26	10.13 847	9.90 787	10	59
2	9.76 957	17	9.86 179	27	10.13 821	9.90 777	9	58
3	9.76 974	17	9.86 206	26	10.13 794	9.90 768	9	57
4	9.76 991	18	9.86 232	27	10.13 768	9.90 759	9	56
5	9.77 009	17	9.86 259	26	10.13 741	9.90 750	9	55
6	9.77 026	17	9.86 285	27	10.13 715	9.90 741	10	54
7	9.77 043	18	9.86 312	26	10.13 688	9.90 731	9	53
8	9.77 061	17	9.86 338	27	10.13 662	9.90 722	9	52
9	9.77 078	17	9.86 365	27	10.13 635	9.90 713	9	51
10	9.77 095	17	9.86 392	26	10.13 608	9.90 704	10	50
11	9.77 112	18	9.86 418	27	10.13 582	9.90 694	9	49
12	9.77 130	17	9.86 445	26	10.13 555	9.90 685	9	48
13	9.77 147	17	9.86 471	27	10.13 529	9.90 676	9	47
14	9.77 164	17	9.86 498	26	10.13 502	9.90 667	10	46
15	9.77 181	18	9.86 524	27	10.13 476	9.90 657	9	45
16	9.77 199	17	9.86 551	26	10.13 449	9.90 648	9	44
17	9.77 216	17	9.86 577	26	10.13 423	9.90 639	9	43
18	9.77 233	17	9.86 603	27	10.13 397	9.90 630	10	42
19	9.77 250	18	9.86 630	26	10.13 370	9.90 620	9	41
20	9.77 268	17	9.86 656	27	10.13 344	9.90 611	9	40
21	9.77 285	17	9.86 683	26	10.13 317	9.90 602	10	39
22	9.77 302	17	9.86 709	27	10.13 291	9.90 592	9	38
23	9.77 319	17	9.86 736	26	10.13 264	9.90 583	9	37
24	9.77 336	17	9.86 762	27	10.13 238	9.90 574	9	36
25	9.77 353	17	9.86 789	26	10.13 211	9.90 565	10	35
26	9.77 370	17	9.86 815	27	10.13 185	9.90 555	9	34
27	9.77 387	18	9.86 842	26	10.13 158	9.90 546	9	33
28	9.77 405	17	9 86 868	26	10.13 132	9.90 537	10	32
29	9.77 422	17	9.86 894	27	10.13 106	9.90 527	9	31
30	9.77 439	17	9.86 921	26	10.13 079	9.90 518	9	30
31	9.77 456	17	9.86 947	27	10.13 053	9.90 509	10	29
32	9.77 473	17	9.86 974	26	10.13 026	9.90 499	9	28
33	9.77 490	17	9.87 000	27	10.13 000	9.90 490	10	27
34	9.77 507	17	9.87 027	26	10.12 973	9.90 480	9	26
35	9.77 524	17	9.87 053	26	10.12 947	9.90 471	9	25
36	9.77 541	17	9.87 079	27	10.12 921	9.90 462	10	24
37	9.77 558	17	9.87 106	26	10.12 894	9.90 452	9	23
38	9.77 575	17	9.87 132	26	10.12 868	9.90 443	9	22
39	9.77 592	17	9.87 158	27	10.12 842	9.90 434	10	21
40	9.77 609	17	9.87 185	26	10.12 815	9.90 424	9	20
41	9.77 626	17	9.87 211	27	10.12 789	9.90 415	10	19
42	9.77 643	17	9.87 238	26	10.12 762	9.90 405	9	18
43	9.77 660	17	9.87 264	26	10.12 736	9.90 396	10	17
44	9.77 677	17	9.87 290	27	10.12 710	9.90 386	9	16
45	9.77 694	17	9.87 317	26	10.12 683	9.90 377	9	15
46	9.77 711	17	9.87 343	26	10.12 657	9.90 368	10	14
47	9.77 728	16	9.87 369	27	10.12 631	9.90 358	9	13
48	9.77 744	17	9.87 396	26	10.12 604	9.90 349	10	12
49	9.77 761	17	9.87 422	26	10.12 578	9.90 339	9	11
50	9.77 778	17	9.87 448	27	10.12 552	9.90 330	10	10
51	9.77 795	17	9.87 475	26	10.12 525	9.90 320	9	9
52	9.77 812	17	9.87 501	26	10.12 499	9.90 311	10	8
53	9.77 829	17	9.87 527	27	10.12 473	9.90 301	9	7
54	9.77 846	16	9.87 554	26	10.12 446	9.90 292	10	6
55	9.77 862	17	9.87 580	26	10.12 420	9.90 282	9	5
56	9.77 879	17	9.87 606	27	10.12 394	9.90 273	10	4
57	9.77 896	17	9.87 633	26	10.12 367	9.90 263	9	3
58	9.77 913	17	9.87 659	26	10.12 341	9.90 254	10	2
59	9.77 930	16	9.87 685	26	10.12 315	9.90 244	9	1
60	9.77 946		9.87 711		10.12 289	9.90 235		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

53°

Prop. Pts.

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	18	17	16
1	1.8	1.7	1.6
2	3.6	3.4	3.2
3	5.4	5.1	4.8
4	7.2	6.8	6.4
5	9.0	8.5	8.0
6	10.8	10.2	9.6
7	12.6	11.9	11.2
8	14.4	13.6	12.8
9	16.2	15.3	14.4

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

37°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.77 946	17	9.87 711	27	10.12 289	9.90 235	10	60
1	9.77 963	17	9.87 738	26	10.12 262	9.90 225	9	59
2	9.77 980	17	9.87 764	26	10.12 236	9.90 216	10	58
3	9.77 997	16	9.87 790	27	10.12 210	9.90 206	9	57
4	9.78 013	17	9.87 817	26	10.12 183	9.90 197	10	56
5	9.78 030	17	9.87 843	26	10.12 157	9.90 187	9	55
6	9.78 047	16	9.87 869	26	10.12 131	9.90 178	10	54
7	9.78 063	17	9.87 895	27	10.12 105	9.90 168	9	53
8	9.78 080	17	9.87 922	26	10.12 078	9.90 159	10	52
9	9.78 097	16	9.87 948	26	10.12 052	9.90 149	10	51
10	9.78 113	17	9.87 974	26	10.12 026	9.90 139	9	50
11	9.78 130	17	9.88 000	27	10.12 000	9.90 130	10	49
12	9.78 147	16	9.88 027	26	10.11 973	9.90 120	9	48
13	9.78 163	17	9.88 053	26	10.11 947	9.90 111	10	47
14	9.78 180	17	9.88 079	26	10.11 921	9.90 101	10	46
15	9.78 197	16	9.88 105	26	10.11 895	9.90 091	9	45
16	9.78 213	17	9.88 131	27	10.11 869	9.90 082	10	44
17	9.78 230	16	9.88 158	26	10.11 842	9.90 072	9	43
18	9.78 246	17	9.88 184	26	10.11 816	9.90 063	10	42
19	9.78 263	17	9.88 210	26	10.11 790	9.90 053	10	41
20	9.78 280	16	9.88 236	26	10.11 764	9.90 043	9	40
21	9.78 296	17	9.88 262	27	10.11 738	9.90 034	10	39
22	9.78 313	16	9.88 289	26	10.11 711	9.90 024	10	38
23	9.78 329	17	9.88 315	26	10.11 685	9.90 014	9	37
24	9.78 346	16	9.88 341	26	10.11 659	9.90 005	10	36
25	9.78 362	17	9.88 367	26	10.11 633	9.89 995	10	35
26	9.78 379	16	9.88 393	27	10.11 607	9.89 985	9	34
27	9.78 395	17	9.88 420	26	10.11 580	9.89 976	10	33
28	9.78 412	16	9.88 446	26	10.11 554	9.89 966	10	32
29	9.78 428	17	9.88 472	26	10.11 528	9.89 956	9	31
30	9.78 445	16	9.88 498	26	10.11 502	9.89 947	10	30
31	9.78 461	17	9.88 524	26	10.11 476	9.89 937	10	29
32	9.78 478	16	9.88 550	27	10.11 450	9.89 927	9	28
33	9.78 494	16	9.88 577	26	10.11 423	9.89 918	10	27
34	9.78 510	17	9.88 603	26	10.11 397	9.89 908	10	26
35	9.78 527	16	9.88 629	26	10.11 371	9.89 898	10	25
36	9.78 543	17	9.88 655	26	10.11 345	9.89 888	9	24
37	9.78 560	16	9.88 681	26	10.11 319	9.89 879	10	23
38	9.78 576	16	9.88 707	26	10.11 293	9.89 869	10	22
39	9.78 592	17	9.88 733	26	10.11 267	9.89 859	10	21
40	9.78 609	16	9.88 759	27	10.11 241	9.89 849	9	20
41	9.78 625	17	9.88 786	26	10.11 214	9.89 840	10	19
42	9.78 642	16	9.88 812	26	10.11 188	9.89 830	10	18
43	9.78 658	16	9.88 838	26	10.11 162	9.89 820	10	17
44	9.78 674	17	9.88 864	26	10.11 136	9.89 810	9	16
45	9.78 691	16	9.88 890	26	10.11 110	9.89 801	10	15
46	9.78 707	16	9.88 916	26	10.11 084	9.89 791	10	14
47	9.78 723	16	9.88 942	26	10.11 058	9.89 781	10	13
48	9.78 739	17	9.88 968	26	10.11 032	9.89 771	10	12
49	9.78 756	16	9.88 994	26	10.11 006	9.89 761	9	11
50	9.78 772	16	9.89 020	26	10.10 980	9.89 752	10	10
51	9.78 788	17	9.89 046	27	10.10 954	9.89 742	10	9
52	9.78 805	16	9.89 073	26	10.10 927	9.89 732	10	8
53	9.78 821	16	9.89 099	26	10.10 901	9.89 722	10	7
54	9.78 837	16	9.89 125	26	10.10 875	9.89 712	10	6
55	9.78 853	16	9.89 151	26	10.10 849	9.89 702	9	5
56	9.78 869	17	9.89 177	26	10.10 823	9.89 693	10	4
57	9.78 886	16	9.89 203	26	10.10 797	9.89 683	10	3
58	9.78 902	16	9.89 229	26	10.10 771	9.89 673	10	2
59	9.78 918	16	9.89 255	26	10.10 745	9.89 663	10	1
60	9.78 934		9.89 281		10.10 719	9.89 653		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	27	26
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	17	16
1	1.7	1.6
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

Prop. Pts.

52°

Table 3-2. Common logarithms of trigonometric functions (continued)

38°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.78 934	16	9.89 281	26	10.10 719	9.89 653	10	**60**
1	9.78 950	17	9.89 307	26	10.10 693	9.89 643	10	59
2	9.78 967	16	9.89 333	26	10.10 667	9.89 633	9	58
3	9.78 983	16	9.89 359	26	10.10 641	9.89 624	10	57
4	9.78 999	16	9.89 385	26	10.10 615	9.89 614	10	56
5	9.79 015	16	9.89 411	26	10.10 589	9.89 604	10	55
6	9.79 031	16	9.89 437	26	10.10 563	9.89 594	10	54
7	9.79 047	16	9.89 463	26	10.10 537	9.89 584	10	53
8	9.79 063	16	9.89 489	26	10.10 511	9.89 574	10	52
9	9.79 079	16	9.89 515	26	10.10 485	9.89 564	10	51
10	9.79 095	16	9.89 541	26	10.10 459	9.89 554	10	**50**
11	9.79 111	17	9.89 567	26	10.10 433	9.89 544	10	49
12	9.79 128	16	9.89 593	26	10.10 407	9.89 534	10	48
13	9.79 144	16	9.89 619	26	10.10 381	9.89 524	10	47
14	9.79 160	16	9.89 645	26	10.10 355	9.89 514	10	46
15	9.79 176	16	9.89 671	26	10.10 329	9.89 504	9	45
16	9.79 192	16	9.89 697	26	10.10 303	9.89 495	10	44
17	9.79 208	16	9.89 723	26	10.10 277	9.89 485	10	43
18	9.79 224	16	9.89 749	26	10.10 251	9.89 475	10	42
19	9.79 240	16	9.89 775	26	10.10 225	9.89 465	10	41
20	9.79 256	16	9.89 801	26	10.10 199	9.89 455	10	**40**
21	9.79 272	16	9.89 827	26	10.10 173	9.89 445	10	39
22	9.79 288	16	9.89 853	26	10.10 147	9.89 435	10	38
23	9.79 304	15	9.89 879	26	10.10 121	9.89 425	10	37
24	9.79 319	16	9.89 905	26	10.10 095	9.89 415	10	36
25	9.79 335	16	9.89 931	26	10.10 069	9.89 405	10	35
26	9.79 351	16	9.89 957	26	10.10 043	9.89 395	10	34
27	9.79 367	16	9.89 983	26	10.10 017	9.89 385	10	33
28	9.79 383	16	9.90 009	26	10.09 991	9.89 375	11	32
29	9.79 399	16	9.90 035	26	10.09 965	9.89 364	10	31
30	9.79 415	16	9.90 061	25	10.09 939	9.89 354	10	**30**
31	9.79 431	16	9.90 086	26	10.09 914	9.89 344	10	29
32	9.79 447	16	9.90 112	26	10.09 888	9.89 334	10	28
33	9.79 463	15	9.90 138	26	10.09 862	9.89 324	10	27
34	9.79 478	16	9.90 164	26	10.09 836	9.89 314	10	26
35	9.79 494	16	9.90 190	26	10.09 810	9.89 304	10	25
36	9.79 510	16	9.90 216	26	10.09 784	9.89 294	10	24
37	9.79 526	16	9.90 242	26	10.09 758	9.89 284	10	23
38	9.79 542	16	9.90 268	26	10.09 732	9.89 274	10	22
39	9.79 558	15	9.90 294	26	10.09 706	9.89 264	10	21
40	9.79 573	16	9.90 320	26	10.09 680	9.89 254	10	**20**
41	9.79 589	16	9.90 346	25	10.09 654	9.89 244	11	19
42	9.79 605	16	9.90 371	26	10.09 629	9.89 233	10	18
43	9.79 621	15	9.90 397	26	10.09 603	9.89 223	10	17
44	9.79 636	16	9.90 423	26	10.09 577	9.89 213	10	16
45	9.79 652	16	9.90 449	26	10.09 551	9.89 203	10	15
46	9.79 668	16	9.90 475	26	10.09 525	9.89 193	10	14
47	9.79 684	15	9.90 501	26	10.09 499	9.89 183	10	13
48	9.79 699	16	9.90 527	26	10.09 473	9.89 173	11	12
49	9.79 715	16	9.90 553	25	10.09 447	9.89 162	10	11
50	9.79 731	15	9.90 578	26	10.09 422	9.89 152	10	**10**
51	9.79 746	16	9.90 604	26	10.09 396	9.89 142	10	9
52	9.79 762	16	9.90 630	26	10.09 370	9.89 132	10	8
53	9.79 778	15	9.90 656	26	10.09 344	9.89 122	10	7
54	9.79 793	16	9.90 682	26	10.09 318	9.89 112	11	6
55	9.79 809	16	9.90 708	26	10.09 292	9.89 101	10	5
56	9.79 825	15	9.90 734	25	10.09 266	9.89 091	10	4
57	9.79 840	16	9.90 759	26	10.09 241	9.89 081	10	3
58	9.79 856	16	9.90 785	26	10.09 215	9.89 071	11	2
59	9.79 872	15	9.90 811	26	10.09 189	9.89 060	10	1
60	9.79 887		9.90 837		10.09 163	9.89 050		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

51°

Prop. Pts.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	17	16	15
1	1.7	1.6	1.5
2	3.4	3.2	3.0
3	5.1	4.8	4.5
4	6.8	6.4	6.0
5	8.5	8.0	7.5
6	10.2	9.6	9.0
7	11.9	11.2	10.5
8	13.6	12.8	12.0
9	15.3	14.4	13.5

	11	10	9
1	1.1	1.0	0.9
2	2.2	2.0	1.8
3	3.3	3.0	2.7
4	4.4	4.0	3.6
5	5.5	5.0	4.5
6	6.6	6.0	5.4
7	7.7	7.0	6.3
8	8.8	8.0	7.2
9	9.9	9.0	8.1

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

39°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.79 887	16	9.90 837	26	10.09 163	9.89 050	10	**60**
1	9.79 903	15	9.90 863	26	10.09 137	9.89 040	10	59
2	9.79 918	16	9.90 889	25	10.09 111	9.89 030	10	58
3	9.79 934	16	9.90 914	26	10.09 086	9.89 020	11	57
4	9.79 950	15	9.90 940	26	10.09 060	9.89 009	10	56
5	9.79 965	16	9.90 966	26	10.09 034	9.88 999	10	55
6	9.79 981	15	9.90 992	26	10.09 008	9.88 989	11	54
7	9.79 996	16	9.91 018	25	10.08 982	9.88 978	10	53
8	9.80 012	15	9.91 043	26	10.08 957	9.88 968	10	52
9	9.80 027	16	9.91 069	26	10.08 931	9.88 958	10	51
10	9.80 043	15	9.91 095	26	10.08 905	9.88 948	11	**50**
11	9.80 058	16	9.91 121	26	10.08 879	9.88 937	10	49
12	9.80 074	15	9.91 147	25	10.08 853	9.88 927	10	48
13	9.80 089	16	9.91 172	26	10.08 828	9.88 917	11	47
14	9.80 105	15	9.91 198	26	10.08 802	9.88 906	10	46
15	9.80 120	16	9.91 224	26	10.08 776	9.88 896	10	45
16	9.80 136	15	9.91 250	26	10.08 750	9.88 886	11	44
17	9.80 151	15	9.91 276	25	10.08 724	9.88 875	10	43
18	9.80 166	16	9.91 301	26	10.08 699	9.88 865	10	42
19	9.80 182	15	9.91 327	26	10.08 673	9.88 855	11	41
20	9.80 197	16	9.91 353	26	10.08 647	9.88 844	10	**40**
21	9.80 213	15	9.91 379	25	10.08 621	9.88 834	10	39
22	9.80 228	16	9.91 404	26	10.08 596	9.88 824	11	38
23	9.80 244	15	9.91 430	26	10.08 570	9.88 813	10	37
24	9.80 259	15	9.91 456	26	10.08 544	9.88 803	10	36
25	9.80 274	16	9.91 482	25	10.08 518	9.88 793	11	35
26	9.80 290	15	9.91 507	26	10.08 493	9.88 782	10	34
27	9.80 305	15	9.91 533	26	10.08 467	9.88 772	11	33
28	9.80 320	16	9.91 559	26	10.08 441	9.88 761	10	32
29	9.80 336	15	9.91 585	25	10.08 415	9.88 751	10	31
30	9.80 351	15	9.91 610	26	10.08 390	9.88 741	11	**30**
31	9.80 366	16	9.91 636	26	10.08 364	9.88 730	10	29
32	9.80 382	15	9.91 662	26	10.08 338	9.88 720	11	28
33	9.80 397	15	9.91 688	25	10.08 312	9.88 709	10	27
34	9.80 412	16	9.91 713	26	10.08 287	9.88 699	11	26
35	9.80 428	15	9.91 739	26	10.08 261	9.88 688	10	25
36	9.80 443	15	9.91 765	26	10.08 235	9.88 678	10	24
37	9.80 458	15	9.91 791	25	10.08 209	9.88 668	11	23
38	9.80 473	16	9.91 816	26	10.08 184	9.88 657	10	22
39	9.80 489	15	9.91 842	26	10.08 158	9.88 647	11	21
40	9.80 504	15	9.91 868	25	10.08 132	9.88 636	10	**20**
41	9.80 519	15	9.91 893	26	10.08 107	9.88 626	11	19
42	9.80 534	16	9.91 919	26	10.08 081	9.88 615	10	18
43	9.80 550	15	9.91 945	26	10.08 055	9.88 605	11	17
44	9.80 565	15	9.91 971	25	10.08 029	9.88 594	10	16
45	9.80 580	15	9.91 996	26	10.08 004	9.88 584	11	15
46	9.80 595	15	9.92 022	26	10.07 978	9.88 573	10	14
47	9.80 610	15	9.92 048	25	10.07 952	9.88 563	11	13
48	9.80 625	16	9.92 073	26	10.07 927	9.88 552	10	12
49	9.80 641	15	9.92 099	26	10.07 901	9.88 542	11	11
50	9.80 656	15	9.92 125	25	10.07 875	9.88 531	10	**10**
51	9.80 671	15	9.92 150	26	10.07 850	9.88 521	11	9
52	9.80 686	15	9.92 176	26	10.07 824	9.88 510	11	8
53	9.80 701	15	9.92 202	25	10.07 798	9.88 499	10	7
54	9.80 716	15	9.92 227	26	10.07 773	9.88 489	11	6
55	9.80 731	15	9.92 253	26	10.07 747	9.88 478	10	5
56	9.80 746	16	9.92 279	25	10.07 721	9.88 468	11	4
57	9.80 762	15	9.92 304	26	10.07 696	9.88 457	10	3
58	9.80 777	15	9.92 330	26	10.07 670	9.88 447	11	2
59	9.80 792	15	9.92 356	25	10.07 644	9.88 436	11	1
60	9.80 807		9.92 381		10.07 619	9.88 425		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

50°

Prop. Pts.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	16	15
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

Prop. Pts.

Table 3-2. Common logarithms of trigonometric functions (continued)

40°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.80 807	15	9.92 381	26	10.07 619	9.88 425	10	**60**
1	9.80 822	15	9.92 407	26	10.07 593	9.88 415	11	59
2	9.80 837	15	9.92 433	25	10.07 567	9.88 404	10	58
3	9.80 852	15	9.92 458	26	10.07 542	9.88 394	11	57
4	9.80 867	15	9.92 484	26	10.07 516	9.88 383	11	56
5	9.80 882	15	9.92 510	25	10.07 490	9.88 372	10	55
6	9.80 897	15	9.92 535	26	10.07 465	9.88 362	11	54
7	9.80 912	15	9.92 561	26	10.07 439	9.88 351	11	53
8	9.80 927	15	9.92 587	25	10.07 413	9.88 340	10	52
9	9.80 942	15	9.92 612	26	10.07 388	9.88 330	11	51
10	9.80 957	15	9.92 638	25	10.07 362	9.88 319	11	**50**
11	9.80 972	15	9.92 663	26	10.07 337	9.88 308	10	49
12	9.80 987	15	9.92 689	26	10.07 311	9.88 298	11	48
13	9.81 002	15	9.92 715	25	10.07 285	9.88 287	11	47
14	9.81 017	15	9.92 740	26	10.07 260	9.88 276	10	46
15	9.81 032	15	9.92 766	26	10.07 234	9.88 266	11	45
16	9.81 047	14	9.92 792	25	10.07 208	9.88 255	11	44
17	9.81 061	15	9.92 817	26	10.07 183	9.88 244	10	43
18	9.81 076	15	9.92 843	25	10.07 157	9.88 234	11	42
19	9.81 091	15	9.92 868	26	10.07 132	9.88 223	11	41
20	9.81 106	15	9.92 894	26	10.07 106	9.88 212	11	**40**
21	9.81 121	15	9.92 920	25	10.07 080	9.88 201	10	39
22	9.81 136	15	9.92 945	26	10.07 055	9.88 191	11	38
23	9.81 151	15	9.92 971	25	10.07 029	9.88 180	11	37
24	9.81 166	14	9.92 996	26	10.07 004	9.88 169	11	36
25	9.81 180	15	9.93 022	26	10.06 978	9.88 158	10	35
26	9.81 195	15	9.93 048	25	10.06 952	9.88 148	11	34
27	9.81 210	15	9.93 073	26	10.06 927	9.88 137	11	33
28	9.81 225	15	9.93 099	25	10.06 901	9.88 126	11	32
29	9.81 240	14	9.93 124	26	10.06 876	9.88 115	10	31
30	9.81 254	15	9.93 150	25	10.06 850	9.88 105	11	**30**
31	9.81 269	15	9.93 175	26	10.06 825	9.88 094	11	29
32	9.81 284	15	9.93 201	26	10.06 799	9.88 083	11	28
33	9.81 299	15	9.93 227	25	10.06 773	9.88 072	11	27
34	9.81 314	14	9.93 252	26	10.06 748	9.88 061	10	26
35	9.81 328	15	9.93 278	25	10.06 722	9.88 051	11	25
36	9.81 343	15	9.93 303	26	10.06 697	9.88 040	11	24
37	9.81 358	14	9.93 329	25	10.06 671	9.88 029	11	23
38	9.81 372	15	9.93 354	26	10.06 646	9.88 018	11	22
39	9.81 387	15	9.93 380	26	10.06 620	9.88 007	11	21
40	9.81 402	15	9.93 406	25	10.06 594	9.87 996	11	**20**
41	9.81 417	14	9.93 431	26	10.06 569	9.87 985	10	19
42	9.81 431	15	9.93 457	25	10.06 543	9.87 975	11	18
43	9.81 446	15	9.93 482	26	10.06 518	9.87 964	11	17
44	9.81 461	14	9.93 508	25	10.06 492	9.87 953	11	16
45	9.81 475	15	9.93 533	26	10.06 467	9.87 942	11	15
46	9.81 490	15	9.93 559	25	10.06 441	9.87 931	11	14
47	9.81 505	14	9.93 584	26	10.06 416	9.87 920	11	13
48	9.81 519	15	9.93 610	26	10.06 390	9.87 909	11	12
49	9.81 534	15	9.93 636	25	10.06 364	9.87 898	11	11
50	9.81 549	14	9.93 661	26	10.06 339	9.87 887	10	**10**
51	9.81 563	15	9.93 687	25	10.06 313	9.87 877	11	9
52	9.81 578	14	9.93 712	26	10.06 288	9.87 866	11	8
53	9.81 592	15	9.93 738	25	10.06 262	9.87 855	11	7
54	9.81 607	15	9.93 763	26	10.06 237	9.87 844	11	6
55	9.81 622	14	9.93 789	25	10.06 211	9.87 833	11	5
56	9.81 636	15	9.93 814	26	10.06 186	9.87 822	11	4
57	9.81 651	14	9.93 840	25	10.06 160	9.87 811	11	3
58	9.81 665	15	9.93 865	26	10.06 135	9.87 800	11	2
59	9.81 680	14	9.93 891	25	10.06 109	9.87 789	11	1
60	9.81 694		9.93 916		10.06 084	9.87 778		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

Prop. Pts.

49°

Table 3-2. Common logarithms of trigonometric functions (continued)

41°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.81 694	15	9.93 916	26	10.06 084	9.87 778	11	**60**
1	9.81 709	14	9.93 942	25	10.06 058	9.87 767	11	59
2	9.81 723	15	9.93 967	26	10.06 033	9.87 756	11	58
3	9.81 738	14	9.93 993	25	10.06 007	9.87 745	11	57
4	9.81 752	15	9.94 018	26	10.05 982	9.87 734	11	56
5	9.81 767	14	9.94 044	25	10.05 956	9.87 723	11	55
6	9.81 781	15	9.94 069	26	10.05 931	9.87 712	11	54
7	9.81 796	14	9.94 095	25	10.05 905	9.87 701	11	53
8	9.81 810	15	9.94 120	26	10.05 880	9.87 690	11	52
9	9.81 825	14	9.94 146	25	10.05 854	9.87 679	11	51
10	9.81 839	15	9.94 171	26	10.05 829	9.87 668	11	**50**
11	9.81 854	14	9.94 197	25	10.05 803	9.87 657	11	49
12	9.81 868	14	9.94 222	26	10.05 778	9.87 646	11	48
13	9.81 882	15	9.94 248	25	10.05 752	9.87 635	11	47
14	9.81 897	14	9.94 273	26	10.05 727	9.87 624	11	46
15	9.81 911	15	9.94 299	25	10.05 701	9.87 613	12	45
16	9.81 926	14	9.94 324	26	10.05 676	9.87 601	11	44
17	9.81 940	15	9.94 350	25	10.05 650	9.87 590	11	43
18	9.81 955	14	9.94 375	26	10.05 625	9.87 579	11	42
19	9.81 969	14	9.94 401	25	10.05 599	9.87 568	11	41
20	9.81 983	15	9.94 426	26	10.05 574	9.87 557	11	**40**
21	9.81 998	14	9.94 452	25	10.05 548	9.87 546	11	39
22	9.82 012	14	9.94 477	26	10.05 523	9.87 535	11	38
23	9.82 026	15	9.94 503	25	10.05 497	9.87 524	11	37
24	9.82 041	14	9.94 528	26	10.05 472	9.87 513	12	36
25	9.82 055	14	9.94 554	25	10.05 446	9.87 501	11	35
26	9.82 069	15	9.94 579	25	10.05 421	9.87 490	11	34
27	9.82 084	14	9.94 604	26	10.05 396	9.87 479	11	33
28	9.82 098	14	9.94 630	25	10.05 370	9.87 468	11	32
29	9.82 112	14	9.94 655	26	10.05 345	9.87 457	11	31
30	9.82 126	15	9.94 681	25	10.05 319	9.87 446	12	**30**
31	9.82 141	14	9.94 706	26	10.05 294	9.87 434	11	29
32	9.82 155	14	9.94 732	25	10.05 268	9.87 423	11	28
33	9.82 169	15	9.94 757	26	10.05 243	9.87 412	11	27
34	9.82 184	14	9.94 783	25	10.05 217	9.87 401	11	26
35	9.82 198	14	9.94 808	26	10.05 192	9.87 390	12	25
36	9.82 212	14	9.94 834	25	10.05 166	9.87 378	11	24
37	9.82 226	14	9.94 859	25	10.05 141	9.87 367	11	23
38	9.82 240	15	9.94 884	26	10.05 116	9.87 356	11	22
39	9.82 255	14	9.94 910	25	10.05 090	9.87 345	11	21
40	9.82 269	14	9.94 935	26	10.05 065	9.87 334	12	**20**
41	9.82 283	14	9.94 961	25	10.05 039	9.87 322	11	19
42	9.82 297	14	9.94 986	26	10.05 014	9.87 311	11	18
43	9.82 311	15	9.95 012	25	10.04 988	9.87 300	12	17
44	9.82 326	14	9.95 037	25	10.04 963	9.87 288	11	16
45	9.82 340	14	9.95 062	26	10.04 938	9.87 277	11	15
46	9.82 354	14	9.95 088	25	10.04 912	9.87 266	11	14
47	9.82 368	14	9.95 113	26	10.04 887	9.87 255	12	13
48	9.82 382	14	9.95 139	25	10.04 861	9.87 243	11	12
49	9.82 396	14	9.95 164	26	10.04 836	9.87 232	11	11
50	9.82 410	14	9.95 190	25	10.04 810	9.87 221	12	**10**
51	9.82 424	15	9.95 215	25	10.04 785	9.87 209	11	9
52	9.82 439	14	9.95 240	26	10.04 760	9.87 198	11	8
53	9.82 453	14	9.95 266	25	10.04 734	9.87 187	12	7
54	9.82 467	14	9.95 291	26	10.04 709	9.87 175	11	6
55	9.82 481	14	9.95 317	25	10.04 683	9.87 164	11	5
56	9.82 495	14	9.95 342	26	10.04 658	9.87 153	12	4
57	9.82 509	14	9.95 368	25	10.04 632	9.87 141	11	3
58	9.82 523	14	9.95 393	25	10.04 607	9.87 130	11	2
59	9.82 537	14	9.95 418	26	10.04 582	9.87 119	12	1
60	9.82 551		9.95 444		10.04 556	9.87 107		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	25
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	12	11
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

Prop. Pts.

48°

Table 3-2. Common logarithms of trigonometric functions (continued)

42°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.82 551	14	9.95 444	25	10.04 556	9.87 107	11	**60**
1	9.82 565	14	9.95 469	26	10.04 531	9.87 096	11	59
2	9.82 579	14	9.95 495	25	10.04 505	9.87 085	12	58
3	9.82 593	14	9.95 520	25	10.04 480	9.87 073	11	57
4	9.82 607	14	9.95 545	26	10.04 455	9.87 062	12	56
5	9.82 621	14	9.95 571	25	10.04 429	9.87 050	11	55
6	9.82 635	14	9.95 596	26	10.04 404	9.87 039	11	54
7	9.82 649	14	9.95 622	25	10.04 378	9.87 028	12	53
8	9.82 663	14	9.95 647	25	10.04 353	9.87 016	11	52
9	9.82 677	14	9.95 672	26	10.04 328	9.87 005	12	51
10	9.82 691	14	9.95 698	25	10.04 302	9.86 993	11	**50**
11	9.82 705	14	9.95 723	25	10.04 277	9.86 982	12	49
12	9.82 719	14	9.95 748	26	10.04 252	9.86 970	11	48
13	9.82 733	14	9.95 774	25	10.04 226	9.86 959	12	47
14	9.82 747	14	9.95 799	26	10.04 201	9.86 947	11	46
15	9.82 761	14	9.95 825	25	10.04 175	9.86 936	12	45
16	9.82 775	13	9.95 850	25	10.04 150	9.86 924	11	44
17	9.82 788	14	9.95 875	26	10.04 125	9.86 913	11	43
18	9.82 802	14	9.95 901	25	10.04 099	9.86 902	12	42
19	9.82 816	14	9.95 926	26	10.04 074	9.86 890	11	41
20	9.82 830	14	9.95 952	25	10.04 048	9.86 879	12	**40**
21	9.82 844	14	9.95 977	25	10.04 023	9.86 867	12	39
22	9.82 858	14	9.96 002	26	10.03 998	9.86 855	11	38
23	9.82 872	13	9.96 028	25	10.03 972	9.86 844	12	37
24	9.82 885	14	9.96 053	25	10.03 947	9.86 832	11	36
25	9.82 899	14	9.96 078	26	10.03 922	9.86 821	12	35
26	9.82 913	14	9.96 104	25	10.03 896	9.86 809	11	34
27	9.82 927	14	9.96 129	26	10.03 871	9.86 798	12	33
28	9.82 941	14	9.96 155	25	10.03 845	9.86 786	11	32
29	9.82 955	13	9.96 180	25	10.03 820	9.86 775	12	31
30	9.82 968	14	9.96 205	26	10.03 795	9.86 763	11	**30**
31	9.82 982	14	9.96 231	25	10.03 769	9.86 752	12	29
32	9.82 996	14	9.96 256	25	10.03 744	9.86 740	12	28
33	9.83 010	13	9.96 281	26	10.03 719	9.86 728	11	27
34	9.83 023	14	9.96 307	25	10.03 693	9.86 717	12	26
35	9.83 037	14	9.96 332	25	10.03 668	9.86 705	11	25
36	9.83 051	14	9.96 357	26	10.03 643	9.86 694	12	24
37	9.83 065	13	9.96 383	25	10.03 617	9.86 682	12	23
38	9.83 078	14	9.96 408	25	10.03 592	9.86 670	11	22
39	9.83 092	14	9.96 433	26	10.03 567	9.86 659	12	21
40	9.83 106	14	9.96 459	25	10.03 541	9.86 647	12	**20**
41	9.83 120	13	9.96 484	26	10.03 516	9.86 635	11	19
42	9.83 133	14	9.96 510	25	10.03 490	9.86 624	12	18
43	9.83 147	14	9.96 535	25	10.03 465	9.86 612	12	17
44	9.83 161	13	9.96 560	26	10.03 440	9.86 600	11	16
45	9.83 174	14	9.96 586	25	10.03 414	9.86 589	12	15
46	9.83 188	14	9.96 611	25	10.03 389	9.86 577	12	14
47	9.83 202	13	9.96 636	26	10.03 364	9.86 565	11	13
48	9.83 215	14	9.96 662	25	10.03 338	9.86 554	12	12
49	9.83 229	13	9.96 687	25	10.03 313	9.86 542	12	11
50	9.83 242	14	9.96 712	26	10.03 288	9.86 530	12	**10**
51	9.83 256	14	9.96 738	25	10.03 262	9.86 518	11	9
52	9.83 270	13	9.96 763	25	10.03 237	9.86 507	12	8
53	9.83 283	14	9.96 788	26	10.03 212	9.86 495	12	7
54	9.83 297	13	9.96 814	25	10.03 186	9.86 483	11	6
55	9.83 310	14	9.96 839	25	10.03 161	9.86 472	12	5
56	9.83 324	14	9.96 864	26	10.03 136	9.86 460	12	4
57	9.83 338	13	9.96 890	25	10.03 110	9.86 448	12	3
58	9.83 351	14	9.96 915	25	10.03 085	9.86 436	11	2
59	9.83 365	13	9.96 940	26	10.03 060	9.86 425	12	1
60	9.83 378		9.96 966		10.03 034	9.86 413		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	**13**
1	1.4	1.3
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	**11**
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

Prop. Pts.

47°

Table 3-2. Common logarithms of trigonometric functions (continued)

43°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.83 378	14	9.96 966	25	10.03 034	9.86 413	12	**60**
1	9.83 392	13	9.96 991	25	10.03 009	9.86 401	12	59
2	9.83 405	14	9.97 016	26	10.02 984	9.86 389	12	58
3	9.83 419	13	9.97 042	25	10.02 958	9.86 377	11	57
4	9.83 432	14	9.97 067	25	10.02 933	9.86 366	12	56
5	9.83 446	13	9.97 092	26	10.02 908	9.86 354	12	55
6	9.83 459	14	9.97 118	25	10.02 882	9.86 342	12	54
7	9.83 473	13	9.97 143	25	10.02 857	9.86 330	12	53
8	9.83 486	14	9.97 168	25	10.02 832	9.86 318	12	52
9	9.83 500	13	9.97 193	26	10.02 807	9.86 306	11	51
10	9.83 513	14	9.97 219	25	10.02 781	9.86 295	12	**50**
11	9.83 527	13	9.97 244	25	10.02 756	9.86 283	12	49
12	9.83 540	14	9.97 269	26	10.02 731	9.86 271	12	48
13	9.83 554	13	9.97 295	25	10.02 705	9.86 259	12	47
14	9.83 567	14	9.97 320	25	10.02 680	9.86 247	12	46
15	9.83 581	13	9.97 345	26	10.02 655	9.86 235	12	45
16	9.83 594	14	9.97 371	25	10.02 629	9.86 223	12	44
17	9.83 608	13	9.97 396	25	10.02 604	9.86 211	11	43
18	9.83 621	13	9.97 421	26	10.02 579	9.86 200	12	42
19	9.83 634	14	9.97 447	25	10.02 553	9.86 188	12	41
20	9.83 648	13	9.97 472	25	10.02 528	9.86 176	12	**40**
21	9.83 661	13	9.97 497	26	10.02 503	9.86 164	12	39
22	9.83 674	14	9.97 523	25	10.02 477	9.86 152	12	38
23	9.83 688	13	9.97 548	25	10.02 452	9.86 140	12	37
24	9.83 701	14	9.97 573	25	10.02 427	9.86 128	12	36
25	9.83 715	13	9.97 598	26	10.02 402	9.86 116	12	35
26	9.83 728	13	9.97 624	25	10.02 376	9.86 104	12	34
27	9.83 741	14	9.97 649	25	10.02 351	9.86 092	12	33
28	9.83 755	13	9.97 674	26	10.02 326	9.86 080	12	32
29	9.83 768	13	9.97 700	25	10.02 300	9.86 068	12	31
30	9.83 781	14	9.97 725	25	10.02 275	9.86 056	12	**30**
31	9.83 795	13	9.97 750	26	10.02 250	9.86 044	12	29
32	9.83 808	13	9.97 776	25	10.02 224	9.86 032	12	28
33	9.83 821	13	9.97 801	25	10.02 199	9.86 020	12	27
34	9.83 834	14	9.97 826	25	10.02 174	9.86 008	12	26
35	9.83 848	13	9.97 851	26	10.02 149	9.85 996	12	25
36	9.83 861	13	9.97 877	25	10.02 123	9.85 984	12	24
37	9.83 874	13	9.97 902	25	10.02 098	9.85 972	12	23
38	9.83 887	14	9.97 927	26	10.02 073	9.85 960	12	22
39	9.83 901	13	9.97 953	25	10.02 047	9.85 948	12	21
40	9.83 914	13	9.97 978	25	10.02 022	9.85 936	12	**20**
41	9.83 927	13	9.98 003	26	10.01 997	9.85 924	12	19
42	9.83 940	14	9.98 029	25	10.01 971	9.85 912	12	18
43	9.83 954	13	9.98 054	25	10.01 946	9.85 900	12	17
44	9.83 967	13	9.98 079	25	10.01 921	9.85 888	12	16
45	9.83 980	13	9.98 104	26	10.01 896	9.85 876	12	15
46	9.83 993	13	9.98 130	25	10.01 870	9.85 864	13	14
47	9.84 006	14	9.98 155	25	10.01 845	9.85 851	12	13
48	9.84 020	13	9.98 180	26	10.01 820	9.85 839	12	12
49	9.84 033	13	9.98 206	25	10.01 794	9.85 827	12	11
50	9.84 046	13	9.98 231	25	10.01 769	9.85 815	12	**10**
51	9.84 059	13	9.98 256	25	10.01 744	9.85 803	12	9
52	9.84 072	13	9.98 281	26	10.01 719	9.85 791	12	8
53	9.84 085	13	9.98 307	25	10.01 693	9.85 779	13	7
54	9.84 098	14	9.98 332	25	10.01 668	9.85 766	12	6
55	9.84 112	13	9.98 357	26	10.01 643	9.85 754	12	5
56	9.84 125	13	9.98 383	25	10.01 617	9.85 742	12	4
57	9.84 138	13	9.98 408	25	10.01 592	9.85 730	12	3
58	9.84 151	13	9.98 433	25	10.01 567	9.85 718	12	2
59	9.84 164	13	9.98 458	26	10.01 542	9.85 706	13	1
60	9.84 177		9.98 484		10.01 516	9.85 693		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	**13**
1	1.4	1.3
2	2.8	2.6
3	4.2	3.9
4	5.6	5.2
5	7.0	6.5
6	8.4	7.8
7	9.8	9.1
8	11.2	10.4
9	12.6	11.7

	12	**11**
1	1.2	1.1
2	2.4	2.2
3	3.6	3.3
4	4.8	4.4
5	6.0	5.5
6	7.2	6.6
7	8.4	7.7
8	9.6	8.8
9	10.8	9.9

Prop. Pts.

46°

Table 3-2. Common logarithms of trigonometric functions (continued)

44°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.84 177		9.98 484		10.01 516	9.85 693		**60**
1	9.84 190	13	9.98 509	25	10.01 491	9.85 681	12	59
2	9.84 203	13	9.98 534	25	10.01 466	9.85 669	12	58
3	9.84 216	13	9.98 560	26	10.01 440	9.85 657	12	57
4	9.84 229	13	9.98 585	25	10.01 415	9.85 645	12	56
5	9.84 242	13	9.98 610	25	10.01 390	9.85 632	13	55
6	9.84 255	13	9.98 635	25	10.01 365	9.85 620	12	54
7	9.84 269	14	9.98 661	26	10.01 339	9.85 608	12	53
8	9.84 282	13	9.98 686	25	10.01 314	9.85 596	12	52
9	9.84 295	13	9.98 711	25	10.01 289	9.85 583	13	51
10	9.84 308	13	9.98 737	26	10.01 263	9.85 571	12	**50**
11	9.84 321	13	9.98 762	25	10.01 238	9.85 559	12	49
12	9.84 334	13	9.98 787	25	10.01 213	9.85 547	12	48
13	9.84 347	13	9.98 812	25	10.01 188	9.85 534	13	47
14	9.84 360	13	9.98 838	26	10.01 162	9.85 522	12	46
15	9.84 373	13	9.98 863	25	10.01 137	9.85 510	12	45
16	9.84 385	12	9.98 888	25.	10.01 112	9.85 497	13	44
17	9.84 398	13	9.98 913	25	10.01 087	9.85 485	12	43
18	9.84 411	13	9.98 939	26	10.01 061	9.85 473	12	42
19	9.84 424	13	9.98 964	25	10.01 036	9.85 460	13	41
20	9.84 437	13	9.98 989	25	10.01 011	9.85 448	12	**40**
21	9.84 450	13	9.99 015	26	10.00 985	9.85 436	12	39
22	9.84 463	13	9.99 040	25	10.00 960	9.85 423	13	38
23	9.84 476	13	9.99 065	25	10.00 935	9.85 411	12	37
24	9.84 489	13	9.99 090	25	10.00 910	9.85 399	12	36
25	9.84 502	13	9.99 116	26	10.00 884	9.85 386	13	35
26	9.84 515	13	9.99 141	25	10.00 859	9.85 374	12	34
27	9.84 528	13	9.99 166	25	10.00 834	9.85 361	13	33
28	9.84 540	12	9.99 191	25	10.00 809	9.85 349	12	32
29	9.84 553	13	9.99 217	26	10.00 783	9.85 337	12	31
30	9.84 566	13	9.99 242	25	10.00 758	9.85 324	13	**30**
31	9.84 579	13	9.99 267	25	10.00 733	9.85 312	12	29
32	9.84 592	13	9.99 293	26	10.00 707	9.85 299	13	28
33	9.84 605	13	9.99 318	25	10.00 682	9.85 287	12	27
34	9.84 618	13	9.99 343	25	10.00 657	9.85 274	13	26
35	9.84 630	12	9.99 368	25	10.00 632	9.85 262	12	25
36	9.84 643	13	9.99 394	26	10.00 606	9.85 250	12	24
37	9.84 656	13	9.99 419	25	10.00 581	9.85 237	13	23
38	9.84 669	13	9.99 444	25	10.00 556	9.85 225	12	22
39	9.84 682	13	9.99 469	25	10.00 531	9.85 212	13	21
40	9.84 694	12	9.99 495	26	10.00 505	9.85 200	12	**20**
41	9.84 707	13	9.99 520	25	10.00 480	9.85 187	13	19
42	9.84 720	13	9.99 545	25	10.00 455	9.85 175	12	18
43	9.84 733	13	9.99 570	25	10.00 430	9.85 162	13	17
44	9.84 745	12	9.99 596	26	10.00 404	9.85 150	12	16
45	9.84 758	13	9.99 621	25	10.00 379	9.85 137	13	15
46	9.84 771	13	9.99 646	25	10.00 354	9.85 125	12	14
47	9.84 784	13	9.99 672	26	10.00 328	9.85 112	13	13
48	9.84 796	12	9.99 697	25	10.00 303	9.85 100	12	12
49	9.84 809	13	9.99 722	25	10.00 278	9.85 087	13	11
50	9.84 822	13	9.99 747	25	10.00 253	9.85 074	13	**10**
51	9.84 835	13	9.99 773	26	10.00 227	9.85 062	12	9
52	9.84 847	12	9.99 798	25	10.00 202	9.85 049	13	8
53	9.84 860	13	9.99 823	25	10.00 177	9.85 037	12	7
54	9.84 873	13	9.99 848	25	10.00 152	9.85 024	13	6
55	9.84 885	12	9.99 874	26	10.00 126	9.85 012	12	5
56	9.84 898	13	9.99 899	25	10.00 101	9.84 999	13	4
57	9.84 911	13	9.99 924	25	10.00 076	9.84 986	13	3
58	9.84 923	12	9.99 949	25	10.00 051	9.84 974	12	2
59	9.84 936	13	9.99 975	26	10.00 025	9.84 961	13	1
60	9.84 949	13	10.00 000	25	10.00 000	9.84 949	12	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Pts.

	26	**25**
1	2.6	2.5
2	5.2	5.0
3	7.8	7.5
4	10.4	10.0
5	13.0	12.5
6	15.6	15.0
7	18.2	17.5
8	20.8	20.0
9	23.4	22.5

	14	**13**	**12**
1	1.4	1.3	1.2
2	2.8	2.6	2.4
3	4.2	3.9	3.6
4	5.6	5.2	4.8
5	7.0	6.5	6.0
6	8.4	7.8	7.2
7	9.8	9.1	8.4
8	11.2	10.4	9.6
9	12.6	11.7	10.8

Prop. Pts.

45°

section 4

MISCELLANEOUS NUMERICAL TABLES

Table 4-1. Common logarithms of trigonometric functions (radian measure)*

Radians	sin	cos	tan	cot	log sin	log cos	log tan
.00	0	1.0000	0			10.00000	
.01	.0100	1.0000	.0100	99.9967	8.0000	10.0000	8.0000
.02	.0200	.9998	.0200	49.9933	.3010	9.9999	.3011
.03	.0300	.9996	.0300	33.3233	.4771	.9998	.4773
.04	.0400	.9992	.0400	24.9867	.6019	.9997	.6023
.05	.0500	.9988	.0500	19.9833	8.6988	9.9995	8.6993
.06	.0600	.9982	.0601	16.6467	.7779	.9992	.7787
.07	.0699	.9976	.0701	14.2624	.8447	.9989	.8458
.08	.0799	.9968	.0802	12.4733	.9026	.9986	.9040
.09	.0899	.9960	.0902	11.0811	.9537	.9982	.9554
.10	.0998	.9950	.1003	9.9666	8.9993	9.9978	9.0015
.11	.1098	.9940	.1104	9.0542	9.0405	.9974	.0431
.12	.1197	.9928	.1206	8.2933	.0781	.9969	.0813
.13	.1296	.9916	.1307	7.6489	.1127	.9963	.1164
.14	.1395	.9902	.1409	7.0961	.1447	.9957	.1490
.15	.1494	.9888	.1511	6.6166	9.1745	9.9951	9.1794
.16	.1593	.9872	.1614	6.1966	.2023	.9944	.2078
.17	.1692	.9856	.1717	5.8256	.2284	.9937	.2347
.18	.1790	.9838	.1820	5.4954	.2529	.9929	.2600
.19	.1889	.9820	.1923	5.1997	.2761	.9921	.2840
.20	.1987	.9801	.2027	4.9332	9.2981	9.9913	9.3069
.21	.2085	.9780	.2131	4.6917	.3190	.9904	.3287
.22	.2182	.9759	.2236	4.4719	.3389	.9894	.3495
.23	.2280	.9737	.2341	4.2709	.3579	.9884	.3695
.24	.2377	.9713	.2447	4.0864	.3760	.9874	.3887
.25	.2474	.9689	.2553	3.9163	9.3934	9.9863	9.4071
.26	.2571	.9664	.2660	3.7591	.4101	.9852	.4249
.27	.2667	.9638	.2768	3.6133	.4261	.9840	.4421
.28	.2764	.9611	.2876	3.4776	.4415	.9827	.4587
.29	.2860	.9582	.2984	3.3511	.4563	.9815	.4748
.30	.2955	.9553	.3093	3.2327	9.4706	9.9802	9.4904
.31	.3051	.9523	.3203	3.1218	.4844	.9788	.5056
.32	.3146	.9492	.3314	3.0176	.4977	.9774	.5203
.33	.3240	.9460	.3425	2.9195	.5106	.9759	.5347
.34	.3335	.9428	.3537	2.8270	.5231	.9744	.5487
.35	.3429	.9394	.3650	2.7395	9.5352	9.9728	9.5623
.36	.3523	.9359	.3764	2.6567	.5469	.9712	.5757
.37	.3616	.9323	.3879	2.5782	.5582	.9696	.5887
.38	.3709	.9287	.3994	2.5037	.5693	.9679	.6014
.39	.3802	.9249	.4111	2.4328	.5800	.9661	.6139
.40	.3894	.9211	.4228	2.3652	9.5904	9.9643	9.6261
.41	.3986	.9171	.4346	2.3008	.6005	.9624	.6381
.42	.4078	.9131	.4466	2.2393	.6104	.9605	.6499
.43	.4169	.9090	.4586	2.1804	.6200	.9585	.6615
.44	.4259	.9048	.4708	2.1241	.6293	.9565	.6728
.45	.4350	.9004	.4831	2.0702	9.6385	9.9545	.6840
.46	.4439	.8961	.4954	2.0184	.6473	.9523	.6950
.47	.4529	.8916	.5080	1.9686	.6560	.9502	.7058
.48	.4618	.8870	.5206	1.9208	.6644	.9479	.7165
.49	.4706	.8823	.5334	1.8748	.6727	.9456	.7270
.50	.4794	.8776	.5463	1.8305	9.6807	9.9433	9.7374
.51	.4882	.8727	.5594	1.7878	.6886	.9409	.7477
.52	.4969	.8678	.5726	1.7465	.6963	.9384	.7578
.53	.5055	.8628	.5859	1.7067	.7038	.9359	.7678
.54	.5141	.8577	.5994	1.6683	.7111	.9333	.7777

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* From E. S. Allen, "Six-Place Tables," McGraw-Hill Book Company, New York, 1947.

Table 4-1. Common logarithms of trigonometric functions (radian measure) (continued)

Radians	sin	cos	tan	cot	log sin	log cos	log tan
.55	.5227	.8525	.6131	1.6310	9.7182	9.9307	9.7875
.56	.5312	.8473	.6269	1.5940	.7252	.9280	.7972
.57	.5396	.8419	.6410	1.5601	.7321	.9253	.8068
.58	.5480	.8365	.6552	1.5263	.7388	.9224	.8164
.59	.5564	.8309	.6696	1.4935	.7454	.9196	.8258
.60	.5646	.8253	.6841	1.4617	9.7518	9.9166	9.8351
.61	.5729	.8196	.6989	1.4308	.7581	.9136	.8444
.62	.5810	.8139	.7139	1.4007	.7642	.9106	.8536
.63	.5891	.8080	.7291	1.3715	.7702	.9074	.8628
.64	.5972	.8021	.7445	1.3431	.7761	.9042	.8719
.65	.6052	.7961	.7602	1.3154	9.7819	9.9010	9.8809
.66	.6131	.7900	.7761	1.2885	.7875	.8976	.8899
.67	.6210	.7838	.7923	1.2622	.7931	.8942	.8989
.68	.6288	.7776	.8087	1.2366	.7985	.8907	.9078
.69	.6365	.7712	.8253	1.2116	.8038	.8872	.9166
.70	.6442	.7648	.8423	1.1872	9.8090	9.8836	9.9255
.71	.6518	.7584	.8595	1.1634	.8141	.8799	.9343
.72	.6594	.7518	.8771	1.1402	.8191	.8761	.9430
.73	.6669	.7452	.8949	1.1174	.8240	.8723	.9518
.74	.6743	.7385	.9131	1.0952	.8288	.8683	.9605
.75	.6816	.7317	.9316	1.0734	9.8336	9.8643	9.9692
.76	.6889	.7248	.9505	1.0521	.8382	.8602	.9779
.77	.6961	.7179	.9697	1.0313	.8427	.8561	.9866
.78	.7033	.7109	0.9893	1.0109	.8471	.8518	.9953
.79	.7104	.7038	1.0092	0.9908	.8515	.8475	0.0040
.80	.7174	.6967	1.0296	.9712	9.8557	9.8431	.0127
.81	.7243	.6895	1.0505	.9520	.8599	.8385	.0214
.82	.7311	.6822	1.0717	.9331	.8640	.8339	.0301
.83	.7379	.6749	1.0934	.9146	.8680	.8292	.0388
.84	.7446	.6675	1.1156	.8964	.8719	.8244	.0475
.85	.7513	.6600	1.1383	.8785	9.8758	9.8195	.0563
.86	.7578	.6524	1.1616	.8609	.8796	.8145	.0650
.87	.7643	.6448	1.1853	.8437	.8833	.8094	.0738
.88	.7707	.6372	1.2097	.8267	.8869	.8042	.0827
.89	.7771	.6294	1.2346	.8100	.8905	.7989	.0915
.90	.7833	.6216	1.2602	.7936	9.8939	9.7935	.1004
.91	.7895	.6137	1.2864	.7774	.8974	.7880	.1094
.92	.7956	.6058	1.3133	.7615	.9007	.7823	.1184
.93	.8016	.5978	1.3409	.7458	.9040	.7766	.1274
.94	.8076	.5898	1.3692	.7303	.9072	.7707	.1365
.95	.8134	.5817	1.3984	.7151	9.9103	9.7647	.1456
.96	.8192	.5735	1.4284	.7001	.9134	.7585	.1548
.97	.8249	.5653	1.4592	.6853	.9164	.7523	.1641
.98	.8305	.5570	1.4910	.6707	.9193	.7459	.1735
.99	.8360	.5487	1.5237	.6563	.9222	.7393	.1829
1.00	.8415	.5403	1.5574	.6421	9.9250	9.7326	.1924
1.01	.8468	.5319	1.5922	.6281	.9278	.7258	.2020
1.02	.8521	.5234	1.6281	.6142	.9305	.7188	.2117
1.03	.8573	.5148	1.6652	.6005	.9331	.7117	.2215
1.04	.8624	.5062	1.7036	.5870	.9357	.7043	.2314
1.05	.8674	.4976	1.7433	.5736	9.9382	9.6969	.2414
1.06	.8724	.4889	1.7844	.5604	.9407	.6892	.2515
1.07	.8772	.4801	1.8270	.5473	.9431	.6814	.2617
1.08	.8820	.4713	1.8712	.5344	.9454	.6733	.2721
1.09	.8866	.4625	1.9171	.5216	.9477	.6651	.2826

Table 4-1. Common logarithms of trigonometric functions (radian measure) (continued)

Radians	sin	cos	tan	cot	log sin	log cos	log tan
1.10	.8912	.4536	1.9648	.5090	9.9500	9.6567	.2933
1.11	.8957	.4447	2.0143	.4964	.9522	.6480	.3041
1.12	.9001	.4357	2.0660	.4840	.9543	.6392	.3151
1.13	.9044	.4267	2.1198	.4718	.9564	.6301	.3263
1.14	.9086	.4176	2.1759	.4596	.9584	.6208	.3376
1.15	.9128	.4085	2.2345	.4475	9.9604	9.6112	.3492
1.16	.9168	.3993	2.2958	.4356	.9623	.6013	.3609
1.17	.9208	.3902	2.3600	.4237	.9641	.5912	.3729
1.18	.9246	.3809	2.4273	.4120	.9660	.5808	.3851
1.19	.9284	.3717	2.4979	.4003	.9677	.5701	.3976
1.20	.9320	.3624	2.5722	.3888	9.9694	9.5591	.4103
1.21	.9356	.3530	2.6503	.3773	.9711	.5478	.4233
1.22	.9391	.3436	2.7328	.3659	.9727	.5361	.4366
1.23	.9425	.3342	2.8198	.3546	.9743	.5241	.4502
1.24	.9458	.3248	2.9119	.3434	.9758	.5116	.4642
1.25	.9490	.3153	3.0096	.3323	9.9773	9.4988	.4785
1.26	.9521	.3058	3.1133	.3212	.9787	.4855	.4932
1.27	.9551	.2963	3.2236	.3102	.9800	.4717	.5083
1.28	.9580	.2867	3.3413	.2993	.9814	.4575	.5239
1.29	.9608	.2771	3.4672	.2884	.9826	.4427	.5400
1.30	.9636	.2675	3.6021	.2776	9.9839	9.4273	.5566
1.31	.9662	.2579	3.7471	.2669	.9851	.4114	.5737
1.32	.9687	.2482	3.9033	.2562	.9862	.3948	.5914
1.33	.9711	.2385	4.0721	.2456	.9873	.3774	.6098
1.34	.9735	.2288	4.2556	.2350	.9883	.3594	.6290
1.35	.9757	.2190	4.4552	.2245	9.9893	9.3405	.6489
1.36	.9779	.2092	4.6734	.2140	.9903	.3206	.6696
1.37	.9799	.1994	4.9131	.2035	.9912	.2998	.6914
1.38	.9819	.1896	5.1774	.1931	.9920	.2779	.7141
1.39	.9837	.1798	5.4707	.1828	.9929	.2548	.7380
1.40	.9854	.1700	5.7979	.1725	9.9936	9.2304	.7633
1.41	.9871	.1601	6.1654	.1622	.9944	.2044	.7900
1.42	.9887	.1502	6.5811	.1519	.9950	.1767	.8183
1.43	.9901	.1403	7.0555	.1417	.9957	.1472	.8485
1.44	.9915	.1304	7.6018	.1315	.9963	.1154	.8809
1.45	.9927	.1205	8.2381	.1214	9.9969	9.0810	.9158
1.46	.9939	.1106	8.9886	.1113	.9973	.0436	.9537
1.47	.9949	.1006	9.8874	.1011	.9978	.0027	0.9951
1.48	.9959	.0907	10.9834	.0910	.9982	8.9575	1.0407
1.49	.9967	.0807	12.3499	.0810	.9986	.9069	.0917
1.50	.9975	.0707	14.1014	.0709	9.9989	8.8496	1.1493
1.51	.9982	.0608	16.4281	.0609	.9992	.7836	.2156
1.52	.9987	.0508	19.6695	.0508	.9994	.7056	.2938
1.53	.9992	.0408	24.4996	.0408	.9996	.6105	.3891
1.54	.9995	.0308	32.4611	.0308	.9998	.4884	.5114
1.55	.9998	.0208	48.0785	.0208	9.9999	8.3180	1.6820
1.56	.9999	.0108	92.6204	.0108	10.0000	8.0333	1.9667
1.57	1.0000	.0008	1255.7656	.0008	10.0000	6.9011	3.0989
1.58	1.0000	−.0092	−108.6492	−.0092	10.0000	7.9640n	2.0360n
1.59	.9998	−.0192	−52.0669	−.0192	9.9999	8.2834n	1.7166n
1.60	.9996	−.0292	−34.2325	−.0292	9.9998	8.4654n	1.5344n
1.61	.9992	−.0392	−25.4947	−.0392	.9997	.5932n	.4064n
1.62	.9988	−.0492	−20.3073	−.0492	.9995	.6918n	.3076n
1.63	.9982	−.0592	−16.8710	−.0593	.9992	.7721n	.2271n
1.64	.9976	−.0691	−14.4270	−.0693	.9990	.8398n	.1592n

Table 4-1. Common logarithms of trigonometric functions (radian measure) (continued)

Radians	sin	cos	tan	cot	log sin	log cos	log tan
1.65	.9969	−.0791	−12.5993	−.0794	9.9986	8.8983n	1.1003n
1.66	.9960	−.0891	−11.1806	−.0894	.9983	.9498n	.0485n
1.67	.9951	−.0990	−10.0472	−.0995	.9979	.9958n	1.0020n
1.68	.9940	−.1090	−9.1208	−.1096	.9974	.0374n	0.9600n
1.69	.9929	−.1189	−8.3492	−.1198	.9969	.0753n	.9216n
1.70	.9917	−.1288	−7.6966	−.1299	9.9964	9.1101n	.8863n
1.71	.9903	−.1388	−7.1373	−.1401	.9958	.1422n	.8535n
1.72	.9889	−.1487	−6.6524	−.1503	.9951	.1722n	.8230n
1.73	.9874	−.1585	−6.2281	−.1606	.9945	.2001n	.7944n
1.74	.9857	−.1684	−5.8535	−.1708	.9938	.2263n	.7674n
1.75	.9840	−.1782	−5.5204	−.1811	9.9930	9.2510n	.7420n
1.76	.9822	−.1881	−5.2221	−.1915	.9922	.2743n	.7178n
1.77	.9802	−.1979	−4.9534	−.2019	.9913	.2964n	.6949n
1.78	.9782	−.2077	−4.7101	−.2123	.9904	.3174n	.6730n
1.79	.9761	−.2175	−4.4887	−.2228	.9895	.3374n	.6521n
1.80	.9738	−.2272	−4.2862	−.2333	9.9885	9.3564n	.6321n
1.81	.9715	−.2369	−4.1005	−.2439	.9875	.3746n	.6128n
1.82	.9691	−.2466	−3.9294	−.2545	.9864	.3920n	.5943n
1.83	.9666	−.2563	−3.7712	−.2652	.9852	.4088n	.5765n
1.84	.9640	−.2660	−3.6245	−.2759	.9841	.4247n	.5597n
1.85	.9613	−.2756	−3.4881	−.2867	9.9828	9.4403n	.5426n
1.86	.9585	−.2852	−3.3608	−.2975	.9816	.4551n	.5265n
1.87	.9556	−.2948	−3.2419	−.3085	.9803	.4695n	.5108n
1.88	.9526	−.3043	−3.1304	−.2194	.9789	.4833n	.4956n
1.89	.9495	−.3138	−3.0257	−.3305	.9775	.4967n	.4808n
1.90	.9463	−.3233	−2.9271	−.3416	9.9760	9.5096n	.4664n
1.91	.9430	−.3327	−2.8341	−.3528	.9745	.5221n	.4524n
1.92	.9396	−.3421	−2.7463	−.3641	.9730	.5342n	.4387n
1.93	.9362	−.3515	−2.6632	−.3755	.9714	.5460n	.4254n
1.94	.9326	−.3609	−2.5843	−.3869	.9697	.5574n	.4123n
1.95	.9290	−.3702	−2.5095	−.3985	9.9680	9.5684n	.3996n
1.96	.9252	−.3795	−2.4383	−.4101	.9662	.5792n	.3871n
1.97	.9214	−.3887	−2.3705	−.4219	.9644	.5896n	.3748n
1.98	.9174	−.3979	−2.3058	−.4337	.9626	.5998n	.3628n
1.99	.9134	−.4070	−2.2426	−.4456	.9607	.6096n	.3510n
2.00	.9093	−.4161	−2.1850	−.4577	9.9587	9.6192n	.3395n
2.01	.9051	−.4252	−2.1285	−.4698	.9567	.6286n	.3281n
2.02	.9008	−.4342	−2.0744	−.4821	.9546	.6377n	.3169n
2.03	.8964	−.4432	−2.0224	−.4945	.9525	.6466n	.3059n
2.04	.8919	−.4522	−1.9725	−.5070	.9503	.6553n	.2950n
2.05	.8874	−.4611	−1.9246	−.5196	9.9481	9.6638n	.2843n
2.06	.8827	−.4699	−1.8784	−.5324	.9458	.6720n	.2738n
2.07	.8780	−.4787	−1.8340	−.5453	.9435	.6801n	.2634n
2.08	.8731	−.4875	−1.7911	−.5583	.9411	.6880n	.2531n
2.09	.8682	−.4962	−1.7498	−.5715	.9386	.6956n	.2430n
2.10	.8632	−.5048	−1.7097	−.5848	9.9361	9.7032n	.2330n
2.11	.8581	−.5135	−1.6713	−.5983	.9335	.7105n	.2230n
2.12	.8529	−.5220	−1.6340	−.6120	.9309	.7177n	.2132n
2.13	.8477	−.5305	−1.5979	−.6258	.9282	.7247n	.2035n
2.14	.8423	−.5390	−1.5629	−.6398	.9255	.7316n	.1939n
2.15	.8369	−.5474	−1.5290	−.6540	9.9227	9.7383n	.1844n
2.16	.8314	−.5557	−1.4961	−.6684	.9198	.7448n	.1750n
2.17	.8258	−.5640	−1.4642	−.6830	.9169	.7513n	.1656n
2.18	.8201	−.5722	−1.4332	−.6977	.9139	.7576n	.1563n
2.19	.8143	−.5804	−1.4031	−.7127	.9108	.7638n	.1470n

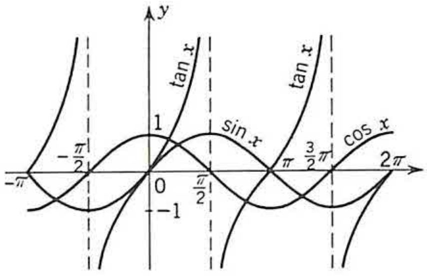
Table 4-1. Common logarithms of trigonometric functions (radian measure) (continued)

Radi-ans	sin	cos	tan	cot	log sin	log cos	log tan
2.20	.8085	−.5885	−1.3738	−.7279	9.9077	9.7697n	.1379n
2.21	.8026	−.5966	−1.3453	−.7433	.9045	.7757n	.1288n
2.22	.7967	−.6046	−1.3176	−.7589	.9012	.7814n	.1198n
2.23	.7905	−.6125	−1.2906	−.7748	.8979	.7871n	.1108n
2.24	.7843	−.6204	−1.2643	−.7910	.8945	.7926n	.1018n
2.25	.7781	−.6282	−1.2386	−.8073	9.8910	9.7981n	.0929n
2.26	.7718	−.6359	−1.2136	−.8240	.8875	.8034n	.0841n
2.27	.7654	−.6436	−1.1892	−.8409	.8839	.8086n	.0752n
2.28	.7589	−.6512	−1.1653	−.8581	.8802	.8137n	.0664n
2.29	.7523	−.6588	−1.1420	−.8757	.8764	.8187n	.0577n
2.30	.7457	−.6663	−1.1192	−.8935	9.8726	9.8237n	.0489n
2.31	.7390	−.6737	−1.0969	−.9117	.8686	.8285n	.0402n
2.32	.7322	−.6811	−1.0751	−.9301	.8646	.8332n	.0315n
2.33	.7254	−.6883	−1.0538	−.9489	.8606	.8378n	.0228n
2.34	.7185	−.6956	−1.0329	−.9681	.8564	.8423n	.0141n
2.35	.7115	−.7027	−1.0125	−.9877	9.8522	9.8468n	0.0054n
2.36	.7044	−.7098	−.9924	−1.0076	.8478	.8511n	9.9967n
2.37	.6973	−.7168	−.9728	−1.0280	.8434	.8554n	.9880n
2.38	.6901	−.7237	−.9535	−1.0488	.8389	.8596n	.9793n
2.39	.6828	−.7306	−.9346	−1.0700	.8343	.8637n	.9706n
2.40	.6755	−.7374	−.9160	−1.0917	9.8296	9.8677n	9.9619n
2.41	.6681	−.7441	−.8978	−1.1138	.8248	.8716n	.9532n
2.42	.6606	−.7508	−.8799	−1.1365	.8199	.8755n	.9444n
2.43	.6530	−.7573	−.8623	−1.1597	.8149	.8793n	.9357n
2.44	.6454	−.7638	−.8450	−1.1834	.8099	.8830n	.9269n
2.45	.6378	−.7702	−.8280	−1.2077	9.8047	9.8866n	9.9180n
2.46	.6300	−.7766	−.8113	−1.2326	.7994	.8902n	.9092n
2.47	.6222	−.7828	−.7948	−1.2581	.7940	.8937n	.9003n
2.48	.6144	−.7890	−.7787	−1.2843	.7884	.8971n	.8913n
2.49	.6065	−.7951	−.7627	−1.3101	.7828	.9004n	.8824n
2.50	.5985	−.8011	−.7470	−1.3386	9.7770	9.9037n	9.8733n
2.51	.5904	−.8071	−.7316	−1.3669	.7712	.9069n	.8642n
2.52	.5823	−.8130	−.7163	−1.3960	.7652	.9101n	.8551n
2.53	.5742	−.8187	−.7013	−1.4259	.7590	.9131n	.8459n
2.54	.5660	−.8244	−.6865	−1.4567	.7528	.9162n	.8366n
2.55	.5577	−.8301	−.6719	−1.4884	9.7464	9.9191n	9.8273n
2.56	.5494	−.8356	−.6574	−1.5210	.7399	.9220n	.8179n
2.57	.5410	−.8410	−.6432	−1.5547	.7332	.9248n	.8084n
2.58	.5325	−.8464	−.6292	−1.5894	.7263	.9276n	.7988n
2.59	.5240	−.8517	−.6153	−1.6252	.7194	.9303n	.7891n
2.60	.5155	−.8569	−.6016	−1.6622	9.7122	9.9329n	9.7793n
2.61	.5069	−.8620	−.5881	−1.7005	.7049	.9355n	.7694n
2.62	.4983	−.8670	−.5747	−1.7401	.6975	.9380n	.7594n
2.63	.4896	−.8720	−.5615	−1.7811	.6898	.9405n	.7493n
2.64	.4808	−.8768	−.5484	−1.8236	.6820	.9429n	.7391n
2.65	.4720	−.8816	−.5354	−1.8676	9.6740	9.9453n	9.7287n
2.66	.4632	−.8863	−.5226	−1.9134	.6658	.9476n	.7182n
2.67	.4543	−.8908	−.5100	−1.9609	.6573	.9498n	.7075n
2.68	.4454	−.8953	−.4974	−2.0103	.6487	.9520n	.6967n
2.69	.4364	−.8998	−.4850	−2.0618	.6399	.9541n	.6858n
2.70	.4274	−.9041	−.4727	−2.1154	9.6308	9.9562n	9.6746n
2.71	.4183	−.9083	−.4606	−2.1713	.6215	.9582n	.6633n
2.72	.4092	−.9124	−.4485	−2.2297	.6119	.9602n	.6517n
2.73	.4001	−.9165	−.4365	−2.2918	.6021	.9621n	.6400n
2.74	.3909	−.9204	−.4247	−2.3548	.5920	.9640n	.6281n

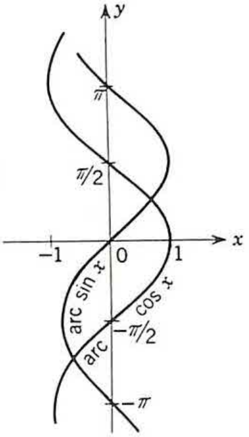
Table 4-1. Common logarithms of trigonometric functions (radian measure) (continued)

Radians	sin	cos	tan	cot	log sin	log cos	log tan
2.75	.3817	−.9243	−.4129	−2.4218	9.5817	9.9658n	9.6159n
2.76	.3724	−.9281	−.4013	−2.4921	.5710	.9676n	.6034n
2.77	.3631	−.9318	−.3897	−2.5661	.5600	.9693n	.5907n
2.78	.3538	−.9353	−.3782	−2.6439	.5487	.9710n	.5777n
2.79	.3444	−.9388	−.3668	−2.7260	.5371	.9726n	.5645n
2.80	.3350	−.9422	−.3555	−2.8107	9.5250	9.9742n	9.5509n
2.81	.3255	−.9455	−.3443	−2.9044	.5126	.9757n	.5369n
2.82	.3161	−.9487	−.3332	−3.0016	.4998	.9771n	.5227n
2.83	.3066	−.9518	−.3221	−3.1048	.4865	.9786n	.5080n
2.84	.2970	−.9549	−.3111	−3.2146	.4728	.9799n	.4929n
2.85	.2875	−.9578	−.3001	−3.3317	9.4586	9.9813n	9.4773n
2.86	.2779	−.9606	−.2893	−3.4569	.4439	.9826n	.4613n
2.87	.2683	−.9633	−.2785	−3.5910	.4286	.9838n	.4448n
2.88	.2586	−.9660	−.2677	−3.7351	.4127	.9850n	.4277n
2.89	.2489	−.9685	−.2570	−3.8904	.3961	.9861n	.4100n
2.90	.2392	−.9710	−.2464	−4.0583	9.3789	9.9872n	9.3917n
2.91	.2295	−.9733	−.2358	−4.2404	.3608	.9882n	.3726n
2.92	.2198	−.9755	−.2253	−4.4387	.3420	.9892n	.3527n
2.93	.2100	−.9777	−.2148	−4.6553	.3223	.9902n	.3321n
2.94	.2002	−.9797	−.2044	−4.8931	.3015	.9911n	.3104n
2.95	.1904	−.9817	−.1940	−5.1554	9.2797	9.9920n	9.2877n
2.96	.1806	−.9836	−.1836	−5.4462	.2567	.9928n	.2639n
2.97	.1708	−.9853	−.1733	−5.7704	.2324	.9936n	.2388n
2.98	.1609	−.9870	−.1630	−6.1344	.2065	.9943n	.2122n
2.99	.1510	−.9885	−.1528	−6.5460	.1760	.9950n	.1840n
3.00	.1411	−.9900	−.1425	−7.0152	9.1496	9.9956n	9.1540n
3.01	.1312	−.9913	−.1324	−7.5553	.1180	.9962n	.1218n
3.02	.1213	−.9926	−.1222	−8.1836	.0838	.9968n	.0871n
3.03	.1114	−.9938	−.1121	−8.9239	.0467	.9973n	.0494n
3.04	.1014	−.9948	−.1019	−9.8093	9.0061	.9978n	9.0084n
3.05	.0915	−.9958	−.0918	−10.8874	8.9613	9.9982n	8.9631n
3.06	.0815	−.9967	−.0818	−12.2288	.9112	.9986n	.9126n
3.07	.0715	−.9974	−.0717	−13.9440	.8545	.9989n	.8556n
3.08	.0616	−.9981	−.0617	−16.2152	.7893	.9992n	.7901n
3.09	.0516	−.9987	−.0516	−19.3654	.7124	.9994n	.7130n
3.10	.0416	−.9991	−.0416	−24.0288	8.6189	9.9996n	8.6193n
3.11	.0316	−.9995	−.0316	−31.6424	.4995	.9998n	.4997n
3.12	.0216	−.9998	−.0216	−46.3049	.3343	9.9999n	.3344n
3.13	.0116	−.9999	−.0116	−86.2578	8.0642	10.0000n	8.0642n
3.14	.0016	−1.0000	−.0016	−627.8825	7.2021	10.0000n	7.2021n
3.15	−.0084	−1.0000	.0084	118.9408	7.9247	10.0000n	7.9247
3.16	−.0184	−.9998	.0184	54.3200	8.2650n	9.9999n	8.2650
3.17	−.0284	−.9996	.0284	35.1927	.4535n	.9998n	.4536
3.18	−.0384	−.9993	.0384	26.0239	.5843n	.9997n	.5846
3.19	−.0484	−.9988	.0484	20.6419	.6847n	.9995n	.6853
3.20	−.0584	−.9983	.0585	15.1017	8.7662n	9.9993n	8.7670

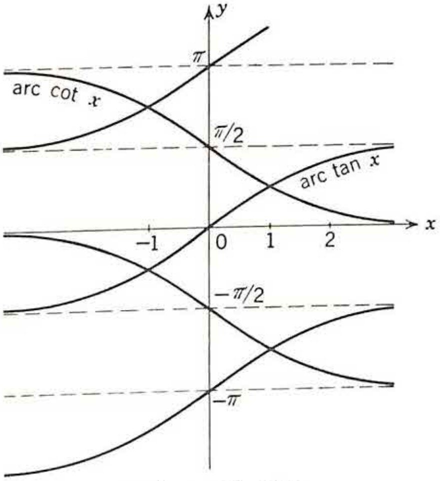
Trigonometric functions and inverse trigonometric functions



sin x, cos x, tan x



arc sin x, arc cos x



arc tan x, arc cot x

Table 4-2*a*. Degrees, minutes, and seconds to radians*

Degrees						Minutes		Seconds	
0°	0.00000 00	**60°**	1.04719 76	**120°**	2.09439 51	**0′**	0.00000 00	**0″**	0.00000 00
1	0.01745 33	61	1.06465 08	121	2.11184 84	1	0.00029 09	1	0.00000 48
2	0.03490 66	62	1.08210 41	122	2.12930 17	2	0.00058 18	2	0.00000 97
3	0.05235 99	63	1.09955 74	123	2.14675 50	3	0.00087 27	3	0.00001 45
4	0.06981 32	64	1.11701 07	124	2.16420 83	4	0.00116 36	4	0.00001 94
5	0.08726 6$\overline{5}$	**65**	1.13446 40	**125**	2.18166 16	**5**	0.00145 44	**5**	0.00002 42
6	0.10471 98	66	1.15191 73	126	2.19911 49	6	0.00174 53	6	0.00002 91
7	0.12217 30	67	1.16937 06	127	2.21656 82	7	0.00203 62	7	0.00003 39
8	0.13962 63	68	1.18682 39	128	2.23402 14	8	0.00232 71	8	0.00003 88
9	0.15707 96	69	1.20427 72	129	2.25147 47	9	0.00261 80	9	0.00004 36
10	0.17453 29	**70**	1.22173 0$\overline{5}$	**130**	2.26892 80	**10**	0.00290 89	**10**	0.00004 8$\overline{5}$
11	0.19198 62	71	1.23918 38	131	2.28638 13	11	0.00319 98	11	0.00005 33
12	0.20943 95	72	1.25663 71	132	2.30383 46	12	0.00349 07	12	0.00005 82
13	0.22689 28	73	1.27409 04	133	2.32128 79	13	0.00378 15	13	0.00006 30
14	0.24434 61	74	1.29154 36	134	2.33874 12	14	0.00407 24	14	0.00006 79
15	0.26179 94	**75**	1.30899 69	**135**	2.35619 4$\overline{5}$	**15**	0.00436 33	**15**	0.00007 27
16	0.27925 27	76	1.32645 02	136	2.37364 78	16	0.00465 42	16	0.00007 76
17	0.29670 60	77	1.34390 35	137	2.39110 11	17	0.00494 51	17	0.00008 24
18	0.31415 93	78	1.36135 68	138	2.40855 44	18	0.00523 60	18	0.00008 73
19	0.33161 26	79	1.37881 01	139	2.42600 77	19	0.00552 69	19	0.00009 21
20	0.34906 59	**80**	1.39626 34	**140**	2.44346 10	**20**	0.00581 78	**20**	0.00009 70
21	0.36651 91	81	1.41371 67	141	2.46091 42	21	0.00610 87	21	0.00010 18
22	0.38397 24	82	1.43117 00	142	2.47836 75	22	0.00639 95	22	0.00010 67
23	0.40142 57	83	1.44862 33	143	2.49582 08	23	0.00669 04	23	0.00011 15
24	0.41887 90	84	1.46607 66	144	2.51327 41	24	0.00698 13	24	0.00011 64
25	0.43633 23	85	1.48352 99	**145**	2.53072 74	**25**	0.00727 22	**25**	0.00012 12
26	0.45378 56	86	1.50098 32	146	2.54818 07	26	0.00756 31	26	0.00012 61
27	0.47123 89	87	1.51843 64	147	2.56563 40	27	0.00785 40	27	0.00013 09
28	0.48869 22	88	1.53588 97	148	2.58308 73	28	0.00814 49	28	0.00013 57
29	0.50614 55	89	1.55334 30	149	2.60054 06	29	0.00843 58	29	0.00014 06
30	0.52359 88	**90**	1.57079 63	**150**	2.61799 39	**30**	0.00872 66	**30**	0.00014 54
31	0.54105 21	91	1.58824 96	151	2.63544 72	31	0.00901 75	31	0.00015 03
32	0.55850 54	92	1.60570 29	152	2.65290 05	32	0.00930 84	32	0.00015 51
33	0.57595 87	93	1.62315 62	153	2.67035 38	33	0.00959 93	33	0.00016 00
34	0.59341 19	94	1.64060 9$\overline{5}$	154	2.68780 70	34	0.00989 02	34	0.00016 48
35	0.61086 52	**95**	1.65806 28	**155**	2.70526 03	**35**	0.01018 11	**35**	0.00016 97
36	0.62831 85	96	1.67551 61	156	2.72271 36	36	0.01047 20	36	0.00017 45
37	0.64577 18	97	1.69296 94	157	2.74016 69	37	0.01076 29	37	0.00017 94
38	0.66322 51	98	1.71042 27	158	2.75762 02	38	0.01105 38	38	0.00018 42
39	0.68067 84	99	1.72787 60	159	2.77507 35	39	0.01134 46	39	0.00018 91
40	0.69813 17	**100**	1.74532 93	**160**	2.79252 68	**40**	0.01163 55	**40**	0.00019 39
41	0.71558 50	101	1.76278 25	161	2.80998 01	41	0.01192 64	41	0.00019 88
42	0.73303 83	102	1.78023 58	162	2.82743 34	42	0.01221 73	42	0.00020 36
43	0.75049 16	103	1.79768 91	163	2.84488 67	43	0.01250 82	43	0.00020 85
44	0.76794 49	104	1.81514 24	164	2.86234 00	44	0.01279 91	44	0.00021 33
45	0.78539 82	**105**	1.83259 57	**165**	2.87979 33	**45**	0.01309 00	**45**	0.00021 82
46	0.80285 15	106	1.85004 90	166	2.89724 66	46	0.01338 09	46	0.00022 30
47	0.82030 47	107	1.86750 23	167	2.91469 99	47	0.01367 17	47	0.00022 79
48	0.83775 80	108	1.88495 56	168	2.93215 31	48	0.01396 26	48	0.00023 27
49	0.85521 13	109	1.90240 89	169	2.94960 64	49	0.01425 35	49	0.00023 76
50	0.87266 46	**110**	1.91986 22	**170**	2.96705 97	**50**	0.01454 44	**50**	0.00024 24
51	0.89011 79	111	1.93731 5$\overline{5}$	171	2.98451 30	51	0.01483 53	51	0.00024 73
52	0.90757 12	112	1.95476 88	172	3.00196 63	52	0.01512 62	52	0.00025 21
53	0.92502 45	113	1.97222 21	173	3.01941 96	53	0.01541 71	53	0.00025 70
54	0.94247 78	114	1.98967 53	174	3.03687 29	54	0.01570 80	54	0.00026 18
55	0.95993 11	**115**	2.00712 86	**175**	3.05432 62	**55**	0.01599 89	**55**	0.00026 66
56	0.97738 44	116	2.02458 19	176	3.07177 95	56	0.01628 97	56	0.00027 15
57	0.99483 77	117	2.04203 52	177	3.08923 28	57	0.01658 06	57	0.00027 63
58	1.01229 10	118	2.05948 85	178	3.10668 61	58	0.01687 15	58	0.00028 12
59	1.02974 43	119	2.07694 18	179	3.12413 94	59	0.01716 24	59	0.00028 60
60	1.04719 76	**120**	2.09439 51	**180**	3.14159 27	**60**	0.01745 33	**60**	0.00029 09
Degrees						Minutes		Seconds	

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* From E. S. Allen, "Six-Place Tables," McGraw-Hill Book Company, New York, 1947.

Table 4-2*b*. Radians to degrees, minutes, and seconds*

Radians	Degrees	Degrees, min., sec.	Radians	Degrees	Degrees, min., sec.
.00001	.000573	0° 0′ 2.1″	.01	.572958	34′ 22.6″
2	1146	4.1″	2	1.145916	1° 8′ 45.3″
3	1719	6.2″	3	1.718873	1° 43′ 7.9″
4	2292	8.3″	4	2.291831	2° 17′ 30.6″
.00005	.002865	0° 0′ 10.3″	.05	2.864789	2° 51′ 53.2″
6	3438	12.4″	6	3.437747	3° 26′ 15.9″
7	4011	14.4″	7	4.010705	4° 0′ 38.5″
8	4584	16.5″	8	4.583662	4° 35′ 1.2″
9	5157	18.6″	9	5.156620	5° 9′ 23.8″
.00010	.005730	0° 0′ 20.6″	.10	5.729578	5° 43′ 46.5″
2	11459	41.3″	.2	11.459156	11° 27′ 33.0″
3	17189	1′ 1.9″	.3	17.188734	17° 11′ 19.4″
4	22918	1′ 22.5″	.4	22.918312	22° 55′ 5.9″
.0005	.028648	0° 1′ 43.1″	.5	28.647890	28° 38′ 52.4″
6	34377	2′ 3.8″	.6	34.377468	34° 22′ 38.9″
7	40107	2′ 24.4″	.7	40.107046	40° 6′ 25.4″
8	45837	2′ 45.0″	.8	45.836624	45° 50′ 11.8″
9	51566	3′ 5.6″	.9	51.566202	51° 33′ 58.3″
.0010	.057296	0° 3′ 26.3″	1.0	57.295780	57° 17′ 44.8″
2	.114592	6′ 52.5″	2	114.591559	114° 35 29.6″
3	.171887	10′ 18.8″	3	171.887339	171° 53′ 14.4″
4	.229183	13′ 45.1″	4	229.183118	229° 10′ 59.2″
.005	.286479	0° 17′ 11.3″	5	286.478898	286° 28′ 44.0″
6	.343775	20′ 37.6″	6	343.774677	343° 46′ 28.8″
7	.401070	24′ 3.9″	7	401.070457	401° 4′ 13.6″
8	.458366	27′ 30.1″	8	458.366236	458° 21′ 58.4″
9	.515662	30′ 56.4″	9	515.662016	515° 39′ 43.3″

* From E. S. Allen, "Six-Place Tables," McGraw-Hill Book Company, New York, 1947.

Table 4-3. Decimal equivalents of minutes, seconds, and common fractions*

From minutes and seconds into decimal parts of a degree			
0′	0°.0000	0″	0°.0000
1	.0167	1	.0003
2	.0333	2	.0006
3	.05	3	.0008
4	.0667	4	.0011
5′	.0833	5″	.0014
6	.10	6	.0017
7	.1167	7	.0019
8	.1333	8	.0022
9	.15	9	.0025
10′	0°.1667	10″	0°.0028
1	.1833	1	.0031
2	.20	2	.0033
3	.2167	3	.0036
4	.2333	4	.0039
15′	.25	15″	.0042
6	.2667	6	.0044
7	.2833	7	.0047
8	.30	8	.005
9	.3167	9	.0053
20′	0°.3333	20″	0°.0056
1	.35	1	.0058
2	.3667	2	.0061
3	.3833	3	.0064
4	.40	4	.0067
25′	.4167	25″	.0069
6	.4333	6	.0072
7	.45	7	.0075
8	.4667	8	.0078
9	.4833	9	.0081
30′	0°.50	30″	0°.0083
1	.5167	1	.0086
2	.5333	2	.0089
3	.55	3	.0092
4	.5667	4	.0094
35′	.5833	35″	.0097
6	.60	6	.01
7	.6167	7	.0103
8	.6333	8	.0106
9	.65	9	.0108
40′	0°.6667	40″	0°.0111
1	.6833	1	.0114
2	.70	2	.0117
3	.7167	3	.0119
4	.7333	4	.0122
45′	.75	45″	.0125
6	.7667	6	.0128
7	.7833	7	.0131
8	.80	8	.0133
9	.8167	9	.0136
50′	0°.8333	50″	0°.0139
1	.85	1	.0142
2	.8667	2	.0144
3	.8833	3	.0147
4	.90	4	.015
55′	.9167	55″	.0153
6	.9333	6	.0156
7	.95	7	.0158
8	.9667	8	.0161
9	.9833	9	.0164
60′	1.00	60″	0°.0167

From decimal parts of a degree into minutes and seconds (exact values)			
0°.00	0′	0°.50	30′
1	0′ 36″	1	30′ 36″
2	1′ 12″	2	31′ 12″
3	1′ 48″	3	31′ 48″
4	2′ 24″	4	32′ 24″
0°.05	3′	0°.55	33′
6	3′ 36″	6	33′ 36″
7	4′ 12″	7	34′ 12″
8	4′ 48″	8	34′ 48″
9	5′ 24″	9	35′ 24″
0°.10	6′	0°.60	36′
1	6′ 36″	1	36′ 36″
2	7′ 12″	2	37′ 12″
3	7′ 48″	3	37′ 48″
4	8′ 24″	4	38′ 24″
0°.15	9′	0°.65	39′
6	9′ 36″	6	39′ 36″
7	10′ 12″	7	40′ 12″
8	10′ 48″	8	40′ 48″
9	11′ 24″	9	41′ 24″
0°.20	12′	0°.70	42′
1	12′ 36″	1	42′ 36″
2	13′ 12″	2	43′ 12″
3	13′ 48″	3	43′ 48″
4	14′ 24″	4	44′ 24″
0°.25	15′	0°.75	45′
6	15′ 36″	6	45′ 36″
7	16′ 12″	7	46′ 12″
8	16′ 48″	8	46′ 48″
9	17′ 24″	9	47′ 24″
0°.30	18′	0°.80	48′
1	18′ 36″	1	48′ 36″
2	19′ 12″	2	49′ 12″
3	19′ 48″	3	49′ 48″
4	20′ 24″	4	50′ 24″
0°.35	21′	0°.85	51′
6	21′ 36″	6	51′ 36″
7	22′ 12″	7	52′ 12″
8	22′ 48″	8	52′ 48″
9	23′ 24″	9	53′ 24″
0°.40	24′	0°.90	54′
1	24′ 36″	1	54′ 36″
2	25′ 12″	2	55′ 12″
3	25′ 48″	3	55′ 48″
4	26′ 24″	4	56′ 24″
0°.45	27′	0°.95	57′
6	27′ 36″	6	57′ 36″
7	28′ 12″	7	58′ 12″
8	28′ 48″	8	58′ 48″
9	29′ 24″	9	59′ 24″
0°.50	30′	1°.00	60′

0°.000	0″.0
1	3″.6
2	7″.2
3	10″.8
4	14″.4
0°.005	18″
6	21″.6
7	25″.2
8	28″.8
9	32″.4
0°.010	36″

Common fractions				
8 ths	16 ths	32 nds	64 ths	Exact decimal values
			1	.01 5625
		1	2	.03 125
			3	.04 6875
	1	2	4	.06 25
			5	.07 8125
		3	6	.09 375
			7	.10 9375
1	2	4	8	.12 5
			9	.14 0625
		5	10	.15 625
			11	.17 1875
	3	6	12	.18 75
			13	.20 3125
		7	14	.21 875
			15	.23 4375
2	4	8	16	.25
			17	.26 5625
		9	18	.28 125
			19	.29 6875
	5	10	20	.31 25
			21	.32 8125
		11	22	.34 375
			23	.35 9375
3	6	12	24	.37 5
			25	.39 0625
		13	26	.40 625
			27	.42 1875
	7	14	28	.43 75
			29	.45 3125
		15	30	.46 875
			31	.48 4375
4	8	16	32	.50
			33	.51 5625
		17	34	.53 125
			35	.54 6875
	9	18	36	.56 25
			37	.57 8125
		19	38	.59 375
			39	.60 9375
5	10	20	40	.62 5
			41	.64 0625
		21	42	.65 625
			43	.67 1875
	11	22	44	.68 75
			45	.70 3125
		23	46	.71 875
			47	.73 4375
6	12	24	48	.75
			49	.76 5625
		25	50	.78 125
			51	.79 6875
	13	26	52	.81 25
			53	.82 8125
		27	54	.84 375
			55	.85 9375
7	14	28	56	.87 5
			57	.89 0625
		29	58	.90 625
			59	.92 1875
	15	30	60	.93 75
			61	.95 3125
		31	62	.96 875
			63	.98 4375

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* From L. S. Marks, "Mechanical Engineers Handbook," McGraw-Hill Book Company, New York, 1958.

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas*

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
1	1	1	1.0000	1.0000	1.000000000	3.1416	0.7854
2	4	8	1.4142	1.2599	.500000000	6.2832	3.1416
3	9	27	1.7321	1.4422	.333333333	9.4248	7.0686
4	16	64	2.0000	1.5874	.250000000	12.5664	12.5664
5	25	125	2.2361	1.7100	.200000000	15.7080	19.635
6	36	216	2.4495	1.8171	.166666667	18.850	28.274
7	49	343	2.6458	1.9129	.142857143	21.991	38.485
8	64	512	2.8284	2.0000	.125000000	25.133	50.266
9	81	729	3.0000	2.0801	.111111111	28.274	63.617
10	100	1,000	3.1623	2.1544	.100000000	31.416	78.540
11	121	1,331	3.3166	2.2240	.090909091	34.558	95.033
12	144	1,728	3.4641	2.2894	.083333333	37.699	113.10
13	169	2,197	3.6056	2.3513	.076923077	40.841	132.73
14	196	2,744	3.7417	2.4101	.071428571	43.982	153.94
15	225	3,375	3.8730	2.4662	.066666667	47.124	176.71
16	256	4,096	4.0000	2.5198	.062500000	50.265	201.06
17	289	4,913	4.1231	2.5713	.058823529	53.407	226.98
18	324	5,832	4.2426	2.6207	.055555556	56.549	254.47
19	361	6,859	4.3589	2.6684	.052631579	59.690	283.53
20	400	8,000	4.4721	2.7144	.050000000	62.832	314.16
21	441	9,261	4.5826	2.7589	.047619048	65.973	346.36
22	484	10,648	4.6904	2.8020	.045454545	69.115	380.13
23	529	12,167	4.7958	2.8439	.043478261	72.257	415.48
24	576	13,824	4.8990	2.8845	.041666667	75.398	452.39
25	625	15,625	5.0000	2.9240	.040000000	78.540	490.87
26	676	17,576	5.0990	2.9625	.038461538	81.681	530.93
27	729	19,683	5.1962	3.0000	.037037037	84.823	572.56
28	784	21,952	5.2915	3.0366	.035714286	87.965	615.75
29	841	24,389	5.3852	3.0723	.034482759	91.106	660.52
30	900	27,000	5.4772	3.1072	.033333333	94.248	706.86
31	961	29,791	5.5678	3.1414	.032258065	97.389	754.77
32	1,024	32,768	5.6569	3.1748	.031250000	100.53	804.25
33	1,089	35,937	5.7446	3.2075	.030303030	103.67	855.30
34	1,156	39,304	5.8310	3.2396	.029411765	106.81	907.92
35	1,225	42,875	5.9161	3.2711	.028571429	109.96	962.11
36	1,296	46,656	6.0000	3.3019	.027777778	113.10	1,017.88
37	1,369	50,653	6.0828	3.3322	.027027027	116.24	1,075.21
38	1,444	54,872	6.1644	3.3620	.026315789	119.38	1,134.11
39	1,521	59,319	6.2450	3.3912	.025641026	122.52	1,194.59
40	1,600	64,000	6.3246	3.4200	.025000000	125.66	1,256.64
41	1,681	68,921	6.4031	3.4482	.024390244	128.81	1,320.25
42	1,764	74,088	6.4807	3.4760	.023809524	131.95	1,385.44
43	1,849	79,507	6.5574	3.5034	.023255814	135.09	1,452.20
44	1,936	85,184	6.6332	3.5303	.022727273	138.23	1,520.53
45	2,025	91,125	6.7082	3.5569	.022222222	141.37	1,590.43
46	2,116	97,336	6.7823	3.5830	.021739130	144.51	1,661.90
47	2,209	103,823	6.8557	3.6088	.021276596	147.65	1,734.94
48	2,304	110,592	6.9282	3.6342	.020833333	150.80	1,809.56
49	2,401	117,649	7.0000	3.6593	.020408163	153.94	1,885.74
50	2,500	125,000	7.0711	3.6840	.020000000	157.08	1,963.50
51	2,601	132,651	7.1414	3.7084	.019607843	160.22	2,042.82
52	2,704	140,608	7.2111	3.7325	.019230769	163.36	2,123.72
53	2,809	148,877	7.2801	3.7563	.018867925	166.50	2,206.18
54	2,916	157,464	7.3485	3.7798	.018518519	169.65	2,290.22
55	3,025	166,375	7.4162	3.8030	.018181818	172.79	2,375.83

* From E. S. Allen, "Six-Place Tables," McGraw-Hill Book Company, New York, 1947.

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum	Area
56	3,136	175,616	7.4833	3.8259	.017857143	175.93	2,463.01
57	3,249	185,193	7.5498	3.8485	.017543860	179.07	2,551.76
58	3,364	195,112	7.6158	3.8709	.017241379	182.21	2,642.08
59	3,481	205,379	7.6811	3.8930	.016949153	185.35	2,733.97
60	3,600	216,000	7.7460	3.9149	.016666667	188.50	2,827.43
61	3,721	226,981	7.8102	3.9365	.016393443	191.64	2,922.47
62	3,844	238,328	7.8740	3.9579	.016129032	194.78	3,019.07
63	3,969	250,047	7.9373	3.9791	.015873016	197.92	3,117.25
64	4,096	262,144	8.0000	4.0000	.015625000	201.06	3,216.99
65	4,225	274,625	8.0623	4.0207	.015384615	204.20	3,318.31
66	4,356	287,496	8.1240	4.0412	.015151515	207.34	3,421.19
67	4,489	300,763	8.1854	4.0615	.014925373	210.49	3,525.65
68	4,624	314,432	8.2462	4.0817	.014705882	213.63	3,631.68
69	4,761	328,509	8.3066	4.1016	.014492754	216.77	3,739.28
70	4,900	343,000	8.3666	4.1213	.014285714	219.91	3,848.45
71	5,041	357,911	8.4261	4.1408	.014084507	223.05	3,959.19
72	5,184	373,248	8.4853	4.1602	.013888889	226.19	4,071.50
73	5,329	389,017	8.5440	4.1793	.013698630	229.34	4,185.39
74	5,476	405,224	8.6023	4.1983	.013513514	232.48	4,300.84
75	5,625	421,875	8.6603	4.2172	.013333333	235.62	4,417.86
76	5,776	438,976	8.7178	4.2358	.013157895	238.76	4,536.46
77	5,929	456,533	8.7750	4.2543	.012987013	241.90	4,656.63
78	6,084	474,552	8.8318	4.2727	.012820513	245.04	4,778.36
79	6,241	493,039	8.8882	4.2908	.012658228	248.19	4,901.67
80	6,400	512,000	8.9443	4.3089	.012500000	251.33	5,026.55
81	6,561	531,441	9.0000	4.3267	.012345679	254.47	5,153.00
82	6,724	551,368	9.0554	4.3445	.012195122	257.61	5,281.02
83	6,889	571,787	9.1104	4.3621	.012048193	260.75	5,410.61
84	7,056	592,704	9.1652	4.3795	.011904762	263.89	5,541.77
85	7,225	614,125	9.2195	4.3968	.011764706	267.04	5,674.50
86	7,396	636,056	9.2736	4.4140	.011627907	270.18	5,808.80
87	7,569	658,503	9.3274	4.4310	.011494253	273.32	5,944.68
88	7,744	681,472	9.3808	4.4480	.011363636	276.46	6,082.12
89	7,921	704,969	9.4340	4.4647	.011235955	279.60	6,221.14
90	8,100	729,000	9.4868	4.4814	.011111111	282.74	6,361.73
91	8,281	753,571	9.5394	4.4979	.010989011	285.88	6,503.88
92	8,464	778,688	9.5917	4.5144	.010869565	289.03	6,647.61
93	8,649	804,357	9.6437	4.5307	.010752688	292.17	6,792.91
94	8,836	830,584	9.6954	4.5468	.010638298	295.31	6,939.78
95	9,025	857,375	9.7468	4.5629	.010526316	298.45	7,088.22
96	9,216	884,736	9.7980	4.5789	.010416667	301.59	7,238.23
97	9,409	912,673	9.8489	4.5947	.010309278	304.73	7,389.81
98	9,604	941,192	9.8995	4.6104	.010204082	307.88	7,542.96
99	9,801	970,299	9.9499	4.6261	.010101010	311.02	7,697.69
100	10,000	1,000,000	10.0000	4.6416	.010000000	314.16	7,853.98
101	10,201	1,030,301	10.0499	4.6570	.009900990	317.30	8,011.85
102	10,404	1,061,208	10.0995	4.6723	.009803922	320.44	8,171.28
103	10,609	1,092,727	10.1489	4.6875	.009708738	323.58	8,332.29
104	10,816	1,124,864	10.1980	4.7027	.009615385	326.73	8,494.87
105	11,025	1,157,625	10.2470	4.7177	.009523810	329.87	8,659.01
106	11,236	1,191,016	10.2956	4.7326	.009433962	333.01	8,824.73
107	11,449	1,225,043	10.3441	4.7475	.009345794	336.15	8,992.02
108	11,664	1,259,712	10.3923	4.7622	.009259259	339.29	9,160.88
109	11,881	1,295,029	10.4403	4.7769	.009174312	342.43	9,331.32
110	12,100	1,331,000	10.4881	4.7914	.009090909	345.58	9,503.32
111	12,321	1,367,631	10.5357	4.8059	.009009009	348.72	9,676.89
112	12,544	1,404,928	10.5830	4.8203	.008928571	351.86	9,852.03
113	12,769	1,442,897	10.6301	4.8346	.008849558	355.00	10,028.75
114	12,996	1,481,544	10.6771	4.8488	.008771930	358.14	10,207.03
115	13,225	1,520,875	10.7238	4.8629	.008695652	361.28	10,386.89
116	13,456	1,560,896	10.7703	4.8770	.008620690	364.42	10,568.32
117	13,689	1,601,613	10.8167	4.8910	.008547009	367.57	10,751.32
118	13,924	1,643,032	10.8628	4.9049	.008474576	370.71	10,935.88

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
119	14,161	1,685,159	10.9087	4.9187	.008403361	373.85	11,122.02
120	14,400	1,728,000	10.9545	4.9324	.008333333	376.99	11,309.73
121	14,641	1,771,561	11.0000	4.9461	.008264463	380.13	11,499:01
122	14,884	1,815,848	11.0454	4.9597	.008196721	383.27	11,689.87
123	15,129	1,860,867	11.0905	4.9732	.008130081	386.42	11,882.29
124	15,376	1,906,624	11.1355	4.9866	.008064516	389.56	12,076.28
125	15,625	1,953,125	11.1803	5.0000	.008000000	392.70	12,271.85
126	15,876	2,000,376	11.2250	5.0133	.007936508	395.84	12,468.98
127	16,129	2,048,383	11.2694	5.0265	.007874016	398.98	12,667.69
128	16,384	2,097,152	11.3137	5.0397	.007812500	402.12	12,867.96
129	16,641	2,146,689	11.3578	5.0528	.007751938	405.27	13,069.81
130	16,900	2,197,000	11.4018	5.0658	.007692308	408.41	13,273.23
131	17,161	2,248.091	11.4455	5.0788	.007633588	411.55	13,478.22
132	17,424	2,299.968	11.4891	5.0916	.007575758	414.69	13,684.78
133	17,689	2,352,637	11.5326	5.1045	.007518797	417.83	13,892.91
134	17,956	2,406.104	11.5758	5.1172	.007462687	420.97	14,102.61
135	18,225	2,460,375	11.6190	5.1299	.007407407	424.12	14,313.88
136	18,496	2,515,456	11.6619	5.1426	.007352941	427.26	14,526.72
137	18,769	2,571,353	11.7047	5.1551	.007299270	430.40	14,741.14
138	19,044	2,628,072	11.7473	5.1676	.007246377	433.54	14,957.12
139	19,321	2,685,619	11.7898	5.1801	.007194245	436.68	15,174.68
140	19,600	2,744,000	11.8322	5.1925	.007142857	439.82	15,393.80
141	19,881	2,803,221	11.8743	5.2048	.007092199	442.96	15,614.50
142	20,164	2,863,288	11.9164	5.2171	.007042254	446.11	15,836.77
143	20,449	2,924,207	11.9583	5.2293	.006993007	449.25	16,060.61
144	20,736	2,985,984	12.0000	5.2415	.006944444	452.39	16,286.02
145	21,025	3,048,625	12.0416	5.2536	.006896552	455.53	16,513.00
146	21,316	3,112,136	12.0830	5.2656	.006849315	458.67	16,741.55
147	21,609	3,176,523	12.1244	5.2776	.006802721	461.81	16,971.67
148	21,904	3,241,792	12.1655	5.2896	.006756757	464.96	17,203.36
149	22,201	3,307,949	12.2066	5.3015	.006711409	468.10	17,436.62
150	22,500	3,375,000	12.2474	5.3133	.006666667	471.24	17,671.46
151	22,801	3,442,951	12.2882	5.3251	.006622517	474.38	17,907.86
152	23,104	3,511,808	12.3288	5.3368	.006578947	477.52	18,145.84
153	23,409	3,581,577	12.3693	5.3485	.006535948	480.66	18,385.39
154	23,716	3,652,264	12.4097	5.3601	.006493506	483.81	18,626.50
155	24,025	3,723,875	12.4499	5.3717	.006451613	486.95	18,869.19
156	24,336	3,796,416	12.4900	5.3832	.006410256	490.09	19,113.45
157	24,649	3,869,893	12.5300	5.3947	.006369427	493.23	19,359.28
158	24,964	3,944,312	12.5698	5.4061	.006329114	496.37	19,606.68
159	25,281	4,019,679	12.6095	5.4175	.006289308	499.51	19,855.65
160	25,600	4,096.000	12.6491	5.4288	.006250000	502.65	20,106.19
161	25,921	4,173,281	12.6886	5.4401	.006211180	505.80	20,358.31
162	26,244	4,251,528	12.7279	5.4514	.006172840	508.94	20,611.99
163	26,569	4,330.747	12.7671	5.4626	.006134969	512.08	20,867.24
164	26,896	4,410,944	12.8062	5.4737	.006097561	515.22	21,124.07
165	27,225	4,492,125	12.8452	5.4848	.006060606	518.36	21,382.46
166	27,556	4,574,296	12.8841	5.4959	.006024096	521.50	21,642.43
167	27,889	4,657,463	12.9228	5.5069	.005988024	524.65	21,903.97
168	28,224	4,741,632	12.9615	5.5178	.005952381	527.79	22,167.08
169	28,561	4,826,809	13.0000	5.5288	.005917160	530.93	22,431.76
170	28,900	4,913,000	13.0384	5.5397	.005882353	534.07	22,698.01
171	29,241	5,000,211	13.0767	5.5505	.005847953	537.21	22,965.83
172	29,584	5,088,448	13.1149	5.5613	.005813953	540.35	23,235.22
173	29,929	5,177,717	13.1529	5.5721	.005780347	543.50	23,506.18
174	30,276	5,268,024	13.1909	5.5828	.005747126	546.64	23,778.71
175	30,625	5,359,375	13.2288	5.5934	.005714286	549.78	24,052.82
176	30,976	5,451,776	13.2665	5.6041	.005681818	552.92	24,328.49
177	31,329	5,545,233	13.3041	5.6147	.005649718	556.06	24,605.74
178	31,684	5,639,752	13.3417	5.6252	.005617978	559.20	24,884.56
179	32,041	5,735,339	13.3791	5.6357	.005586592	562.35	25,164.94
180	32,400	5,832,000	13.4164	5.6462	.005555556	565.49	25,446.90
181	32,761	5,929,741	13.4536	5.6567	.005524862	568.63	25,730.48

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
182	33,124	6,028,568	13.4907	5.6671	.005494505	571.77	26,015.53
183	33,489	6,128,487	13.5277	5.6774	.005464481	574.91	26,302.20
184	33,856	6,229,504	13.5647	5.6877	.005434783	578.05	26,590.44
185	34,225	6,331,625	13.6015	5.6980	.005405405	581.19	26,880.25
186	34,596	6,434,856	13.6382	5.7083	.005376344	584.34	27,171.63
187	34,969	6,539,203	13.6748	5.7185	.005347594	587.48	27,464.59
188	35,344	6,644,672	13.7113	5.7287	.005319149	590.62	27,759.11
189	35,721	6,751,269	13.7477	5.7388	.005291005	593.76	28,055.21
190	36,100	6,859,000	13.7840	5.7489	.005263158	596.90	28,352.87
191	36,481	6,967,871	13.8203	5.7590	.005235602	600.04	28,652.11
192	36,864	7,077,888	13.8564	5.7690	.005208333	603.19	28,952.92
193	37,249	7,189,057	13.8924	5.7790	.005181347	606.33	29,255.30
194	37,636	7,301,384	13.9284	5.7890	.005154639	609.47	29,559.25
195	38,025	7,414,875	13.9642	5.7989	.005128205	612.61	29,864.77
196	38,416	7,529,536	14.0000	5.8088	.005102041	615.75	30,171.86
197	38,809	7,645,373	14.0357	5.8186	.005076142	618.89	30,480.52
198	39,204	7,762,392	14.0712	5.8285	.005050505	622.04	30,790.75
199	39,601	7,880,599	14.1067	5.8383	.005025126	625.18	31,102.55
200	40,000	8,000,000	14.1421	5.8480	.005000000	628.32	31,415.93
201	40,401	8,120,601	14.1774	5.8578	.004975124	631.46	31,730.87
202	40,804	8,242,408	14.2127	5.8675	.004950495	634.60	32,047.39
203	41,209	8,365,427	14.2478	5.8771	.004926108	637.74	32,365.47
204	41,616	8,489,664	14.2829	5.8868	.004901961	640.88	32,685.13
205	42,025	8,615,125	14.3178	5.8964	.004878049	644.03	33,006.36
206	42,436	8,741,816	14.3527	5.9059	.004854369	647.17	33,329.16
207	42,849	8,869,743	14.3875	5.9155	.004830918	650.31	33,653.53
208	43,264	8,998,912	14.4222	5.9250	.004807692	653.45	33,979.47
209	43,681	9,129,329	14.4568	5.9345	.004784689	656.59	34,306.98
210	44,100	9,261,000	14.4914	5.9439	.004761905	659.73	34,636.06
211	44,521	9,393,931	14.5258	5.9533	.004739336	662.88	34,966.71
212	44,944	9,528.128	14.5602	5.9627	.004716981	666.02	35,298.94
213	45,369	9,663,597	14.5945	5.9721	.004694836	669.16	35,632.73
214	45,796	9,800.344	14.6287	5.9814	.004672897	672.30	35,968.09
215	46,225	9,938,375	14.6629	5.9907	.004651163	675.44	36,305.03
216	46,656	10,077,696	14.6969	6.0000	.004629630	678.58	36,643.54
217	47,089	10,218,313	14.7309	6.0092	.004608295	681.73	36,983.61
218	47,524	10,360,232	14.7648	6.0185	.004587156	684.87	37,325.26
219	47,961	10,503,459	14.7986	6.0277	.004566210	688.01	37,668.48
220	48,400	10,648,000	14.8324	6.0368	.004545455	691.15	38,013.27
221	48,841	10,793,861	14.8661	6.0459	.004524887	694.29	38,359.63
222	49,284	10,941,048	14.8997	6.0550	.004504505	697.43	38,707.56
223	49,729	11,089,567	14.9332	6.0641	.004484305	700.58	39,057.07
224	50,176	11,239,424	14.9666	6.0732	.004464286	703.72	39,408.14
225	50,625	11,390,625	15.0000	6.0822	.004444444	706.86	39,760.78
226	51,076	11,543,176	15.0333	6.0912	.004424779	710.00	40,115.00
227	51,529	11,697,083	15.0665	6.1002	.004405286	713.14	40,470.78
228	51,984	11,852,352	15.0997	6.1091	.004385965	716.28	40,828.14
229	52,441	12,008,989	15.1327	6.1180	.004366812	719.42	41,187.07
230	52,900	12,167,000	15.1658	6.1269	.004347826	722.57	41,547.56
231	53,361	12,326,391	15.1987	6.1358	.004329004	725.71	41,909.63
232	53,824	12,487,168	15.2315	6.1446	.004310345	728.85	42,273.27
233	54.289	12,649,337	15.2643	6.1534	.004291845	731.99	42,638.48
234	54,756	12,812.904	15.2971	6.1622	.004273504	735.13	43,005.26
235	55,225	12,977,875	15.3297	6.1710	.004255319	738.27	43,373.61
236	55,696	13,144.256	15.3623	6.1797	.004237288	741.42	43,743.54
237	56,169	13,312,053	15.3948	6.1885	.004219409	744.56	44,115.03
238	56,644	13,481,272	15.4272	6.1972	.004201681	747.70	44,488.09
239	57,121	13,651,919	15.4596	6.2058	.004184100	750.84	44,862.73
240	57,600	13,824,000	15.4919	6.2145	.004166667	753.98	45,238.93
241	58,081	13,997,521	15.5242	6.2231	.004149378	757.12	45,616.71
242	58,564	14,172.488	15.5563	6.2317	.004132231	760.27	45,996.06
243	59,049	14,348,907	15.5885	6.2403	.004115226	763.41	46,376.98
244	59,536	14,526,784	15.6205	6.2488	.004098361	766.55	46,759.47

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
245	60,025	14,706,125	15.6525	6.2573	.004081633	769.69	47,143.52
246	60,516	14,886,936	15.6844	6.2658	.004065041	772.83	47,529.16
247	61,009	15,069,223	15.7162	6.2743	.004048583	775.97	47,916.36
248	61,504	15,252,992	15.7480	6.2828	.004032258	779.11	48,305.13
249	62,001	15,438,249	15.7797	6.2912	.004016064	782.26	48,695.47
250	62,500	15,625,000	15.8114	6.2996	.004000000	785.40	49,087.39
251	63,001	15,813,251	15.8430	6.3080	.003984064	788.54	49,480.87
252	63,504	16,003,008	15.8745	6.3164	.003968254	791.68	49,875.92
253	64,009	16,194,277	15.9060	6.3247	.003952569	794.82	50,272.55
254	64,516	16,387,064	15.9374	6.3330	.003937008	797.96	50,670.75
255	65,025	16,581,375	15.9687	6.3413	.003921569	801.11	51,070.52
256	65,536	16,777,216	16.0000	6.3496	.003906250	804.25	51,471.85
257	66,049	16,974,593	16.0312	6.3579	.003891051	807.39	51,874.76
258	66,564	17,173,512	16.0624	6.3661	.003875969	810.53	52,279.24
259	67,081	17,373,979	16.0935	6.3743	.003861004	813.67	52,685.29
260	67,600	17,576,000	16.1245	6.3825	.003846154	816.81	53,092.92
261	68,121	17,779,581	16.1555	6.3907	.003831418	819.96	53,502.11
262	68,644	17,984,728	16.1864	6.3988	.003816794	823.10	53,912.87
263	69,169	18.191,447	16.2173	6.4070	.003802281	826.24	54,325.21
264	69,696	18,399,744	16.2481	6.4151	.003787879	829.38	54,739.11
265	70,225	18,609,625	16.2788	6.4232	.003773585	832.52	55,154.59
266	70,756	18,821.096	16.3095	6.4312	.003759398	835.66	55,571.63
267	71,289	19,034,163	16.3401	6.4393	.003745318	838.81	55,990.25
268	71,824	19,248,832	16.3707	6.4473	.003731343	841.95	56,410.44
269	72,361	19,465,109	16.4012	6.4553	.003717472	845.09	56,832.20
270	72,900	19,683,000	16.4317	6.4633	.003703704	848.23	57,255.53
271	73,441	19,902,511	16.4621	6.4713	.003690037	851.37	57,680.43
272	73,984	20,123,648	16.4924	6.4792	.003676471	854.51	58,106.90
273	74,529	20,346,417	16.5227	6.4872	.003663004	857.65	58,534.94
274	75,076	20,570,824	16.5529	6.4951	.003649635	860.80	58,964.55
275	75,625	20,796,875	16.5831	6.5030	.003636364	863.94	59,395.74
276	76,176	21,024,576	16.6132	6.5108	.003623188	867.08	59,828.49
277	76,729	21,253,933	16.6433	6.5187	.003610108	870.22	60,262.82
278	77,284	21,484,952	16.6733	6.5265	.003597122	873.36	60,698.71
279	77,841	21,717,639	16 7033	6.5343	.003584229	876.50	61,136.18
280	78,400	21,952,000	16.7332	6.5421	.003571429	879.65	61,575.22
281	78,961	22,188,041	16.7631	6.5499	.003558719	882.79	62,015.82
282	79,524	22,425,768	16.7929	6.5577	.003546099	885.93	62,458.00
283	80,089	22,665,187	16.8226	6.5654	.003533569	889.07	62,901.75
284	80,656	22,906,304	16.8523	6.5731	.003521127	892.21	63,347.07
285	81,225	23,149,125	16.8819	6.5808	.003508772	895.35	63,793.97
286	81,796	23,393,656	16.9115	6.5885	.003496503	898.50	64,242.43
287	82.369	23,639,903	16.9411	6.5962	.003484321	901.64	64,692.46
288	82,944	23,887,872	16.9706	6.6039	.003472222	904.78	65,144.07
289	83,521	24,137,569	17.0000	6.6115	.003460208	907.92	65,597.24
290	84,100	24,389,000	17.0294	6.6191	.003448276	911.06	66,051.99
291	84,681	24,642,171	17.0587	6.6267	.003436426	914.20	66,508.30
292	85,264	24,897,088	17.0880	6.6343	.003424658	917.35	66,966.19
293	85,849	25,153,757	17.1172	6.6419	.003412969	920.49	67,425.65
294	86,436	25,412,184	17.1464	6.6494	.003401361	923.63	67,886.68
295	87,025	25,672,375	17.1756	6.6569	.003389831	926.77	68,349.28
296	87,616	25,934,336	17.2047	6.6644	.003378378	929.91	68,813.45
297	88,209	26,198,073	17.2337	6.6719	.003367003	933.05	69,279.19
298	88,804	26,463,592	17.2627	6.6794	.003355705	936.19	69,746.50
299	89,401	26,730,899	17.2916	6.6869	.003344482	939.34	70,215.38
300	90,000	27,000,000	17.3205	6.6943	.003333333	942.48	70,685.83
301	90,601	27,270,901	17.3494	6.7018	.003322259	945.62	71,157.86
302	91,204	27,543,608	17.3781	6.7092	.003311258	948.76	71,631.45
303	91,809	27,818,127	17.4069	6.7166	.003300330	951.90	72,106.62
304	92,416	28,094,464	17.4356	6.7240	.003289474	955.04	72,583.36
305	93,025	28,372,625	17.4642	6.7313	.003278689	958.19	73,061.66
306	93,636	28,652,616	17.4929	6.7387	.003267974	961.33	73,541.54
307	94,249	28,934,443	17.5214	6.7460	.003257329	964.47	74,022.99

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
308	94,864	29,218,112	17.5499	6.7533	.003246753	967.61	74,506.01
309	95,481	29,503,629	17.5784	6.7606	.003236246	970.75	74,990.60
310	96,100	29,791,000	17.6068	6.7679	.003225806	973.89	75,476.76
311	96,721	30,080,231	17.6352	6.7752	.003215434	977.04	75,964.50
312	97,344	30,371,328	17.6635	6.7824	.003205128	980.18	76,453.80
313	97,969	30,664,297	17.6918	6.7897	.003194888	983.32	76,944.67
314	98,596	30,959,144	17.7200	6.7969	.003184713	986.46	77,437.12
315	99,225	31,255,875	17.7482	6.8041	.003174603	989.60	77,931.13
316	99,856	31,554,496	17.7764	6.8113	.003164557	992.74	78,426.72
317	100,489	31,855,013	17.8045	6.8185	.003154574	995.88	78,923.88
318	101,124	32,157,432	17.8326	6.8256	.003144654	999.03	79,422.60
319	101,761	32,461,759	17.8606	6.8328	.003134796	1,002.17	79,922.90
320	102,400	32,768,000	17.8885	6.8399	.003125000	1,005.31	80,424.77
321	103,041	33,076,161	17.9165	6.8470	.003115265	1,008.45	80,928.21
322	103,684	33,386,248	17.9444	6.8541	.003105590	1,011.59	81,433.22
323	104,329	33,698,267	17.9722	6.8612	.003095975	1,014.73	81,939.80
324	104,976	34,012,224	18.0000	6.8683	.003086420	1,017.88	82,447.96
325	105,625	34,328,125	18.0278	6.8753	.003076923	1,021.02	82,957.68
326	106,276	34,645,976	18.0555	6.8824	.003067485	1,024.16	83,468.98
327	106,929	34,965,783	18.0831	6.8894	.003058104	1,027.30	83,981.84
328	107,584	35,287,552	18.1108	6.8964	.003048780	1,030.44	84,496.28
329	108,241	35,611,289	18.1384	6.9034	.003039514	1,033.58	85,012.28
330	108,900	35,937,000	18.1659	6.9104	.003030303	1,036.73	85,529.86
331	109,561	36,264,691	18.1934	6.9174	.003021148	1,039.87	86,049.01
332	110,224	36,594,368	18.2209	6.9244	.003012048	1,043.01	86,569.73
333	110,889	36,926,037	18.2483	6.9313	.003003003	1,046.15	87,092.02
334	111,556	37,259,704	18.2757	6.9382	.002994012	1,049.29	87,615.88
335	112,225	37,595,375	18.3030	6.9451	.002985075	1,052.43	88,141.31
336	112,896	37,933,056	18.3303	6.9521	.002976190	1,055.58	88,668.31
337	113,569	38,272,753	18.3576	6.9589	.002967359	1,058.72	89,196.88
338	114,244	38,614,472	18.3848	6.9658	.002958580	1,061.86	89,727.03
339	114,921	38,958,219	18.4120	6.9727	.002949853	1,065.00	90,258.74
340	115,600	39,304,000	18.4391	6 9795	.002941176	1,068.14	90,792.03
341	116,281	39,651,821	18.4662	6.9864	.002932551	1,071.28	91,326.88
342	116,964	40,001,688	18.4932	6.9932	.002923977	1,074.42	91,863.31
343	117,649	40,353,607	18.5203	7.0000	.002915452	1,077.57	92,401.31
344	118,336	40,707,584	18.5472	7.0068	.002906977	1,080.71	92,940.88
345	119,025	41,063,625	18.5742	7.0136	.002898551	1,083.85	93,482.02
346	119,716	41,421,736	18.6011	7.0203	.002890173	1,086.99	94,024.73
347	120,409	41 781,923	18.6279	7.0271	.002881844	1,090.13	94,569.01
348	121,104	42,144,192	18.6548	7.0338	.002873563	1,093.27	95,114.86
349	121,801	42,508,549	18.6815	7.0406	.002865330	1,096.42	95,662.28
350	122,500	42,875,000	18.7083	7.0473	.002857143	1,099.56	96,211.28
351	123,201	43,243,551	18.7350	7.0540	.002849003	1,102.70	96,761.84
352	123,904	43,614,208	18.7617	7.0607	.002840909	1,105.84	97,313.97
353	124,609	43,986,977	18.7883	7.0674	.002832861	1,108.98	97,867.68
354	125,316	44,361,864	18.8149	7.0740	.002824859	1,112.12	98,422.96
355	126,025	44,738,875	18.8414	7.0807	.002816901	1,115.27	98,979.80
356	126,736	45,118,016	18.8680	7.0873	.002808989	1,118.41	99,538.22
357	127,449	45,499,293	18.8944	7.0940	.002801120	1,121.55	100,098.21
358	128,164	45,882,712	18.9209	7.1006	.002793296	1,124.69	100,659.77
359	128,881	46,268,279	18.9473	7.1072	.002785515	1,127.83	101,222.90
360	129,600	46,656,000	18.9737	7.1138	.002777778	1,130.97	101,787.60
361	130,321	47,045,881	19.0000	7.1204	.002770083	1,134.11	102,353.87
362	131,044	47,437,928	19.0263	7.1269	.002762431	1,137.26	102,921:72
363	131,769	47,832,147	19.0526	7.1335	.002754821	1,140.40	103,491.13
364	132,496	48,228,544	19.0788	7.1400	.002747253	1,143.54	104,062.12
365	133,225	48,627,125	19.1050	7.1466	.002739726	1,146.68	104,634.67
366	133,956	49,027,896	19.1311	7.1531	.002732240	1,149.82	105,208.80
367	134,689	49,430,863	19.1572	7.1596	.002724796	1,152.96	105,784.49
368	135,424	49,836,032	19.1833	7.1661	.002717391	1,156.11	106,361.76
369	136,161	50,243,409	19.2094	7.1726	.002710027	1,159.25	106,940.60
370	136,900	50,653,000	19.2354	7.1791	.002702703	1,162.39	107,521.01

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
371	137,641	51,064,811	19.2614	7:1855	.002695418	1,165.53	108,102.99
372	138,384	51,478,848	19.2873	7.1920	.002688172	1,168.67	108,686.54
373	139,129	51,895,117	19.3132	7.1984	.002680965	1,171.81	109,271.66
374	139,876	52,313,624	19.3391	7.2048	.002673797	1,174.96	109,858.35
375	140,625	52,734,375	19.3649	7.2112	.002666667	1,178.10	110,446.62
376	141,376	53,157.376	19.3907	7.2177	.002659574	1,181.24	111,036.45
377	142,129	53,582,633	19.4165	7.2240	.002652520	1,184.38	111,627.86
378	142,884	54,010,152	19.4422	7.2304	.002645503	1,187.52	112,220.83
379	143,641	54,439,939	19.4679	7.2368	.002638522	1,190.66	112,815.38
380	144,400	54,872,000	19.4936	7.2432	.002631579	1,193.81	113,411.49
381	145,161	55,306,341	19.5192	7.2495	.002624672	1,196.95	114,009.18
382	145,924	55,742,968	19.5448	7.2558	.002617801	1,200.09	114,608.44
383	146,689	56,181,887	19.5704	7.2622	.002610966	1,203.23	115,209.27
384	147,456	56,623,104	19.5959	7.2685	.002604167	1,206.37	115,811.67
385	148,225	57,066,625	19.6214	7.2748	.002597403	1,209.51	116,415.64
386	148,996	57,512,456	19.6469	7.2811	.002590674	1,212.65	117,021.18
387	149,769	57,960,603	19.6723	7.2874	.002583979	1,215.80	117,628.30
388	150,544	58,411,072	19.6977	7.2936	.002577320	1,218.94	118,236.98
389	151,321	58,863,869	19.7231	7.2999	.002570694	1,222.08	118,847.24
390	152,100	59,319,000	19.7484	7.3061	.002564103	1,225.22	119,459.06
391	152,881	59,776,471	19.7737	7.3124	.002557545	1,228.36	120,072.46
392	153,664	60,236,288	19.7990	7.3186	.002551020	1,231.50	120,687.42
393	154,449	60,698,457	19.8242	7.3248	.002544529	1,234.65	121,303.96
394	155,236	61,162,984	19.8494	7.3310	.002538071	1,237.79	121,922.07
395	156,025	61,629,875	19.8746	7.3372	.002531646	1,240.93	122,541.75
396	156,816	62,099,136	19.8997	7.3434	.002525253	1,244.07	123,163.00
397	157,609	62,570,773	19.9249	7.3496	.002518892	1,247.21	123,785.82
398	158,404	63,044,792	19.9499	7.3558	.002512563	1,250.35	124,410.21
399	159,201	63,521,199	19.9750	7.3619	.002506266	1,253.50	125,036.17
400	160,000	64,000,000	20.0000	7.3681	.002500000	1,256.64	125,663.71
401	160,801	64,481,201	20.0250	7.3742	.002493766	1,259.78	126,292.81
402	161,604	64,964,808	20.0499	7.3803	.002487562	1,262.92	126,923.48
403	162,409	65,450,827	20.0749	7.3864	.002481390	1,266.06	127,555.73
404	163,216	65,939,264	20.0998	7.3925	.002475248	1,269.20	128,189.55
405	164,025	66,430,125	20.1246	7.3986	.002469136	1.272.35	128,824.93
406	164,836	66,923,416	20.1494	7.4047	.002463054	1,275.49	129,461.89
407	165,649	67,419,143	20.1742	7.4108	.002457002	1,278.63	130,100.42
408	166,464	67,917,312	20.1990	7.4169	.002450980	1,281.77	130,740.52
409	167,281	68,417,929	20.2237	7.4229	.002444988	1,284.91	131,382.19
410	168,100	68,921,000	20.2485	7.4290	.002439024	1,288.05	132,025.43
411	168,921	69,426,531	20.2731	7.4350	.002433090	1,291.19	132,670.24
412	169,744	69,934,528	20.2978	7.4410	.002427184	1,294.34	133,316.63
413	170,569	70,444,997	20.3224	7.4470	.002421308	1,297.48	133,964.58
414	171,396	70,957,944	20.3470	7.4530	.002415459	1,300.62	134,614.10
415	172,225	71,473,375	20.3715	7.4590	.002409639	1,303.76	135,265.20
416	173,056	71,991,296	20.3961	7.4650	.002403846	1,306.90	135,917.86
417	173,889	72,511,713	20.4206	7.4710	.002398082	1,310.04	136,572.10
418	174,724	73,034,632	20.4450	7.4770	.002392344	1,313.19	137,227.91
419	175,561	73,560,059	20.4695	7.4829	.002386635	1,316.33	137,885.29
420	176,400	74,088,000	20.4939	7.4889	.002380952	1,319.47	138,544.24
421	177,241	74,618,461	20.5183	7.4948	.002375297	1,322.61	139,204.76
422	178,084	75,151,448	20.5426	7.5007	.002369668	1,325.75	139,866.85
423	178,929	75,686,967	20.5670	7.5067	.002364066	1,328.89	140,530.51
424	179,776	76,225,024	20.5913	7.5126	.002358491	1,332.04	141,195.74
425	180,625	76,765,625	20.6155	7.5185	.002352941	1,335.18	141,862.54
426	181,476	77,308,776	20.6398	7.5244	.002347418	1,338.32	142,530.92
427	182,329	77,854,483	20.6640	7.5302	.002341920	1,341.46	143,200.86
428	183,184	78,402,752	20.6882	7.5361	.002336449	1,344.60	143,872.38
429	184,041	78,953,589	20.7123	7.5420	.002331002	1,347.74	144,545.46
430	184.900	79,507,000	20.7364	7.5478	.002325581	1,350.88	145,220.12
431	185,761	80,062,991	20.7605	7.5537	.002320186	1,354.03	145,896.35
432	186,624	80,621,568	20.7846	7.5595	.002314815	1,357.17	146,574.15
433	187,489	81,182,737	20.8087	7.5654	.002309469	1,360.31	147,253.52

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
434	188,356	81,746,504	20.8327	7.5712	.002304147	1,363.45	147,934.46
435	189,225	82,312,875	20.8567	7.5770	.002298851	1,366.59	148,616.97
436	190,096	82,881,856	20.8806	7.5828	.002293578	1,369.73	149,301.05
437	190,969	83,453,453	20.9045	7.5886	.002288330	1,372.88	149,986.70
438	191,844	84,027,672	20.9284	7.5944	.002283105	1,376.02	150,673.93
439	192,721	84,604,519	20.9523	7.6001	.002277904	1,379.16	151,362.72
440	193,600	85,184,000	20.9762	7.6059	.002272727	1,382.30	152,053.08
441	194,481	85,766,121	21.0000	7.6117	.002267574	1,385.44	152,745.02
442	195,364	86,350,888	21.0238	7.6174	.002262443	1,388.58	153,438.53
443	196,249	86,938,307	21.0476	7.6232	.002257336	1,391.73	154,133.60
444	197,136	87,528,384	21.0713	7.6289	.002252252	1,394.87	154,830.25
445	198,025	88,121,125	21.0950	7.6346	.002247191	1,398.01	155,528.47
446	198,916	88,716,536	21.1187	7.6403	.002242152	1,401.15	156,228.26
447	199,809	89,314,623	21.1424	7.6460	.002237136	1,404.29	156,929.62
448	200,704	89,915,392	21.1660	7.6517	.002232143	1,407.43	157,632.55
449	201,601	90,518,849	21.1896	7.6574	.002227171	1,410.58	158,337.06
450	202,500	91,125,000	21.2132	7.6631	.002222222	1,413.72	159,043.13
451	203,401	91,733,851	21.2368	7.6688	.002217295	1,416.86	159,750.77
452	204,304	92,345,408	21.2603	7.6744	.002212389	1,420.00	160,459.99
453	205,209	92,959,677	21.2838	7.6801	.002207506	1,423.14	161,170.77
454	206,116	93,576,664	21.3073	7.6857	.002202643	1,426.28	161,883.13
455	207,025	94,196,375	21.3307	7.6914	.002197802	1,429.42	162,597.05
456	207,936	94,818,816	21.3542	7.6970	.002192982	1,432.57	163,312.55
457	208,849	95,443,993	21.3776	7.7026	.002188184	1,435.71	164,029.62
458	209,764	96,071,912	21.4009	7.7082	.002183406	1,438.85	164,748.26
459	210,681	96,702,579	21.4243	7.7138	.002178649	1,441.99	165,468.47
460	211,600	97,336,000	21.4476	7.7194	.002173913	1,445.13	166,190.25
461	212,521	97,972,181	21.4709	7.7250	.002169197	1,448.27	166,913.60
462	213,444	98,611,128	21.4942	7.7306	.002164502	1,451.42	167,638.53
463	214,369	99,252,847	21.5174	7.7362	.002159827	1,454.56	168,365.02
464	215,296	99,897,344	21.5407	7.7418	.002155172	1,457.70	169,093.08
465	216,225	100,544,625	21.5639	7.7473	.002150538	1,460.84	169,822.72
466	217,156	101,194,696	21.5870	7.7529	.002145923	1,463.98	170,553.92
467	218,089	101,847,563	21.6102	7.7584	.002141328	1,467.12	171,286.70
468	219,024	102,503,232	21.6333	7.7639	.002136752	1,470.27	172,021.05
469	219,961	103,161,709	21.6564	7.7695	.002132196	1,473.41	172,756.97
470	220,900	103,823,000	21.6795	7.7750	.002127660	1,476.55	173,494.45
471	221,841	104,487,111	21.7025	7.7805	.002123142	1,479.69	174,233.51
472	222,784	105,154,048	21.7256	7.7860	.002118644	1,482.83	174,974.14
473	223,729	105,823,817	21.7486	7.7915	.002114165	1,485.97	175,716.35
474	224,676	106,496,424	21.7715	7.7970	.002109705	1,489.11	176,460.12
475	225,625	107,171,875	21.7945	7.8025	.002105263	1,492.26	177,205.46
476	226,576	107,850,176	21.8174	7.8079	.002100840	1,495.40	177,952.37
477	227,529	108,531,333	21.8403	7.8134	.002096436	1,498.54	178,700.86
478	228,484	109,215,352	21.8632	7.8188	.002092050	1,501.68	179,450.91
479	229,441	109,902,239	21.8861	7.8243	.002087683	1,504.82	180,202.54
480	230,400	110,592,000	21.9089	7.8297	.002083333	1,507.96	180,955.74
481	231,361	111,284,641	21.9317	7.8352	.002079002	1,511.11	181,710.50
482	232,324	111,980,168	21.9545	7.8406	.002074689	1,514.25	182,466.84
483	233,289	112,678,587	21.9773	7.8460	.002070393	1.517.39	183,224.75
484	234,256	113,379,904	22.0000	7.8514	.002066116	1,520.53	183,984.23
485	235,225	114,084,125	22.0227	7.8568	.002061856	1,523.67	184,745.28
486	236,196	114,791,256	22.0454	7.8622	.002057613	1,526.81	185,507.90
487	237,169	115,501,303	22.0681	7.8676	.002053388	1,529.96	186,272.10
488	238,144	116,214,272	22.0907	7.8730	.002049180	1,533.10	187,037.86
489	239,121	116,930,169	22.1133	7.8784	.002044990	1,536.24	187,805.19
490	240,100	117,649,000	22.1359	7.8837	.002040816	1,539.38	188,574.10
491	241,081	118,370,771	22.1585	7.8891	.002036660	1,542.52	189,344.57
492	242,064	119,095,488	22.1811	7.8944	.002032520	1,545.66	190,116.62
493	243,049	119,823,157	22.2036	7.8998	.002028398	1,548.81	190,899.24
494	244,036	120,553,784	22.2261	7.9051	.002024291	1,551.95	191,665.43
495	245,025	121,287,375	22.2486	7.9105	.002020202	1,555.09	192,442.18
496	246,016	122,023,936	22.2711	7.9158	.002016129	1,558.23	193,220.51

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
497	247,009	122,763,473	22.2935	7.9211	.002012072	1,561.37	194,000.41
498	248,004	123,505,992	22.3159	7.9264	.002008032	1,564.51	194,781.89
499	249,001	124,251,499	22.3383	7.9317	.002004008	1,567.65	195,564.93
500	250,000	125,000,000	22.3607	7.9370	.002000000	1,570.80	196,349.54
501	251,001	125,751,501	22.3830	7.9423	.001996008	1,573.94	197,135.72
502	252,004	126,506,008	22.4054	7.9476	.001992032	1,577.08	197,923.48
503	253,009	127,263,527	22.4277	7.9528	.001988072	1,580.22	198,712.80
504	254.016	128,024,064	22.4499	7.9581	.001984127	1,583.36	199,503.70
505	255,025	128,787,625	22.4722	7.9634	.001980198	1,586.50	200,296.17
506	256,036	129,554,216	22.4944	7.9686	.001976285	1,589.65	201,090.20
507	257,049	130,323,843	22.5167	7.9739	.001972387	1,592.79	201,885.81
508	258,064	131,096,512	22.5389	7.9791	.001968504	1,595.93	202,682.99
509	259,081	131,872.229	22.5610	7.9843	.001964637	1,599.07	203,481.74
510	260,100	132,651,000	22.5832	7.9896	.001960784	1,602.21	204,282.06
511	261,121	133,432,831	22.6053	7.9948	.001956947	1,605.35	205,083.95
512	262,144	134,217,728	22.6274	8.0000	.001953125	1,608.50	205,887.42
513	263,169	135,005,697	22.6495	8.0052	.001949318	1,611.64	206,692.45
514	264.196	135,796,744	22.6716	8.0104	.001945525	1,614.78	207,499.05
515	265,225	136,590,875	22.6936	8.0156	.001941748	1,617.92	208,307.23
516	266,256	137,388,096	22.7156	8.0208	.001937984	1,621.06	209,116.97
517	267,289	138,188,413	22.7376	8.0260	.001934236	1,624.20	209,928.29
518	268,324	138,991,832	22.7596	8.0311	.001930502	1,627.34	210,741.18
519	269,361	139,798,359	22.7816	8.0363	.001926782	1,630.49	211,555.63
520	270,400	140,608,000	22.8035	8.0415	.001923077	1,633.63	212,371.66
521	271,441	141,420,761	22.8254	8.0466	.001919386	1,636.77	213,189.26
522	272,484	142,236,648	22.8473	8.0517	.001915709	1,639.91	214,008.43
523	273,529	143,055,667	22.8692	8.0569	.001912046	1,643.05	214,829.17
524	274,576	143,877,824	22.8910	8.0620	.001908397	1,646.19	215,651.49
525	275,625	144,703,125	22.9129	8.0671	.001904762	1,649.34	216,475.37
526	276,676	145,531,576	22.9347	8.0723	.001901141	1,652.48	217,300.82
527	277,729	146,363.183	22.9565	8.0774	.001897533	1,655.62	218,127.85
528	278.784	147,197.952	22.9783	8.0825	.001893939	1,658.76	218,956.44
529	279,841	148,035,889	23.0000	8.0876	.001890359	1,661.90	219,786.61
530	280,900	148,877,000	23.0217	8.0927	.001886792	1,665.04	220,618.34
531	281,961	149,721,291	23.0434	8.0978	.001883239	1,668.19	221,451.65
532	283,024	150,568,768	23.0651	8.1028	.001879699	1,671.33	222,286.53
533	284,089	151,419,437	23.0868	8.1079	.001876173	1,674.47	223,122.98
534	285.156	152,273,304	23.1084	8.1130	.001872659	1,677.61	223,961.00
535	286,225	153,130,375	23.1301	8.1180	.001869159	1,680.75	224,800.59
536	287,296	153,990,656	23.1517	8.1231	.001865672	1,683.89	225,641.75
537	288,369	154,854,153	23.1733	8.1281	.001862197	1,687.04	226,484.48
538	289.444	155,720,872	23.1948	8.1332	.001858736	1,690.18	227,328.79
539	290,521	156,590,819	23.2164	8.1382	.001855288	1,693.32	228,174.66
540	291,600	157,464,000	23.2379	8.1433	.001851852	1,696.46	229,022.10
541	292,681	158,340,421	23.2594	8.1483	.001848429	1,699.60	229,871.12
542	293,764	159,220,088	23.2809	8.1533	.001845018	1,702.74	230,721.71
543	294,849	160,103,007	23.3024	8.1583	.001841621	1,705.88	231,573.86
544	295,936	160,989,184	23.3238	8.1633	.001838235	1,709.03	232,427.59
545	297,025	161,878,625	23.3452	8.1683	.001834862	1,712.17	233,282.89
546	298,116	162,771,336	23.3666	8.1733	.001831502	1,715.31	234,139.76
547	299,209	163,667,323	23.3880	8.1783	.001828154	1,718.45	234,998.20
548	300,304	164,566,592	23.4094	8.1833	.001824818	1,721.59	235,858.21
549	301,401	165,469,149	23.4307	8.1882	.001821494	1,724.73	236,719.79
550	302,500	166,375,000	23.4521	8.1932	.001818182	1,727.88	237,582.94
551	303,601	167,284,151	23.4734	8.1982	.001814882	1,731.02	238,447.67
552	304,704	168,196,608	23.4947	8.2031	.001811594	1,734.16	239,313.96
553	305,809	169,112,377	23.5160	8.2081	.001808318	1,737.30	240,181.83
554	306,916	170,031,464	23.5372	8.2130	.001805054	1,740.44	241,051.26
555	308.025	170,953,875	23.5584	8.2180	.001801802	1,743.58	241,922.27
556	309,136	171,879,616	23.5797	8.2229	.001798561	1,746.73	242,794.85
557	310,249	172,808,693	23.6008	8.2278	.001795332	1,749.87	243,668.99
558	311,364	173,741.112	23.6220	8.2327	.001792115	1,753.01	244,544.71
559	312,481	174,676,879	23.6432	8.2377	.001788909	1,756.15	245,422.00

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
560	313,600	175,616,000	23.6643	8.2426	.001785714	1,759.29	246,300.86
561	314,721	176,558,481	23.6854	8.2475	.001782531	1,762.43	247,181.30
562	315,844	177,504,328	23.7065	8.2524	.001779359	1,765.58	248,063.30
563	316,969	178,453,547	23.7276	8.2573	.001776199	1,768.72	248,946.87
564	318,096	179,406,144	23.7487	8.2621	.001773050	1,771.86	249,832.01
565	319,225	180.362,125	23.7697	8.2670	.001769912	1,775.00	250,718.73
566	320,356	181,321,496	23.7908	8.2719	.001766784	1,778.14	251,607.01
567	321,489	182,284,263	23.8118	8.2768	.001763668	1,781.28	252,496.87
568	322,624	183,250,432	23.8328	8.2816	.001760563	1,784.42	253,388.30
569	323,761	184,220,009	23.8537	8.2865	.001757469	1,787.57	254,281.29
570	324,900	185,193,000	23.8747	8.2913	.001754386	1,790.71	255,175.86
571	326,041	186,169,411	23.8956	8.2962	.001751313	1,793.85	256,072.00
572	327,184	187,149,248	23.9165	8.3010	.001748252	1,796.99	256,969.71
573	328,329	188,132,517	23.9374	8.3059	.001745201	1,800.13	257,868.99
574	329,476	189,119,224	23.9583	8.3107	.001742160	1,803.27	258,769.85
575	330,625	190,109,375	23.9792	8.3155	.001739130	1,806.42	259,672.27
576	331,776	191,102,976	24.0000	8.3203	.001736111	1,809.56	260,576.26
577	332,929	192,100,033	24.0208	8.3251	.001733102	1,812.70	261,481.83
578	334,084	193,100,552	24.0416	8.3300	.001730104	1,815.84	262,388.96
579	335,241	194,104,539	24.0624	8.3348	.001727116	1,818.98	263,297.67
580	336,400	195,112,000	24.0832	8.3396	.001724138	1,822.12	264,207.94
581	337,561	196,122,941	24.1039	8.3443	.001721170	1,825.27	265,119.79
582	338,724	197,137,368	24.1247	8.3491	.001718213	1,828.41	266,033.21
583	339,889	198,155,287	24.1454	8.3539	.001715266	1,831.55	266,948.20
584	341,056	199,176,704	24.1661	8.3587	.001712329	1,834.69	267,864.76
585	342,225	200,201,625	24.1868	8.3634	.001709402	1,837.83	268,782.89
586	343,396	201,230,056	24.2074	8.3682	.001706485	1,840.97	269,702.59
587	344,569	202,262,003	24.2281	8.3730	.001703578	1,844.11	270,623.86
588	345,744	203,297,472	24.2487	8.3777	.001700680	1,847.26	271,546.70
589	346,921	204,336,469	24.2693	8.3825	.001697793	1,850.40	272,471.12
590	348,100	205,379,000	24.2899	8.3872	.001694915	1,853.54	273,397.10
591	349,281	206,425,071	24.3105	8.3919	.001692047	1,856.68	274,324.66
592	350,464	207,474,688	24.3311	8.3967	.001689189	1,859.82	275,253.78
593	351,649	208,527,857	24.3516	8.4014	.001686341	1,862.96	276,184.48
594	352,836	209,584,584	24.3721	8.4061	.001683502	1,866.11	277,116.75
595	354,025	210,644,875	24.3926	8.4108	.001680672	1,869.25	278,050.58
596	355,216	211,708,736	24.4131	8.4155	.001677852	1,872.39	278,985.99
597	356,409	212,776,173	24.4336	8.4202	.001675042	1,875.53	279.922.97
598	357,604	213,847,192	24.4540	8.4249	.001672241	1,878.67	280,861.52
599	358,801	214,921,799	24.4745	8.4296	.001669449	1,881.81	281,801.65
600	360,000	216,000,000	24.4949	8.4343	.001666667	1,884.96	282,743.34
601	361,201	217,081,801	24.5153	8.4390	.001663894	1,888.10	283,686.60
602	362,404	218,167,208	24.5357	8.4437	.001661130	1,891.24	284,631.44
603	363.609	219,256,227	24.5561	8.4484	.001658375	1,894.38	285,577.84
604	364,816	220,348,864	24.5764	8.4530	.001655629	1,897.52	286,525.82
605	366,025	221,445,125	24.5968	8.4577	.001652893	1,900.66	287,475.36
606	367,236	222,545,016	24.6171	8.4623	.001650165	1,903.81	288,426.48
607	368,449	223,648,543	24.6374	8.4670	.001647446	1,906.95	289,379.17
608	369.664	224,755,712	24.6577	8.4716	.001644737	1,910.09	290,333.43
609	370,881	225,866,529	24.6779	8.4763	.001642036	1,913.23	291,289.26
610	372.100	226,981,000	24.6982	8.4809	.001639344	1,916.37	292,246.66
611	373,321	228,099,131	24.7184	8.4856	.001636661	1,919.51	293,205.63
612	374.544	229,220,928	24.7386	8.4902	.001633987	1,922.65	294,166.17
613	375,769	230,346,397	24.7588	8.4948	.001631321	1,925.80	295,128.28
614	376,996	231,475,544	24.7790	8.4994	.001628664	1,928.94	296,091.97
615	378,225	232,608,375	24.7992	8.5040	.001626016	1,932.08	297,057.22
616	379.456	233,744,896	24.8193	8.5086	.001623377	1,935.22	298,024.05
617	380.689	234,885,113	24.8395	8.5132	.001620746	1,938.36	298,992.44
618	381,924	236,029,032	24.8596	8.5178	.001618123	1,941.50	299,962.41
619	383,161	237,176,659	24.8797	8.5224	.001615509	1,944.65	300,933.95
620	384.400	238,328,000	24.8998	8.5270	.001612903	1,947.79	301,907.05
621	385.641	239,483,061	24.9199	8.5316	.001610306	1,950.93	302,881.73
622	386 884	240,641,848	24.9399	8.5362	.001607717	1,954.07	303,857.98

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
623	388,129	241,804,367	24.9600	8.5408	.001605136	1,957.21	304,835.80
624	389,376	242,970,624	24.9800	8.5453	.001602564	1,960.35	305,815.20
625	390,625	244,140,625	25.0000	8.5499	.001600000	1,963.50	306,796.16
626	391,876	245,314,376	25.0200	8.5544	.001597444	1,966.64	307,778.69
627	393,129	246,491,883	25.0400	8.5590	.001594896	1,969.78	308,762.79
628	394,384	247,673,152	25.0599	8.5635	.001592357	1,972.92	309,748.47
629	395,641	248,858,189	25.0799	8.5681	.001589825	1,976.06	310,735.71
630	396,900	250,047,000	25.0998	8.5726	.001587302	1,979.20	311,724.53
631	398,161	251,239,591	25.1197	8.5772	.001584786	1,982.35	312,714.92
632	399,424	252,435,968	25.1396	8.5817	.001582278	1,985.49	313,706.88
633	400,689	253,636,137	25.1595	8.5862	.001579779	1,988.63	314,700.40
634	401,956	254,840,104	25.1794	8.5907	.001577287	1,991.77	315,695.50
635	403,225	256,047,875	25.1992	8.5952	.001574803	1,994.91	316,692.17
636	404,496	257,259,456	25.2190	8.5997	.001572327	1,998.05	317,690.42
637	405,769	258,474,853	25.2389	8.6043	.001569859	2,001.19	318,690.23
638	407,044	259,694,072	25.2587	8.6088	.001567398	2,004.34	319,691.61
639	408,321	260,917,119	25.2784	8.6132	.001564945	2,007.48	320,694.56
640	409,600	262,144,000	25.2982	8.6177	.001562500	2,010.62	321,699.09
641	410,881	263,374,721	25.3180	8.6222	.001560062	2,013.76	322,705.18
642	412,164	264,609,283	25.3377	8.6267	.001557632	2,016.90	323,712.85
643	413,449	265,847,707	25.3574	8.6312	.001555210	2,020.04	324,722.09
644	414,736	267,089,984	25.3772	8.6357	.001552795	2,023.19	325,732.89
645	416,025	268,336,125	25.3969	8.6401	.001550388	2,026.33	326,745.27
646	417,316	269,586,136	25.4165	8.6446	.001547988	2,029.47	327,759.22
647	418,609	270,840,023	25.4362	8.6490	.001545595	2,032.61	328,774.74
648	419,904	272,097,792	25.4558	8.6535	.001543210	2,035.75	329,791.83
649	421,201	273,359,449	25.4755	8.6579	.001540832	2,038.89	330,810.49
650	422,500	274,625,000	25.4951	8.6624	.001538462	2,042.04	331,830.72
651	423,801	275,894,451	25.5147	8.6668	.001536098	2,045.18	332,852.53
652	425,104	277,167,808	25.5343	8.6713	.001533742	2,048.32	333,875.90
653	426,409	278,445,077	25.5539	8.6757	.001531394	2,051.46	334,900.85
654	427,716	279,726,264	25.5734	8.6801	.001529052	2,054.60	335,927.36
655	429,025	281,011,375	25.5930	8.6845	.001526718	2,057.74	336,955.45
656	430,336	282,300,416	25.6125	8.6890	.001524390	2,060.88	337,985.10
657	431,649	283,593,393	25.6320	8.6934	.001522070	2,064.03	339,016.33
658	432,964	284,890,312	25.6515	8.6978	.001519757	2,067.17	340,049.13
659	434,281	286,191,179	25.6710	8.7022	.001517451	2,070.31	341,083.50
660	435,600	287,496,000	25.6905	8.7066	.001515152	2,073.45	342,119.44
661	436,921	288,804,781	25.7099	8.7110	.001512859	2,076.59	343,156.95
662	438,244	290,117,528	25.7294	8.7154	.001510574	2,079.73	344,196.03
663	439,569	291,434,247	25.7488	8.7198	.001508296	2,082.88	345,236.69
664	440,896	292,754,944	25.7682	8.7241	.001506024	2,086.02	346,278.91
665	442,225	294,079,625	25.7876	8.7285	.001503759	2,089.16	347,322.70
666	443,556	295,408,296	25.8070	8.7329	.001501502	2,092.30	348,368.07
667	444,889	296,740,963	25.8263	8.7373	.001499250	2,095.44	349,415.00
668	446,224	298,077,632	25.8457	8.7416	.001497006	2,098.58	350,463.51
669	447,561	299,418,309	25.8650	8.7460	.001494768	2,101.73	351,513.59
670	448,900	300,763,000	25.8844	8.7503	.001492537	2,104.87	352,565.24
671	450,241	302,111,711	25.9037	8.7547	.001490313	2,108.01	353,618.45
672	451,584	303,464,448	25.9230	8.7590	.001488095	2,111.15	354,673.24
673	452,929	304,821,217	25.9422	8.7634	.001485884	2,114.29	355,729.60
674	454,276	306,182,024	25.9615	8.7677	.001483680	2,117.43	356,787.54
675	455,625	307,546,875	25.9808	8.7721	.001481481	2,120.58	357,847.04
676	456,976	308,915,776	26.0000	8.7764	.001479290	2,123.72	358,908.11
677	458,329	310,288,733	26.0192	8.7807	.001477105	2,126.86	359,970.75
678	459,684	311,665,752	26.0384	8.7850	.001474926	2,130.00	361,034.97
679	461,041	313,046,839	26.0576	8.7893	.001472754	2,133.14	362,100.75
680	462,400	314,432,000	26.0768	8.7937	.001470588	2,136.28	363,168.11
681	463,761	315,821,241	26.0960	8.7980	.001468429	2,139.42	364,237.04
682	465,124	317,214,568	26.1151	8.8023	.001466276	2,142.57	365,307.54
683	466,489	318,611,987	26.1343	8.8066	.001464129	2,145.71	366,379.60
684	467,856	320,013,504	26.1534	8.8109	.001461988	2,148.85	367,453.24
685	469,225	321,419,125	26.1725	8.8152	.001459854	2,151.99	368,528.45

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
686	470,596	322,828,856	26.1916	8.8194	.001457726	2,155.13	369,605.23
687	471,969	324,242,703	26.2107	8.8237	.001455604	2,158.27	370,683.59
688	473,344	325,660,672	26.2298	8.8280	.001453488	2,161.42	371,763.51
689	474,721	327,082,769	26.2488	8.8323	.001451379	2,164.56	372,845.00
690	476,100	328,509,000	26.2679	8.8366	.001449275	2,167.70	373,928.07
691	477,481	329,939,371	26.2869	8.8408	.001447178	2,170.84	375,012.70
692	478,864	331,373,888	26.3059	8.8451	.001445087	2,173.98	376,098.91
693	480,249	332,812,557	26.3249	8.8493	.001443001	2,177.12	377,186.68
694	481,636	334,255,384	26.3439	8.8536	.001440922	2,180.27	378,276.03
695	483,025	335,702,375	26.3629	8.8578	.001438849	2,183.41	379,366.95
696	484,416	337,153,536	26.3818	8.8621	.001436782	2,186.55	380,459.44
697	485,809	338,608,873	26.4008	8.8663	.001434720	2,189.69	381,553.50
698	487,204	340,068,392	26.4197	8.8706	.001432665	2,192.83	382,649.13
699	488,601	341,532,099	26.4386	8.8748	.001430615	2,195.97	383,746.33
700	490,000	343,000,000	26.4575	8.8790	.001428571	2,199.11	384,845.10
701	491,401	344,472,101	26.4764	8.8833	.001426534	2,202.26	385,945.44
702	492,804	345,948,408	26.4953	8.8875	.001424501	2,205.40	387,047.36
703	494,209	347,428,927	26.5141	8.8917	.001422475	2,208.54	388,150.84
704	495,616	348,913,664	26.5330	8.8959	.001420455	2,211.68	389,255.90
705	497,025	350,402,625	26.5518	8.9001	.001418440	2,214.82	390,362.52
706	498,436	351,895,816	26.5707	8.9043	.001416431	2,217.96	391,470.72
707	499,849	353,393,243	26.5895	8.9085	.001414427	2,221.11	392,580.49
708	501,264	354,894,912	26.6083	8.9127	.001412429	2,224.25	393,691.82
709	502,681	356,400,829	26.6271	8.9169	.001410437	2,227.39	394,804.73
710	504,100	357,911,000	26.6458	8.9211	.001408451	2,230.53	395,919.21
711	505,521	359,425,431	26.6646	8.9253	.001406470	2,233.67	397,035.26
712	506,944	360,944,128	26.6833	8.9295	.001404494	2,236.81	398,152.89
713	508,369	362,467,097	26.7021	8.9337	.001402525	2,239.96	399,272.08
714	509,796	363,994,344	26.7208	8.9378	.001400560	2,243.10	400,392.84
715	511,225	365,525,875	26.7395	8.9420	.001398601	2,246.24	401,515.18
716	512,656	367,061,696	26.7582	8.9462	.001396648	2,249.38	402,639.08
717	514,089	368,601,813	26.7769	8.9503	.001394700	2,252.52	403,764.56
718	515,524	370,146,232	26.7955	8.9545	.001392758	2,255.66	404,891.60
719	516,961	371,694,959	26.8142	8.9587	.001390821	2,258.81	406,020.22
720	518,400	373,248,000	26.8328	8.9628	.001388889	2,261.95	407,150.41
721	519,841	374,805,361	26.8514	8.9670	.001386963	2,265.09	408,282.17
722	521,284	376,367,048	26.8701	8.9711	.001385042	2,268.23	409,415.50
723	522,729	377,933,067	26.8887	8.9752	.001383126	2,271.37	410,550.40
724	524,176	379,503,424	26.9072	8.9794	.001381215	2,274.51	411,686.87
725	525,625	381,078,125	26.9258	8.9835	.001379310	2,277.65	412,824.91
726	527,076	382,657,176	26.9444	8.9876	.001377410	2,280.80	413,964.52
727	528,529	384,240,583	26.9629	8.9918	.001375516	2,283.94	415,105.71
728	529,984	385,828,352	26.9815	8.9959	.001373626	2,287.08	416,248.46
729	531,441	387,420,489	27.0000	9.0000	.001371742	2,290.22	417,392.79
730	532,900	389,017,000	27.0185	9.0041	.001369863	2,293.36	418,538.68
731	534,361	390,617,891	27.0370	9.0082	.001367989	2,296.50	419,686.15
732	535,824	392,223,168	27.0555	9.0123	.001366120	2,299.65	420,835.19
733	537,289	393,832,837	27.0740	9.0164	.001364256	2,302.79	421,985.79
734	538,756	395,446,904	27.0924	9.0205	.001362398	2,305.93	423,137.97
735	540,225	397,065,375	27.1109	9.0246	.001360544	2,309.07	424,291.72
736	541,696	398,688,256	27.1293	9.0287	.001358696	2,312.21	425,447.04
737	543,169	400,315,553	27.1477	9.0328	.001356852	2,315.35	426,603.94
738	544,644	401,947,272	27.1662	9.0369	.001355014	2,318.50	427,762.40
739	546,121	403,583,419	27.1846	9.0410	.001353180	2,321.64	428,922.43
740	547,600	405,224,000	27.2029	9.0450	.001351351	2,324.78	430,084.03
741	549,081	406,869,021	27.2213	9.0491	.001349528	2,327.92	431,247.21
742	550,564	408,518,488	27.2397	9.0532	.001347709	2,331.06	432,411.95
743	552,049	410,172,407	27.2580	9.0572	.001345895	2,334.20	433,578.27
744	553,536	411,830,784	27.2764	9.0613	.001344086	2,337.34	434,746.16
745	555,025	413,493,625	27.2947	9.0654	.001342282	2,340.49	435,915.62
746	556,516	415,160,936	27.3130	9.0694	.001340483	2,343.63	437,086.64
747	558,009	416,832,723	27.3313	9.0735	.001338688	2,346.77	438,259.24
748	559,504	418,508,992	27.3496	9.0775	.001336898	2,349.91	439,433.41

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
749	561,001	420,189,749	27.3679	9.0816	.001335113	2,353.05	440,609.16
750	562,500	421,875,000	27.3861	9.0856	.001333333	2,356.19	441,786.47
751	564,001	423,564,751	27.4044	9.0896	.001331558	2,359.34	442,965.35
752	565,504	425,259,008	27.4226	9.0937	.001329787	2,362.48	444,145.80
753	567,009	426,957,777	27.4408	9.0977	.001328021	2,365.62	445,327.83
754	568,516	428,661,064	27.4591	9.1017	.001326260	2,368.76	446,511.42
755	570,025	430,368,875	27.4773	9.1057	.001324503	2,371.90	447,696.59
756	571,536	432,081,216	27.4955	9.1098	.001322751	2,375.04	448,883.32
757	573,049	433,798,093	27.5136	9.1138	.001321004	2,378.19	450,071.63
758	574,564	435,519,512	27.5318	9.1178	.001319261	2,381.33	451,261.51
759	576,081	437,245,479	27.5500	9.1218	.001317523	2,384.47	452,452.96
760	577,600	438,976,000	27.5681	9.1258	.001315789	2,387.61	453,645.98
761	579,121	440,711,081	27.5862	9.1298	.001314060	2,390.75	454,840.57
762	580,644	442,450,728	27.6043	9.1338	.001312336	2,393.89	456,036.73
763	582,169	444,194,947	27.6225	9.1378	.001310616	2,397.04	457,234.46
764	583,696	445,943,744	27.6405	9.1418	.001308901	2,400.18	458,433.77
765	585,225	447,697,125	27.6586	9.1458	.001307190	2,403.32	459,634.64
766	586,756	449,455,096	27.6767	9.1498	.001305483	2,406.46	460,837.08
767	588,289	451,217,663	27.6948	9.1537	.001303781	2,409.60	462,041.10
768	589,824	452,984,832	27.7128	9.1577	.001302083	2,412.74	463,246.69
769	591,361	454,756,609	27.7308	9.1617	.001300390	2,415.88	464,453.84
770	592,900	456,533,000	27.7489	9.1657	.001298701	2,419.03	465,662.57
771	594,441	458,314,011	27.7669	9.1696	.001297017	2,422.17	466,872.87
772	595,984	460,099,648	27.7849	9.1736	.001295337	2,425.31	468,084.74
773	597,529	461,889,917	27.8029	9.1775	.001293661	2,428.45	469,298.18
774	599,076	463,684,824	27.8209	9.1815	.001291990	2,431.59	470,513.19
775	600,625	465,484,375	27.8388	9.1855	.001290323	2,434.73	471,729.77
776	602,176	467,288,576	27.8568	9.1894	.001288660	2,437.88	472,947.92
777	603,729	469,097,433	27.8747	9.1933	.001287001	2,441.02	474,167.65
778	605,284	470,910,952	27.8927	9.1973	.001285347	2,444.16	475,388.94
779	606,841	472,729,139	27.9106	9.2012	.001283697	2,447.30	476,611.81
780	608,400	474,552,000	27.9285	9.2052	.001282051	2,450.44	477,836.24
781	609,961	476,379,541	27.9464	9.2091	.001280410	2,453.58	479,062.25
782	611,524	478,211,768	27.9643	9.2130	.001278772	2,456.73	480,289.83
783	613,089	480,048,687	27.9821	9.2170	.001277139	2,459.87	481,518.97
784	614,656	481,890,304	28.0000	9.2209	.001275510	2,463.01	482,749.69
785	616,225	483,736,625	28.0179	9.2248	.001273885	2,466.15	483,981.98
786	617,796	485,587,656	28.0357	9.2287	.001272265	2,469.29	485,215.84
787	619,369	487,443,403	28.0535	9.2326	.001270648	2,472.43	486,451.28
788	620,944	489,303,872	28.0713	9.2365	.001269036	2,475.58	487,688.28
789	622,521	491,169,069	28.0891	9.2404	.001267427	2,478.72	488,926.85
790	624,100	493,039,000	28.1069	9.2443	.001265823	2,481.86	490,166.99
791	625,681	494,913,671	28.1247	9.2482	.001264223	2,485.00	491,408.71
792	627,264	496,793,088	28.1425	9.2521	.001262626	2,488.14	492,651.99
793	628,849	498,677,257	28.1603	9.2560	.001261034	2,491.28	493,896.85
794	630,436	500,566,184	28.1780	9.2599	.001259446	2,494.42	495,143.28
795	632,025	502,459,875	28.1957	9.2638	.001257862	2,497.57	496,391.27
796	633,616	504,358,336	28.2135	9.2677	001256281	2,500.71	497,640.84
797	635,209	506,261,573	28.2312	9.2716	.001254705	2,503.85	498,891.98
798	636,804	508,169,592	28.2489	9.2754	.001253133	2,506.99	500,144.69
799	638,401	510,082,399	28.2666	9.2793	.001251564	2,510.13	501,398.97
800	640,000	512,000,000	28.2843	9.2832	.001250000	2,513.27	502,654.82
801	641,601	513,922,401	28.3019	9.2870	.001248439	2,516.42	503,912.25
802	643,204	515,849,608	28.3196	9.2909	.001246883	2,519.56	505,171.24
803	644,809	517,781,627	28.3373	9.2948	.001245330	2,522.70	506,431.80
804	646,416	519,718,464	28.3549	9.2986	.001243781	2,525.84	507,693.94
805	648,025	521,660,125	28.3725	9.3025	.001242236	2,528.98	508,957.64
806	649,636	523,606,616	28.3901	9.3063	.001240695	2,532.12	510,222.92
807	651,249	525,557,943	28.4077	9.3102	.001239157	2,535.27	511,489.77
808	652,864	527,514,112	28.4253	9.3140	.001237624	2,538.41	512,758.19
809	654,481	529,475,129	28.4429	9.3179	.001236094	2,541.55	514,028.18
810	656,100	531,441,000	28.4605	9.3217	.001234568	2,544.69	515,299.74
811	657,721	533,411,731	28.4781	9.3255	.001233046	2,547.83	516,572.87

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
812	659,344	535,387,328	28.4956	9.3294	.001231527	2,550.97	517,847.57
813	660,969	537,367,797	28.5132	9.3332	.001230012	2,554.11	519,123.84
814	662,596	539,353,144	28.5307	9.3370	.001228501	2,557.26	520,401.68
815	664,225	541,343,375	28.5482	9.3408	.001226994	2,560.40	521,681.10
816	665,856	543,338,496	28.5657	9.3447	.001225490	2,563.54	522,962.08
817	667,489	545,338,513	28.5832	9.3485	.001223990	2,566.68	524,244.63
818	669,124	547,343,432	28.6007	9.3523	.001222494	2,569.82	525,528.76
819	670,761	549,353,259	28.6182	9.3561	.001221001	2,572.96	526,814.46
820	672,400	551,368,000	28.6356	9.3599	.001219512	2,576.11	528,101.73
821	674,041	553,387,661	28.6531	9.3637	.001218027	2,579.25	529,390.56
822	675,684	555,412,248	28.6705	9.3675	.001216545	2,582.39	530,680.97
823	677,329	557,441,767	28.6880	9.3713	.001215067	2,585.53	531,972.95
824	678,976	559,476,224	28.7054	9.3751	.001213592	2,588.67	533,266.50
825	680,625	561,515,625	28.7228	9.3789	.001212121	2,591.81	534,561.62
826	682,276	563,559,976	28.7402	9.3827	.001210654	2,594.96	535,858.32
827	683,929	565,609,283	28.7576	9.3865	.001209190	2,598.10	537,156.58
828	685,584	567,663,552	28.7750	9 3902	.001207729	2,601.24	538,456.41
829	687,241	569,722,789	28.7924	9.3940	.001206273	2,604.38	539,757.82
830	688,900	571,787,000	28.8097	9.3978	.001204819	2,607.52	541,060.79
831	690,561	573,856,191	28.8271	9.4016	.001203369	2,610.66	542,365.34
832	692,224	575,930,368	28.8444	9.4053	.001201923	2,613.81	543,671.46
833	693,889	578,009,537	28.8617	9.4091	.001200480	2,616.95	544,979.15
834	695,556	580,093,704	28.8791	9.4129	.001199041	2,620.09	546,288.40
835	697,225	582,182,875	28.8964	9.4166	.001197605	2,623.23	547,599.23
836	698,896	584,277,056	28.9137	9.4204	.001196172	2,626.37	548,911.63
837	700,569	586,376,253	28.9310	9.4241	.001194743	2,629.51	550,225.61
838	702,244	588,480,472	28.9482	9.4279	.001193317	2,632.65	551,541.15
839	703,921	590,589,719	28.9655	9.4316	.001191895	2,635.80	552,858.26
840	705,600	592,704,000	28.9828	9.4354	.001190476	2,638.94	554,176.94
841	707,281	594,823,321	29.0000	9.4391	.001189061	2,642.08	555,497.20
842	708,964	596,947,688	29.0172	9.4429	.001187648	2,645.22	556,819.02
843	710,649	599,077,107	29.0345	9.4466	.001186240	2,648.36	558,142.42
844	712,336	601,211,584	29.0517	9.4503	.001184834	2,651.50	559,467.39
845	714,025	603,351,125	29.0689	9.4541	.001183432	2,654.65	560,793.92
846	715,716	605,495,736	29.0861	9.4578	.001182033	2,657.79	562,122.03
847	717,409	607,645,423	29.1033	9.4615	.001180638	2,660.93	563,451.71
848	719,104	609,800,192	29.1204	9.4652	.001179245	2,664.07	564,782.96
849	720.801	611,960,049	29.1376	9.4690	.001177856	2,667.21	566,115.78
850	722,500	614,125,000	29.1548	9.4727	.001176471	2,670.35	567,450.17
851	724,201	616,295,051	29.1719	9.4764	.001175088	2,673.50	568,786.14
852	725,904	618,470,208	29.1890	9.4801	.001173709	2,676.64	570,123.67
853	727,609	620,650,477	29.2062	9.4838	.001172333	2,679.78	571,462.77
854	729,316	622,835,864	29.2233	9.4875	.001170960	2,682.92	572,803.45
855	731,025	625,026,375	29.2404	9.4912	.001169591	2,686.06	574,145.69
856	732,736	627,222,016	29.2575	9.4949	.001168224	2,689.20	575,489.51
857	734,449	629,422,793	29.2746	9.4986	.001166861	2,692.34	576,834.90
858	736,164	631,628,712	29.2916	9.5023	.001165501	2,695.49	578,181.85
859	737,881	633,839,779	29.3087	9.5060	.001164144	2,698.63	579,530.38
860	739,600	636,056,000	29.3258	9.5097	.001162791	2,701.77	580,880.48
861	741,321	638,277,381	29.3428	9.5134	.001161440	2,704.91	582,232.15
862	743,044	640,503,928	29.3598	9.5171	.001160093	2,708.05	583,585.39
863	744,769	642,735,647	29.3769	9.5207	.001158749	2,711.19	584,940.20
864	746,496	644,972,544	29.3939	9.5244	.001157407	2,714.34	586,296.59
865	748,225	647,214,625	29.4109	9.5281	.001156069	2,717.48	587,654.54
866	749,956	649,461,896	29.4279	9.5317	.001154734	2,720.62	589,014.07
867	751,689	651,714,363	29.4449	9.5354	.001153403	2,723.76	590,375.16
868	753,424	653,972,032	29.4618	9.5391	.001152074	2,726.90	591,737.83
869	755,161	656,234,909	29.4788	9.5427	.001150748	2,730.04	593,102.06
870	756,900	658,503,000	29.4958	9.5464	.001149425	2,733.19	594,467.87
871	758,641	660,776,311	29.5127	9.5501	.001148106	2,736.33	595,835.25
872	760,384	663,054,848	29.5296	9.5537	.001146789	2,739.47	597,204.20
873	762,129	665,338,617	29.5466	9.5574	.001145475	2,742.61	598,574.72
874	763,876	667,627,624	29.5635	9.5610	.001144165	2,745.75	599,946.81

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
875	765,625	669,921,875	29.5804	9.5647	.001142857	2,748.89	601,320.47
876	767,376	672,221,376	29.5973	9.5683	.001141553	2,752.04	602,695.70
877	769,129	674,526,133	29.6142	9.5719	.001140251	2,755.18	604,072.50
878	770,884	676,836,152	29.6311	9.5756	.001138952	2,758.32	605,450.88
879	772,641	679,151,439	29.6479	9.5792	.001137656	2,761.46	606,830.82
880	774,400	681,472,000	29.6648	9.5828	.001136364	2,764.60	608,212.34
881	776,161	683,797.841	29.6816	9.5865	.001135074	2,767.74	609,595.42
882	777,924	686,128,968	29.6985	9.5901	.001133787	2,770.88	610,980.08
883	779.689	688,465.387	29.7153	9.5937	.001132503	2,774.03	612,366.31
884	781,456	690,807,104	29.7321	9.5973	.001131222	2,777.17	613,754.11
885	783,225	693,154,125	29.7489	9.6010	.001129944	2,780.31	615,143.48
886	784,996	695,506,456	29.7658	9.6046	.001128668	2,783.45	616,534.42
887	786,769	697,864,103	29.7825	9.6082	.001127396	2,786.59	617,926.93
888	788,544	700,227,072	29.7993	9.6118	.001126126	2,789.73	619,321.01
889	790,321	702,595,369	29.8161	9.6154	.001124859	2,792.88	620,716.66
890	792,100	704,969,000	29.8329	9.6190	.001123596	2,796.02	622,113.89
891	793,881	707,347,971	29.8496	9.6226	.001122334	2,799.16	623,512.68
892	795,664	709.732,288	29.8664	9.6262	.001121076	2,802.30	624,913.04
893	797,449	712,121,957	29.8831	9.6298	.001119821	2,805.44	626,314.98
894	799,236	714,516,984	29.8998	9.6334	.001118568	2,808.58	627,718.49
895	801,025	716,917,375	29.9166	9.6370	.001117318	2,811.73	629,123.56
896	802,816	719,323,136	29.9333	9.6406	.001116071	2,814.87	630,530.21
897	804,609	721,734,273	29.9500	9.6442	.001114827	2,818.01	631,938.43
898	806,404	724,150,792	29.9666	9.6477	.001113586	2,821.15	633,348.22
899	808,201	726,572,699	29.9833	9.6513	.001112347	2,824.29	634,759.58
900	810,000	729,000.000	30.0000	9.6549	.001111111	2,827.43	636,172.51
901	811,801	731,432,701	30.0167	9.6585	.001109878	2,830.58	637,587.01
902	813,604	733,870,808	30.0333	9.6620	.001108647	2,833.72	639,003.09
903	815,409	736,314,327	30.0500	9.6656	.001107420	2,836.86	640,420.73
904	817,216	738,763,264	30.0666	9.6692	.001106195	2,840.00	641,839.95
905	819.025	741,217,625	30.0832	9.6727	.001104972	2,843.14	643,260.73
906	820,836	743,677,416	30.0998	9.6763	.001103753	2,846.28	644,683.09
907	822,649	746,142,643	30.1164	9.6799	.001102536	2,849.42	646.107.01
908	824.464	748,613,312	30.1330	9.6834	.001101322	2,852.57	647,532.51
909	826.281	751,089,429	30.1496	9.6870	.001100110	2,855.71	648,959.58
910	828.100	753,571,000	30.1662	9.6905	.001098901	2,858.85	650,388.22
911	829,921	756,058,031	30.1828	9.6941	001097695	2,861.99	651,818.43
912	831,744	758,550,528	30.1993	9.6976	.001096491	2,865.13	653,250.21
913	833,569	761,048,497	30.2159	9.7012	.001095290	2,868.27	654,683.56
914	835,396	763,551,944	30.2324	9.7047	.001094092	2,871.42	656,118.48
915	837,225	766,060,875	30.2490	9.7082	.001092896	2,874.56	657,554.98
916	839,056	768,575,296	30.2655	9.7118	.001091703	2,877.70	658,993.04
917	840,889	771,095,213	30.2820	9.7153	.001090513	2,880.84	660,432.68
918	842,724	773,620,632	30.2985	9.7188	.001089325	2,883.98	661,873.88
919	844,561	776,151,559	30.3150	9.7224	.001088139	2.887.12	663,316.66
920	846,400	778,688,000	30.3315	9.7259	.001086957	2,890.27	664,761.01
921	848,241	781,229,961	30.3480	9.7294	.001085776	2,893.41	666,206.92
922	850,084	783,777,448	30.3645	9.7329	.001084599	2,896.55	667,654.41
923	851,929	786,330,467	30.3809	9.7364	.001083424	2,899.69	669,103.47
924	853,776	788,889,024	30.3974	9.7400	.001082251	2,902.83	670,554.10
925	855,625	791,453,125	30.4138	9.7435	.001081081	2,905.97	672.006.30
926	857,476	794,022,776	30.4302	9.7470	.001079914	2,909.11	673,460.08
927	859,329	796,597,983	30.4467	9.7505	.001078749	2,912.26	674,915.42
928	861,184	799,178,752	30.4631	9.7540	.001077586	2,915.40	676.372.33
929	863.041	801,765,089	30.4795	9.7575	.001076426	2,918.54	677,830.82
930	864.900	804,357,000	30.4959	9.7610	.001075269	2,921.68	679,290.87
931	866,761	806,954,491	30.5123	9.7645	.001074114	2,924.82	680,752.50
932	868,624	809,557,568	30.5287	9.7680	.001072961	2,927.96	682.215.69
933	870,489	812,166,237	30.5450	9.7715	.001071811	2,931.11	683.680.46
934	872,356	814,780,504	30.5614	9.7750	.001070664	2,934.25	685.146.80
935	874,225	817,400,375	30.5778	9.7785	.001069519	2,937.39	686.614.71
936	876,096	820,025,856	30.5941	9.7819	.001068376	2,940.53	688,084.19
937	877,969	822,656,953	30.6105	9.7854	.001067236	2,943 67	689,555.24

Table 4-4. Squares, cubes, square and cube roots, circumferences, and areas (continued)

No.	Square	Cube	Sq. Root	Cu. Root	Reciprocal	Circum.	Area
938	879,844	825,293,672	30.6268	9.7889	.001066098	2,946.81	691,027.36
939	881,721	827,936,019	30.6431	9.7924	.001064963	2,949.96	692,502.05
940	883,600	830,584,000	30.6594	9.7959	.001063830	2,953.10	693,977.82
941	885,481	833,237,621	30.6757	9.7993	.001062699	2,956.24	695,455.15
942	887,364	835,896,888	30.6920	9.8028	.001061571	2,959.38	696,934.06
943	889,249	838,561,807	30.7083	9.8063	.001060445	2,962.52	698,414.53
944	891,136	841,232,384	30.7246	9.8097	.001059322	2,965.66	699,896.58
945	893,025	843,908,625	30.7409	9.8132	.001058201	2,968.81	701,380.19
946	894,916	846,590,536	30.7571	9.8167	.001057082	2,971.95	702,865.38
947	896,809	849,278,123	30.7734	9.8201	.001055966	2,975.09	704,352.14
948	898,704	851,971,392	30.7896	9.8236	.001054852	2,978.23	705,840.47
949	900,601	854,670,349	30.8058	9.8270	.001053741	2,981.37	707,330.37
950	902,500	857,375,000	30.8221	9.8305	.001052632	2,984.51	708,821.84
951	904,401	860,085,351	30.8383	9.8339	.001051525	2,987.65	710,314.88
952	906,304	862,801,408	30.8545	9.8374	.001050420	2,990.80	711,809.50
953	908,209	865,523,177	30.8707	9.8408	.001049318	2,993.94	713,305.68
954	910,116	868,250,664	30.8869	9.8443	.001048218	2,997.08	714,803.43
955	912,025	870,983,875	30.9031	9.8477	.001047120	3,000.22	716,302.76
956	913,936	873,722,816	30.9192	9.8511	.001046025	3,003.36	717,803.66
957	915,849	876,467,493	30.9354	9.8546	.001044932	3,006.50	719,306.12
958	917,764	879,217,912	30.9516	9.8580	.001043841	3,009.65	720,810.16
959	919,681	881,974,079	30.9677	9.8614	.001042753	3,012.79	722,315.77
960	921,600	884,736,000	30.9839	9.8648	.001041667	3,015.93	723,822.95
961	923,521	887,503,681	31.0000	9.8683	.001040583	3,019.07	725,331.70
962	925,444	890,277,128	31.0161	9.8717	.001039501	3,022.21	726,842.02
963	927,369	893,056,347	31.0322	9.8751	.001038422	3,025.35	728,353.91
964	929,296	895,841,344	31.0483	9.8785	.001037344	3,028.50	729,867.37
965	931,225	898,632,125	31.0644	9.8819	.001036269	3,031.64	731,382.40
966	933,156	901,428,696	31.0805	9.8854	.001035197	3,034.78	732,899.01
967	935,089	904,231,063	31.0966	9.8888	.001034126	3,037.92	734,417.18
968	937,024	907,039,232	31.1127	9.8922	.001033058	3,041.06	735,936.93
969	938,961	909,853.209	31.1288	9.8956	.001031992	3,044.20	737,458.24
970	940,900	912,673,000	31.1448	9.8990	.001030928	3,047.34	738,981.13
971	942,841	915,498,611	31.1609	9.9024	.001029866	3,050.49	740,505.59
972	944,784	918,330,048	31.1769	9.9058	.001028807	3,053.63	742,031.62
973	946,729	921,167,317	31.1929	9.9092	.001027749	3,056.77	743,559.22
974	948,676	924,010,424	31.2090	9.9126	.001026694	3,059.91	745,088.39
975	950,625	926,859,375	31.2250	9.9160	.001025641	3,063.05	746,619.13
976	952,576	929,714,176	31.2410	9.9194	.001024590	3,066.19	748,151.44
977	954,529	932,574,833	31.2570	9.9227	.001023541	3,069.34	749,685.32
978	956,484	935,441,352	31.2730	9.9261	.001022495	3,072.48	751,220.78
979	958,441	938,313,739	31.2890	9.9295	.001021450	3,075.62	752,757.80
980	960,400	941,192,000	31.3050	9.9329	.001020408	3,078.76	754,296.40
981	962,361	944,076,141	31.3209	9.9363	.001019368	3,081.90	755,836.56
982	964,324	946,966,168	31.3369	9.9396	.001018330	3,085.04	757,378.30
983	966,289	949,862,087	31.3528	9.9430	.001017294	3,088.19	758,921.61
984	968,256	952,763,904	31.3688	9.9464	.001016260	3,091.33	760,466.48
985	970,225	955,671,625	31.3847	9.9497	.001015228	3,094.47	762,012.93
986	972,196	958,585,256	31.4006	9.9531	.001014199	3,097.61	763,560.95
987	974,169	961,504,803	31.4166	9.9565	.001013171	3,100.75	765,110.54
988	976,144	964,430,272	31.4325	9.9598	.001012146	3,103.89	766,661.70
989	978,121	967,361,669	31.4484	9.9632	.001011122	3,107.04	768,214.44
990	980,100	970,299,000	31.4643	9.9666	.001010101	3,110.18	769,768.74
991	982,081	973,242,271	31.4802	9.9699	.001009082	3,113.32	771,324.61
992	984,064	976,191,488	31.4960	9.9733	.001008065	3,116.46	772,882.06
993	986,049	979,146,657	31.5119	9.9766	.001007049	3,119.60	774,441.07
994	988,036	982,107,784	31.5278	9.9800	.001006036	3,122.74	776,001.66
995	990,025	985,074,875	31.5436	9.9833	.001005025	3,125.88	777,563.82
996	992,016	988,047,936	31.5595	9.9866	.001004016	3,129.03	779,127.54
997	994,009	991,026,973	31.5753	9.9900	.001003009	3,132.17	780,692.84
998	996,004	994,011,992	31.5911	9.9933	.001002004	3,135.31	782,259.71
999	998,001	997,002,999	31.6070	9.9967	.001001001	3,138.45	783,828.15
1000	1,000,000	1,000,000,000	31.6228	10.0000	.001000000	3,141.59	785,398.16

Table 4-5. Natural logarithms*

These two pages give the natural or Napierian logarithms (ln) of numbers between 1 and 10, correct to four places. Moving the decimal point n places to the right (or left) in the number is equivalent to adding n times 2.3026 (or n times $\bar{3}.6974$) to the logarithm. Base $e = 2.71828+$.

n	n (2.3026)	n (0.6974–3)
1	2.3026	0.6974–3
2	4.6052	0.3948–5
3	6.9078	0.0922–7
4	9.2103	0.7897–10
5	11.5129	0.4871–12
6	13.8155	0.1845–14
7	16.1181	0.8819–17
8	18.4207	0.5793–19
9	20.7233	0.2767–21

No.	0	1	2	3	4	5	6	7	8	9	Avg. diff.
1.0	0.0000	0100	0198	0296	0392	0488	0583	0677	0770	0862	95
1.1	0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	87
1.2	1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	80
1.3	2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	74
1.4	3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	69
1.5	0.4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	65
1.6	4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	61
1.7	5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	57
1.8	5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	54
1.9	6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	51
2.0	0.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	49
2.1	7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	47
2.2	7885	7930	7975	8020	8065	8109	8154	8198	8242	8286	44
2.3	8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	43
2.4	8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	41
2.5	0.9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	39
2.6	9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	38
2.7	0.9933	9969	*0006	*0043	*0080	*0116	*0152	*0188	*0225	*0260	36
2.8	1.0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	35
2.9	0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	34
3.0	1.0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	33
3.1	1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	32
3.2	1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	31
3.3	1939	1969	2000	2030	2060	2090	2119	2149	2179	2208	30
3.4	2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	29
3.5	1.2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	28
3.6	2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	27
3.7	3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	27
3.8	3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	26
3.9	3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	25
4.0	1.3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	25
4.1	4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	24
4.2	4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	23
4.3	4586	4609	4633	4656	4679	4702	4725	4748	4770	4793	23
4.4	4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	22
4.5	1.5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	22
4.6	5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	21
4.7	5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	21
4.8	5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	20
4.9	5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	20

$\ln x = (2.3026) \log_{10} x \qquad \log_{10} x = (0.4343) \ln x$

where $2.3026 = \ln 10$ and $0.4343 = \log_{10} e$.

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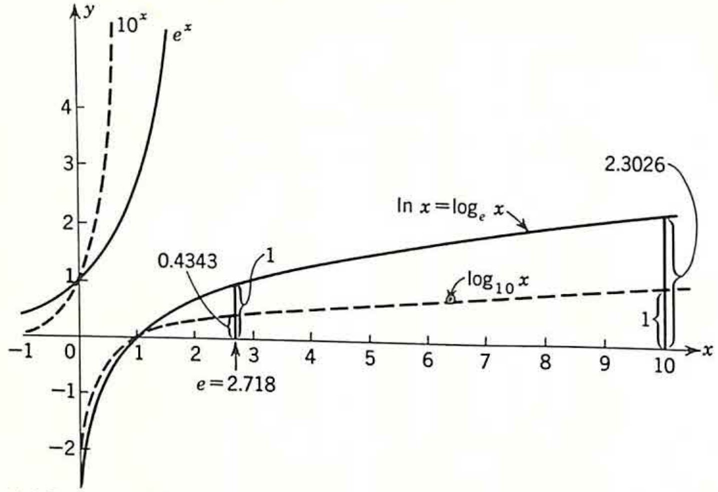
* From W. D. Cockrell, "Industrial Electronics Handbook," McGraw-Hill Book Company, New York, 1958.

Table 4-5. Natural logarithms (continued)

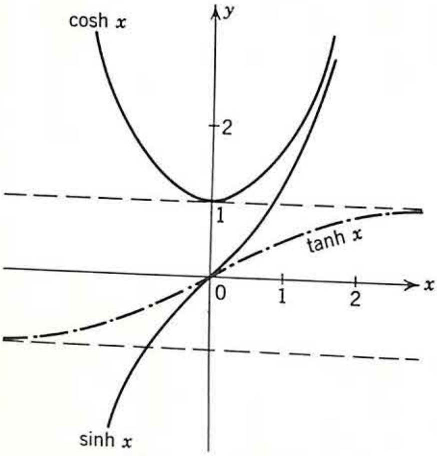
No.	0	1	2	3	4	5	6	7	8	9	Avg. diff.
5.0	1.6094	6114	6134	6154	6174	6194	6214	6233	6253	6273	20
5.1	6292	6312	6332	6351	6371	6390	6409	6429	6448	6467	19
5.2	6487	6506	6525	6544	6563	6582	6601	6620	6639	6658	19
5.3	6677	6696	6715	6734	6752	6771	6790	6808	6827	6845	18
5.4	6864	6882	6901	6919	6938	6956	6974	6993	7011	7029	18
5.5	1.7047	7066	7084	7102	7120	7138	7156	7174	7192	7210	18
5.6	7228	7246	7263	7281	7299	7317	7334	7352	7370	7387	18
5.7	7405	7422	7440	7457	7475	7492	7509	7527	7544	7561	17
5.8	7579	7596	7613	7630	7647	7664	7681	7699	7716	7733	17
5.9	7750	7766	7783	7800	7817	7834	7851	7867	7884	7901	17
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066	16
6.1	8083	8099	8116	8132	8148	8165	8181	8197	8213	8229	16
6.2	8245	8262	8278	8294	8310	8326	8342	8358	8374	8390	16
6.3	8405	8421	8437	8453	8469	8485	8500	8516	8532	8547	16
6.4	8563	8579	8594	8610	8625	8641	8656	8672	8687	8703	15
6.5	1.8718	8733	8749	8764	8779	8795	8810	8825	8840	8856	15
6.6	8871	8886	8901	8916	8931	8946	8961	8976	8991	9006	15
6.7	9021	9036	9051	9066	9081	9095	9110	9125	9140	9155	15
6.8	9169	9184	9199	9213	9228	9242	9257	9272	9286	9301	15
6.9	9315	9330	9344	9359	9373	9387	9402	9416	9430	9445	14
7.0	1.9459	9473	9488	9502	9516	9530	9544	9559	9573	9587	14
7.1	9601	9615	9629	9643	9657	9671	9685	9699	9713	9727	14
7.2	9741	9755	9769	9782	9796	9810	9824	9838	9851	9865	14
7.3	1.9879	9892	9906	9920	9933	9947	9961	9974	9988	*0001	13
7.4	2.0015	0028	0042	0055	0069	0082	0096	0109	0122	0136	13
7.5	2.0149	0162	0176	0189	0202	0215	0229	0242	0255	0268	13
7.6	0281	0295	0308	0321	0334	0347	0360	0373	0386	0399	13
7.7	0412	0425	0438	0451	0464	0477	0490	0503	0516	0528	13
7.8	0541	0554	0567	0580	0592	0605	0618	0631	0643	0656	13
7.9	0669	0681	0694	0707	0719	0732	0744	0757	0769	0782	12
8.0	2.0794	0807	0819	0832	0844	0857	0869	0882	0894	0906	12
8.1	0919	0931	0943	0956	0968	0980	0992	1005	1017	1029	12
8.2	1041	1054	1066	1078	1090	1102	1114	1126	1138	1150	12
8.3	1163	1175	1187	1199	1211	1223	1235	1247	1258	1270	12
8.4	1282	1294	1306	1318	1330	1342	1353	1365	1377	1389	12
8.5	2.1401	1412	1424	1436	1448	1459	1471	1483	1494	1506	12
8.6	1518	1529	1541	1552	1564	1576	1587	1599	1610	1622	12
8.7	1633	1645	1656	1668	1679	1691	1702	1713	1725	1736	11
8.8	1748	1759	1770	1782	1793	1804	1815	1827	1838	1849	11
8.9	1861	1872	1883	1894	1905	1917	1928	1939	1950	1961	11
9.0	2.1972	1983	1994	2006	2017	2028	2039	2050	2061	2072	11
9.1	2083	2094	2105	2116	2127	2138	2148	2159	2170	2181	11
9.2	2192	2203	2214	2225	2235	2246	2257	2268	2279	2289	11
9.3	2300	2311	2322	2332	2343	2354	2364	2375	2386	2396	11
9.4	2407	2418	2428	2439	2450	2460	2471	2481	2492	2502	11
9.5	2.2513	2523	2534	2544	2555	2565	2576	2586	2597	2607	10
9.6	2618	2628	2638	2649	2659	2670	2680	2690	2701	2711	10
9.7	2721	2732	2742	2752	2762	2773	2783	2793	2803	2814	10
9.8	2824	2834	2844	2854	2865	2875	2885	2895	2905	2915	10
9.9	2925	2935	2946	2956	2966	2976	2986	2996	3006	3016	10
10.0	2.3026										

Moving the decimal point *n* places to the right (or left) in the number requires adding *n* times 2.3026 [or *n* times (0.6974–3)] in the body of the table.

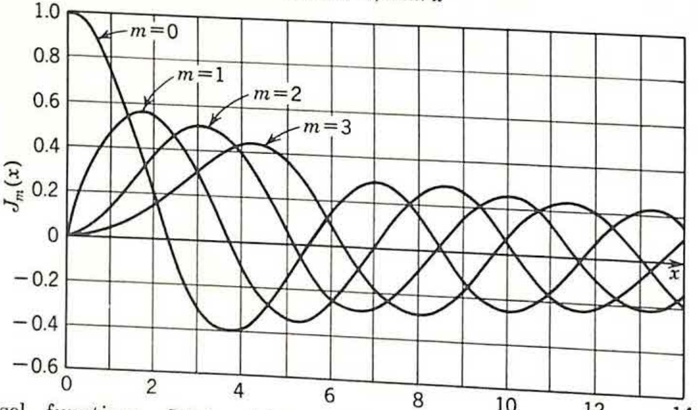
Miscellaneous functions



Exponential function, 10^x or e^x. Logarithmic function, $\log_{10} x$ or $\ln_e x$. (From Marks, *Mechanical Engineer's Handbook*, 6 ed., McGraw-Hill Book Company, New York, 1958.)



sinh x, cosh x, tanh x



The Bessel functions $J_0(x)$, $J_1(x)$, $J_2(x)$, . . . for real arguments. Note that $J_m(-x) = (-1)^m J_m(x)$.

Table 4-6. Values and logarithms of exponential and hyperbolic functions*

x	e^x		e^{-x}	sinh x		cosh x		tanh x
	Value	$\log_{10}$	(value)	Value	$\log_{10}$	Value	$\log_{10}$	(value)
0.00	1.0000	0.00000	1.00000	0.0000	$-\infty$	1.0000	0.00000	0.00000
0.01	1.0101	.00434	0.99005	.0100	$\bar{2}.00001$	1.0001	.00002	.01000
0.02	1.0202	.00869	.98020	.0200	$\bar{2}.30106$	1.0002	.00009	.02000
0.03	1.0305	.01303	.97045	.0300	$\bar{2}.47719$	1.0005	.00020	.02999
0.04	1.0408	.01737	.96079	.0400	$\bar{2}.60218$	1.0008	.00035	.03998
0.05	1.0513	.02171	.95123	.0500	$\bar{2}.69915$	1.0013	.00054	.04996
0.06	1.0618	.02606	.94176	.0600	$\bar{2}.77841$	1.0018	.00078	.05993
0.07	1.0725	.03040	.93239	.0701	$\bar{2}.84545$	1.0025	.00106	.06989
0.08	1.0833	.03474	.92312	.0801	$\bar{2}.90355$	1.0032	.00139	.07983
0.09	1.0942	.03909	.91393	.0901	$\bar{2}.95483$	1.0041	.00176	.08976
0.10	1.1052	.04343	.90484	.1002	$\bar{1}.00072$	1.0050	.00217	.09967
0.11	1.1163	.04777	.89583	.1102	$\bar{1}.04227$	1.0061	.00262	.10956
0.12	1.1275	.05212	.88692	.1203	$\bar{1}.08022$	1.0072	.00312	.11943
0.13	1.1388	.05646	.87809	.1304	$\bar{1}.11517$	1.0085	.00366	.12927
0.14	1.1503	.06080	.86936	.1405	$\bar{1}.14755$	1.0098	.00424	.13909
0.15	1.1618	.06514	.86071	.1506	$\bar{1}.17772$	1.0113	.00487	.14889
0.16	1.1735	.06949	.85214	.1607	$\bar{1}.20597$	1.0128	.00554	.15865
0.17	1.1853	.07383	.84366	.1708	$\bar{1}.23254$	1.0145	.00625	.16838
0.18	1.1972	.07817	.83527	.1810	1.25762	1.0162	.00700	.17808
0.19	1.2092	.08252	.82696	.1911	$\bar{1}.28136$	1.0181	.00779	.18775
0.20	1.2214	.08686	.81873	.2013	$\bar{1}.30392$	1.0201	.00863	.19738
0.21	1.2337	.09120	.81058	.2115	$\bar{1}.32541$	1.0221	.00951	.20697
0.22	1.2461	.09554	.80252	.2218	$\bar{1}.34592$	1.0243	.01043	.21652
0.23	1.2586	.09989	.79453	.2320	$\bar{1}.36555$	1.0266	.01139	.22603
0.24	1.2712	.10423	.78663	.2423	$\bar{1}.38437$	1.0289	.01239	.23550
0.25	1.2840	.10857	.77880	.2526	$\bar{1}.40245$	1.0314	.01343	.24492
0.26	1.2969	.11292	.77105	.2629	$\bar{1}.41986$	1.0340	.01452	.25430
0.27	1.3100	.11726	.76338	.2733	$\bar{1}.43663$	1.0367	.01564	.26362
0.28	1.3231	.12160	.75578	.2837	$\bar{1}.45282$	1.0395	.01681	.27291
0.29	1.3364	.12595	.74826	.2941	$\bar{1}.46847$	1.0423	.01801	.28213
0.30	1.3499	.13029	.74082	.3045	$\bar{1}.48362$	1.0453	.01926	.29131
0.31	1.3634	.13463	.73345	.3150	$\bar{1}.49830$	1.0484	.02054	.30044
0.32	1.3771	.13897	.72615	.3255	$\bar{1}.51254$	1.0516	.02187	.30951
0.33	1.3910	.14332	.71892	.3360	$\bar{1}.52637$	1.0549	.02323	.31852
0.34	1.4049	.14766	.71177	.3466	$\bar{1}.53981$	1.0584	.02463	.32748
0.35	1.4191	.15200	.70469	.3572	$\bar{1}.55290$	1.0619	.02607	.33638
0.36	1.4333	.15635	.69768	.3678	$\bar{1}.56564$	1.0655	.02755	.34521
0.37	1.4477	.16069	.69073	.3785	$\bar{1}.57807$	1.0692	.02907	.35399
0.38	1.4623	.16503	.68386	.3892	$\bar{1}.59019$	1.0731	.03063	.36271
0.39	1.4770	.16937	.67706	.4000	$\bar{1}.60202$	1.0770	.03222	.37136
0.40	1.4918	.17372	.67032	.4108	$\bar{1}.61358$	1.0811	.03385	.37995
0.41	1.5068	.17806	.66365	.4216	$\bar{1}.62488$	1.0852	.03552	.38847
0.42	1.5220	.18240	.65705	.4325	$\bar{1}.63594$	1.0895	.03723	.39693
0.43	1.5373	.18675	.65051	.4434	$\bar{1}.64677$	1.0939	.03897	.40532
0.44	1.5527	.19109	.64404	.4543	$\bar{1}.65738$	1.0984	.04075	.41364
0.45	1.5683	.19543	.63763	.4653	$\bar{1}.66777$	1.1030	.04256	.42190
0.46	1.5841	.19978	.63128	.4764	$\bar{1}.67797$	1.1077	.04441	.43008
0.47	1.6000	.20412	.62500	.4875	1.68797	1.1125	.04630	.43820
0.48	1.6161	.20846	.61878	.4986	$\bar{1}.69779$	1.1174	.04822	.44624
0.49	1.6323	.21280	.61263	.5098	$\bar{1}.70744$	1.1225	.05018	.45422
0.50	1.6487	.21715	.60653	.5211	$\bar{1}.71692$	1.1276	.05217	.46212

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* From R. H. Perry, "Engineering Manual," McGraw-Hill Book Company, New York, 1959.

Table 4-6. Values and logarithms of exponential and hyperbolic functions (continued)

x	e^x		e^{-x}	$\sinh x$		$\cosh x$		$\tanh x$
	Value	$\log_{10}$	(value)	Value	$\log_{10}$	Value	$\log_{10}$	(value)
0.50	1.6487	0.21715	0.60653	0.5211	$\bar{1}.71692$	1.1276	0.05217	0.46212
0.51	1.6653	.22149	.60050	0.5324	$\bar{1}.72624$	1.1329	.05419	.46995
0.52	1.6820	.22583	.59452	0.5438	$\bar{1}.73540$	1.1383	.05625	.47770
0.53	1.6989	.23018	.58860	0.5552	$\bar{1}.74442$	1.1438	.05834	.48538
0.54	1.7160	.23452	.58275	0.5666	$\bar{1}.75330$	1.1494	.06046	.49299
0.55	1.7333	.23886	.57695	0.5782	$\bar{1}.76204$	1.1551	.06262	.50052
0.56	1.7507	.24320	.57121	0.5897	$\bar{1}.77065$	1.1609	.06481	.50798
0.57	1.7683	.24755	.56553	0.6014	$\bar{1}.77914$	1.1669	.06703	.51536
0.58	1.7860	.25189	.55990	0.6131	$\bar{1}.78751$	1.1730	.06929	.52267
0.59	1.8040	.25623	.55433	0.6248	$\bar{1}.79576$	1.1792	.07157	.52990
0.60	1.8221	.26058	.54881	0.6367	$\bar{1}.80390$	1.1855	.07389	.53705
0.61	1.8404	.26492	.54335	0.6485	$\bar{1}.81194$	1.1919	.07624	.54413
0.62	1.8589	.26926	.53794	0.6605	$\bar{1}.81987$	1.1984	.07861	.55113
0.63	1.8776	.27361	.53259	0.6725	$\bar{1}.82770$	1.2051	.08102	.55805
0.64	1.8965	.27795	.52729	0.6846	$\bar{1}.83543$	1.2119	.08346	.56490
0.65	1.9155	.28229	.52205	0.6967	$\bar{1}.84308$	1.2188	.08593	.57167
0.66	1.9348	.28664	.51685	0.7090	$\bar{1}.85063$	1.2258	.08843	.57836
0.67	1.9542	.29098	.51171	0.7213	$\bar{1}.85809$	1.2330	.09095	.58498
0.68	1.9739	.29532	.50662	0.7336	$\bar{1}.86548$	1.2402	.09351	.59152
0.69	1.9937	.29966	.50158	0.7461	$\bar{1}.87278$	1.2476	.09609	.59798
0.70	2.0138	.30401	.49659	0.7586	$\bar{1}.88000$	1.2552	.09870	.60437
0.71	2.0340	.30835	.49164	0.7712	$\bar{1}.88715$	1.2628	.10134	.61068
0.72	2.0544	.31269	.48675	0.7838	$\bar{1}.89423$	1.2706	.10401	.61691
0.73	2.0751	.31703	.48191	0.7966	$\bar{1}.90123$	1.2785	.10670	.62307
0.74	2.0959	.32138	.47711	0.8094	$\bar{1}.90817$	1.2865	.10942	.62915
0.75	2.1170	.32572	.47237	0.8223	$\bar{1}.91504$	1.2947	.11216	.63515
0.76	2.1383	.33006	.46767	0.8353	$\bar{1}.92185$	1.3030	.11493	.64108
0.77	2.1598	.33441	.46301	0.8484	$\bar{1}.92859$	1.3114	.11773	.64693
0.78	2.1815	.33875	.45841	0.8615	$\bar{1}.93527$	1.3199	.12055	.65271
0.79	2.2034	.34309	.45384	0.8748	$\bar{1}.94190$	1.3286	.12340	.65841
0.80	2.2255	.34744	.44933	0.8881	$\bar{1}.94846$	1.3374	.12627	.66404
0.81	2.2479	.35178	.44486	0.9015	$\bar{1}.95498$	1.3464	.12917	.66959
0.82	2.2705	.35612	.44043	0.9150	$\bar{1}.96144$	1.3555	.13209	.67507
0.83	2.2933	.36046	.43605	0.9286	$\bar{1}.96784$	1.3647	.13503	.68048
0.84	2.3164	.36481	.43171	0.9423	$\bar{1}.97420$	1.3740	.13800	.68581
0.85	2.3396	.36915	.42741	0.9561	$\bar{1}.98051$	1.3835	.14099	.69107
0.86	2.3632	.37349	.42316	0.9700	$\bar{1}.98677$	1.3932	.14400	.69626
0.87	2.3869	.37784	.41895	0.9840	$\bar{1}.99299$	1.4029	.14704	.70137
0.88	2.4109	.38218	.41478	0.9981	$\bar{1}.99916$	1.4128	.15009	.70642
0.89	2.4351	.38652	.41066	1.0122	0.00528	1.4229	.15317	.71139
0.90	2.4596	.39087	.40657	1.0265	.01137	1.4331	.15627	.71630
0.91	2.4843	.39521	.40252	1.0409	.01741	1.4434	.15939	.72113
0.92	2.5093	.39955	.39852	1.0554	.02341	1.4539	.16254	.72590
0.93	2.5345	.40389	.39455	1.0700	.02937	1.4645	.16570	.73059
0.94	2.5600	.40824	.39063	1.0847	.03530	1.4753	.16888	.73522
0.95	2.5857	.41258	.38674	1.0995	.04119	1.4862	.17208	.73978
0.96	2.6117	.41692	.38289	1.1144	.04704	1.4973	.17531	.74428
0.97	2.6379	.42127	.37908	1.1294	.05286	1.5085	.17855	.74870
0.98	2.6645	.42561	.37531	1.1446	.05864	1.5199	.18181	.75307
0.99	2.6912	.42995	.37158	1.1598	.06439	1.5314	.18509	.75736
1.00	2.7183	.43429	.36788	1.1752	.07011	1.5431	.18839	.76159

Table 4-6. Values and logarithms of exponential and hyperbolic functions (continued)

x	e^x		e^{-x} (value)	sinh x		cosh x		tanh x (value)
	Value	$\log_{10}$		Value	$\log_{10}$	Value	$\log_{10}$	
1.00	2.7183	0.43429	0.36788	1.1752	0.07011	1.5431	0.18839	0.76159
1.01	2.7456	.43864	.36422	1.1907	.07580	1.5549	.19171	.76576
1.02	2.7732	.44298	.36060	1.2063	.08146	1.5669	.19504	.76987
1.03	2.8011	.44732	.35701	1.2220	.08708	1.5790	.19839	.77391
1.04	2.8292	.45167	.35345	1.2379	.09268	1.5913	.20176	.77789
1.05	2.8577	.45601	.34994	1.2539	.09825	1.6038	.20515	.78181
1.06	2.8864	.46035	.34646	1.2700	.10379	1.6164	.20855	.78566
1.07	2.9154	.46470	.34301	1.2862	.10930	1.6292	.21197	.78946
1.08	2.9447	.46904	.33960	1.3025	.11479	1.6421	.21541	.79320
1.09	2.9743	.47338	.33622	1.3190	.12025	1.6552	.21886	.79688
1.10	3.0042	.47772	.33287	1.3356	.12569	1.6685	.22233	.80050
1.11	3.0344	.48207	.32956	1.3524	.13111	1.6820	.22582	.80406
1.12	3.0649	.48641	.32628	1.3693	.13649	1.6956	.22931	.80757
1.13	3.0957	.49075	.32303	1.3863	.14186	1.7093	.23283	.81102
1.14	3.1268	.49510	.31982	1.4035	.14720	1.7233	.23636	.81441
1.15	3.1582	.49944	.31664	1.4208	.15253	1.7374	.23990	.81775
1.16	3.1899	.50378	.31349	1.4382	.15783	1.7517	.24346	.82104
1.17	3.2220	.50812	.31037	1.4558	.16311	1.7662	.24703	.82427
1.18	3.2544	.51247	.30728	1.4735	.16836	1.7808	.25062	.82745
1.19	3.2871	.51681	.30422	1.4914	.17360	1.7957	.25422	.83058
1.20	3.3201	.52115	.30119	1.5095	.17882	1.8107	.25784	.83365
1.21	3.3535	.52550	.29820	1.5276	.18402	1.8258	.26146	.83668
1.22	3.3872	.52984	.29523	1.5460	.18920	1.8412	.26510	.83965
1.23	3.4212	.53418	.29229	1.5645	.19437	1.8568	.26876	.84258
1.24	3.4556	.53853	.28938	1.5831	.19951	1.8725	.27242	.84546
1.25	3.4903	.54287	.28650	1.6019	.20464	1.8884	.27610	.84828
1.26	3.5254	.54721	.28365	1.6209	.20975	1.9045	.27979	.85106
1.27	3.5609	.55155	.28083	1.6400	.21485	1.9208	.28349	.85380
1.28	3.5966	.55590	.27804	1.6593	.21993	1.9373	.28721	.85648
1.29	3.6328	.56024	.27527	1.6788	.22499	1.9540	.29093	.85913
1.30	3.6693	.56458	.27253	1.6984	.23004	1.9709	.29467	.86172
1.31	3.7062	.56893	.26982	1.7182	.23507	1.9880	.29842	.86428
1.32	3.7434	.57327	.26714	1.7381	.24009	2.0053	.30217	.86678
1.33	3.7810	.57761	.26448	1.7583	.24509	2.0228	.30594	.86925
1.34	3.8190	.58195	.26185	1.7786	.25008	2.0404	.30972	.87167
1.35	3.8574	.58630	.25924	1.7991	.25505	2.0583	.31352	.87405
1.36	3.8962	.59064	.25666	1.8198	.26002	2.0764	.31732	.87639
1.37	3.9354	.59498	.25411	1.8406	.26496	2.0947	.32113	.87869
1.38	3.9749	.59933	.25158	1.8617	.26990	2.1132	.32495	.88095
1.39	4.0149	.60367	.24908	1.8829	.27482	2.1320	.32878	.88317
1.40	4.0552	.60801	.24660	1.9043	.27974	2.1509	.33262	.88535
1.41	4.0960	.61236	.24414	1.9259	.28464	2.1700	.33647	.88749
1.42	4.1371	.61670	.24171	1.9477	.28952	2.1894	.34033	.88960
1.43	4.1787	.62104	.23931	1.9697	.29440	2.2090	.34420	.89167
1.44	4.2207	.62538	.23693	1.9919	.29926	2.2288	.34807	.89370
1.45	4.2631	.62973	.23457	2.0143	.30412	2.2488	.35196	.89569
1.46	4.3060	.63407	.23224	2.0369	.30896	2.2691	.35585	.89765
1.47	4.3492	.63841	.22993	2.0597	.31379	2.2896	.35976	.89958
1.48	4.3929	.64276	.22764	2.0827	.31862	2.3103	.36367	.90147
1.49	4.4371	.64710	.22537	2.1059	.32343	2.3312	.36759	.90332
1.50	4.4817	.65144	.22313	2.1293	.32823	2.3524	.37151	.90515

Table 4-6. Values and logarithms of exponential and hyperbolic functions (continued)

x	e^x		e^{-x} (value)	$\sinh x$		$\cosh x$		$\tanh x$ (value)
	Value	$\log_{10}$		Value	$\log_{10}$	Value	$\log_{10}$	
1.50	4.4817	0.65144	0.22313	2.1293	0.32823	2.3524	0.37151	0.90515
1.51	4.5267	.65578	.22091	2.1529	.33303	2.3738	.37545	.90694
1.52	4.5722	.66013	.21871	2.1768	.33781	2.3955	.37939	.90870
1.53	4.6182	.66447	.21654	2.2008	.34258	2.4174	.38334	.91042
1.54	4.6646	.66881	.21438	2.2251	.34735	2.4395	.38730	.91212
1.55	4.7115	.67316	.21225	2.2496	.35211	2.4619	.39126	.91379
1.56	4.7588	.67750	.21014	2.2743	.35686	2.4845	.39524	.91542
1.57	4.8066	.68184	.20805	2.2993	.36160	2.5073	.39921	.91703
1.58	4.8550	.68619	.20598	2.3245	.36633	2.5305	.40320	.91860
1.59	4.9037	.69053	.20393	2.3499	.37105	2.5538	.40719	.92015
1.60	4.9530	.69487	.20190	2.3756	.37577	2.5775	.41119	.92167
1.61	5.0028	.69921	.19989	2.4015	.38048	2.6013	.41520	.92316
1.62	5.0531	.70356	.19790	2.4276	.38518	2.6255	.41921	.92462
1.63	5.1039	.70790	.19593	2.4540	.38987	2.6499	.42323	.92606
1.64	5.1552	.71224	.19398	2.4806	.39456	2.6746	.42725	.92747
1.65	5.2070	.71659	.19205	2.5075	.39923	2.6995	.43129	.92886
1.66	5.2593	.72093	.19014	2.5346	.40391	2.7247	.43532	.93022
1.67	5.3122	.72527	.18825	2.5620	.40857	2.7502	.43937	.93155
1.68	5.3656	.72961	.18637	2.5896	.41323	2.7760	.44341	.93286
1.69	5.4195	.73396	.18452	2.6175	.41788	2.8020	.44747	.93415
1.70	5.4739	.73830	.18268	2.6456	.42253	2.8283	.45153	.93541
1.71	5.5290	.74264	.18087	2.6740	.42717	2.8549	.45559	.93665
1.72	5.5845	.74699	.17907	2.7027	.43180	2.8818	.45966	.93786
1.73	5.6407	.75133	.17728	2.7317	.43643	2.9090	.46374	.93906
1.74	5.6973	.75567	.17552	2.7609	.44105	2.9364	.46782	.94023
1.75	5.7546	.76002	.17377	2.7904	.44567	2.9642	.47191	.94138
1.76	5.8124	.76436	.17204	2.8202	.45028	2.9922	.47600	.94250
1.77	5.8709	.76870	.17033	2.8503	.45488	3.0206	.48009	.94361
1.78	5.9299	.77304	.16864	2.8806	.45948	3.0492	.48419	.94470
1.79	5.9895	.77739	.16696	2.9112	.46408	3.0782	.48830	.94576
1.80	6.0496	.78173	.16530	2.9422	.46867	3.1075	.49241	.94681
1.81	6.1104	.78607	.16365	2.9734	.47325	3.1371	.49652	.94783
1.82	6.1719	.79042	.16203	3.0049	.47783	3.1669	.50064	.94884
1.83	6.2339	.79476	.16041	3.0367	.48241	3.1972	.50476	.94983
1.84	6.2965	.79910	.15882	3.0689	.48698	3.2277	.50889	.95080
1.85	6.3598	.80344	.15724	3.1013	.49154	3.2585	.51302	.95175
1.86	6.4237	.80779	.15567	3.1340	.49610	3.2897	.51716	.95268
1.87	6.4883	.81213	.15412	3.1671	.50066	3.3212	.52130	.95359
1.88	6.5535	.81647	.15259	3.2005	.50521	3.3530	.52544	.95449
1.89	6.6194	.82082	.15107	3.2341	.50976	3.3852	.52959	.95537
1.90	6.6859	.82516	.14957	3.2682	.51430	3.4177	.53374	.95624
1.91	6.7531	.82950	.14808	3.3025	.51884	3.4506	.53789	.95709
1.92	6.8210	.83385	.14661	3.3372	.52338	3.4838	.54205	.95792
1.93	6.8895	.83819	.14515	3.3722	.52791	3.5173	.54621	.95873
1.94	6.9588	.84253	.14370	3.4075	.53244	3.5512	.55038	.95953
1.95	7.0287	.84687	.14227	3.4432	.53696	3.5855	.55455	.96032
1.96	7.0993	.85122	.14086	3.4792	.54148	3.6201	.55872	.96109
1.97	7.1707	.85556	.13946	3.5156	.54600	3.6551	.56290	.96185
1.98	7.2427	.85990	.13807	3.5523	.55051	3.6904	.56707	.96259
1.99	7.3155	.86425	.13670	3.5894	.55502	3.7261	.57126	.96331
2.00	7.3891	.86859	.13534	3.6269	.55953	3.7622	.57544	.96403

Table 4-6. Values and logarithms of exponential and hyperbolic functions (continued)

x	e^x		e^{-x} (value)	sinh x		cosh x		tanh x (value)
	Value	$\log_{10}$		Value	$\log_{10}$	Value	$\log_{10}$	
2.00	7.3891	0.86859	0.13534	3.6269	0.55953	3.7622	0.57544	0.96403
2.01	7.4633	0.87293	.13399	3.6647	.56403	3.7987	.57963	.96473
2.02	7.5383	0.87727	.13266	3.7028	.56853	3.8355	.58382	.96541
2.03	7.6141	0.88162	.13134	3.7414	.57303	3.8727	.58802	.96609
2.04	7.6906	0.88596	.13003	3.7803	.57753	3.9103	.59221	.96675
2.05	7.7679	0.89030	.12873	3.8196	.58202	3.9483	.59641	.96740
2.06	7.8460	0.89465	.12745	3.8593	.58650	3.9867	.60061	.96803
2.07	7.9248	0.89899	.12619	3.8993	.59099	4.0255	.60482	.96865
2.08	8.0045	0.90333	.12493	3.9398	.59547	4.0647	.60903	.96926
2.09	8.0849	0.90768	.12369	3.9806	.59995	4.1043	.61324	.96986
2.10	8.1662	0.91202	.12246	4.0219	.60443	4.1443	.61745	.97045
2.11	8.2482	0.91636	.12124	4.0635	.60890	4.1847	.62167	.97103
2.12	8.3311	0.92070	.12003	4.1056	.61337	4.2256	.62589	.97159
2.13	8.4149	0.92505	.11884	4.1480	.61784	4.2669	.63011	.97215
2.14	8.4994	0.92939	.11765	4.1909	.62231	4.3085	.63433	.97269
2.15	8.5849	0.93373	.11648	4.2342	.62677	4.3507	.63856	.97323
2.16	8.6711	0.93808	.11533	4.2779	.63123	4.3932	.64278	.97375
2.17	8.7583	0.94242	.11418	4.3221	.63569	4.4362	.64701	.97426
2.18	8.8463	0.94676	.11304	4.3666	.64015	4.4797	.65125	.97477
2.19	8.9352	0.95110	.11192	4.4116	.64460	4.5236	.65548	.97526
2.20	9.0250	0.95545	.11080	4.4571	.64905	4.5679	.65972	.97574
2.21	9.1157	0.95979	.10970	4.5030	.65350	4.6127	.66396	.97622
2.22	9.2073	0.96413	.10861	4.5494	.65795	4.6580	.66820	.97668
2.23	9.2999	0.96848	.10753	4.5962	.66240	4.7037	.67244	.97714
2.24	9.3933	0.97282	.10646	4.6434	.66684	4.7499	.67668	.97759
2.25	9.4877	0.97716	.10540	4.6912	.67128	4.7966	.68093	.97803
2.26	9.5831	0.98151	.10435	4.7394	.67572	4.8437	.68518	.97846
2.27	9.6794	0.98585	.10331	4.7880	.68016	4.8914	.68943	.97888
2.28	9.7767	0.99019	.10228	4.8372	.68459	4.9395	.69368	.97929
2.29	9.8749	0.99453	.10127	4.8868	.68903	4.9881	.69794	.97970
2.30	9.9742	0.99888	.10026	4.9370	.69346	5.0372	.70219	.98010
2.31	10.074	1.00322	.09926	4.9876	.69789	5.0868	.70645	.98049
2.32	10.176	1.00756	.09827	5.0387	.70232	5.1370	.71071	.98087
2.33	10.278	1.01191	.09730	5.0903	.70675	5.1876	.71497	.98124
2.34	10.381	1.01625	.09633	5.1425	.71117	5.2388	.71923	.98161
2.35	10.486	1.02059	.09537	5.1951	.71559	5.2905	.72349	.98197
2.36	10.591	1.02493	.09442	5.2483	.72002	5.3427	.72776	.98233
2.37	10.697	1.02928	.09348	5.3020	.72444	5.3954	.73203	.98267
2.38	10.805	1.03362	.09255	5.3562	.72885	5.4487	.73630	.98301
2.39	10.913	1.03796	.09163	5.4109	.73327	5.5026	.74056	.98335
2.40	11.023	1.04231	.09072	5.4662	.73769	5.5569	.74484	.98367
2.41	11.134	1.04665	.08982	5.5221	.74210	5.6119	.74911	.98400
2.42	11.246	1.05099	.08892	5.5785	.74652	5.6674	.75338	.98431
2.43	11.359	1.05534	.08804	5.6354	.75093	5.7235	.75766	.98462
2.44	11.473	1.05968	.08716	5.6929	.75534	5.7801	.76194	.98492
2.45	11.588	1.06402	.08629	5.7510	.75975	5.8373	.76621	.98522
2.46	11.705	1.06836	.08543	5.8097	.76415	5.8951	.77049	.98551
2.47	11.822	1.07271	.08458	5.8689	.76856	5.9535	.77477	.98579
2.48	11.941	1.07705	.08374	5.9288	.77296	6.0125	.77906	.98607
2.49	12.061	1.08139	.08291	5.9892	.77737	6.0721	.78334	.98635
2.50	12.182	1.08574	.08208	6.0502	.78177	6.1323	.78762	.98661

Table 4-6. Values and logarithms of exponential and hyperbolic functions (continued)

x	e^x		e^{-x} (value)	sinh x		cosh x		tanh x (value)
	Value	$\log_{10}$		Value	$\log_{10}$	Value	$\log_{10}$	
2.50	12.182	1.03574	0.08208	6.0502	0.78177	6.1323	0.78762	0.98661
2.51	12.305	1.09008	.08127	6.1118	.78617	6.1931	.79191	.93598
2.52	12.429	1.09442	.08046	6.1741	.79057	6.2545	.79619	.98714
2.53	12.554	1.09877	.07966	6.2369	.79497	6.3166	.80048	.98739
2.54	12.680	1.10311	.07887	6.3004	.79937	6.3793	.80477	.98764
2.55	12.807	1.10745	.07808	6.3645	.80377	6.4426	.80906	.98788
2.56	12.936	1.11179	.07730	6.4293	.80816	6.5066	.81335	.98812
2.57	13.066	1.11614	.07654	6.4946	.81256	6.5712	.81764	.98835
2.58	13.197	1.12048	.07577	6.5607	.81695	6.6365	.82194	.98858
2.59	13.330	1.12482	.07502	6.6274	.82134	6.7024	.82623	.98881
2.60	13.464	1.12917	.07427	6.6947	.82573	6.7690	.83052	.98903
2.61	13.599	1.13351	.07353	6.7628	.83012	6.8363	.83482	.93924
2.62	13.736	1.13785	.07280	6.8315	.83451	6.9043	.83912	.93946
2.63	13.874	1.14219	.07208	6.9008	.83890	6.9729	.84341	.98966
2.64	14.013	1.14654	.07136	6.9709	.84329	7.0423	.84771	.98987
2.65	14.154	1.15088	.07065	7.0417	.84768	7.1123	.85201	.99007
2.66	14.296	1.15522	.06995	7.1132	.85206	7.1831	.85631	.99026
2.67	14.440	1.15957	.06925	7.1854	.85645	7.2546	.86061	.99045
2.68	14.585	1.16391	.06856	7.2583	.86083	7.3268	.86492	.99064
2.69	14.732	1.16825	.06788	7.3319	.86522	7.3998	.86922	.99083
2.70	14.880	1.17260	.06721	7.4063	.86960	7.4735	.87352	.99101
2.71	15.029	1.17694	.06654	7.4814	.87398	7.5479	.87783	.99118
2.72	15.180	1.18128	.06587	7.5572	.87836	7.6231	.88213	.99136
2.73	15.333	1.18562	.06522	7.6338	.88274	7.6991	.88644	.99153
2.74	15.487	1.18997	.06457	7.7112	.88712	7.7758	.89074	.99170
2.75	15.643	1.19431	.06393	7.7894	.89150	7.8533	.89505	.99186
2.76	15.800	1.19865	.06329	7.8683	.89588	7.9316	.89936	.99202
2.77	15.959	1.20300	.06266	7.9480	.90026	8.0106	.90367	.99218
2.78	16.119	1.20734	.06204	8.0285	.90463	8.0905	.90798	.99233
2.79	16.281	1.21168	.06142	8.1098	.90901	8.1712	.91229	.99248
2.80	16.445	1.21602	.06081	8.1919	.91339	8.2527	.91660	.99263
2.81	16.610	1.22037	.06020	8.2749	.91776	8.3351	.92091	.99278
2.82	16.777	1.22471	.05961	8.3586	.92213	8.4182	.92522	.99292
2.83	16.945	1.22905	.05901	8.4432	.92651	8.5022	.92953	.99306
2.84	17.116	1.23340	.05843	8.5287	.93088	8.5871	.93385	.99320
2.85	17.288	1.23774	.05784	8.6150	.93525	8.6728	.93816	.99333
2.86	17.462	1.24208	.05727	8.7021	.93963	8.7594	.94247	.99346
2.87	17.637	1.24643	.05670	8.7902	.94400	8.8469	.94679	.99359
2.88	17.814	1.25077	.05613	8.8791	.94837	8.9352	.95110	.99372
2.89	17.993	1.25511	.05558	8.9689	.95274	9.0244	.95542	.99384
2.90	18.174	1.25945	.05502	9.0596	.95711	9.1146	.95974	.99396
2.91	18.357	1.26380	.05448	9.1512	.96148	9.2056	.96405	.99408
2.92	18.541	1.26814	.05393	9.2437	.96584	9.2976	.96837	.99420
2.93	18.728	1.27248	.05340	9.3371	.97021	9.3905	.97269	.99431
2.94	18.916	1.27683	.05287	9.4315	.97458	9.4844	.97701	.99443
2.95	19.106	1.28117	.05234	9.5268	.97895	9.5791	.98133	.99454
2.96	19.298	1.28551	.05182	9.6231	.98331	9.6749	.98565	.99464
2.97	19.492	1.28985	.05130	9.7203	.98763	9.7716	.98997	.99475
2.98	19.688	1.29420	.05079	9.8185	.99205	9.8693	.99429	.99485
2.99	19.886	1.29854	.05029	9.9177	.99641	9.9680	.99861	.99496
3.00	20.086	1.30288	.04979	10.018	1.00078	10.068	1.00293	.99505

Table 4-6. Values and logarithms of exponential and hyperbolic functions (continued)

x	e^x		e^{-x} (value)	sinh x		cosh x		tanh x (value)
	Value	log₁₀		Value	log₁₀	Value	log₁₀	
3.00	20.086	1.30288	0.04979	10.018	1.00078	10.068	1.00293	0.99505
3.05	21.115	1.32460	.04736	10.534	1.02259	10.581	1.02454	0.99552
3.10	22.198	1.34631	.04505	11.076	1.04440	11.122	1.04616	0.99595
3.15	23.336	1.36803	.04285	11.647	1.06620	11.690	1.06779	0.99633
3.20	24.533	1.38974	.04076	12.246	1.08799	12.287	1.08943	0.99668
3.25	25.790	1.41146	.03877	12.876	1.10977	12.915	1.11108	0.99700
3.30	27.113	1.43317	.03688	13.538	1.13155	13.575	1.13273	0.99728
3.35	28.503	1.45489	.03508	14.234	1.15332	14.269	1.15439	0 99754
3.40	29.964	1.47660	.03337	14.965	1.17509	14.999	1.17605	0.99777
3.45	31.500	1.49832	.03175	15.734	1.19685	15.766	1.19772	0.99799
3.50	33.115	1.52003	.03020	16.543	1.21860	16.573	1.21940	0.99818
3.55	34.813	1.54175	.02872	17.392	1.24036	17.421	1.24107	0.99835
3.60	36.598	1.56346	.02732	18.286	1.26211	18.313	1.26275	0.99851
3.65	38.475	1.58517	.02599	19.224	1.28385	19.250	1.28444	0.99865
3.70	40.447	1.60689	.02472	20.211	1.30559	20.236	1.30612	0.99878
3.75	42.521	1.62860	.02352	21.249	1.32733	21.272	1.32781	0.99889
3.80	44.701	1.65032	.02237	22.339	1.34907	22.362	1.34951	0.99900
3.85	46.993	1.67203	.02128	23.486	1.37081	23.507	1.37120	0.99909
3.90	49.402	1.69375	.02024	24.691	1.39254	24.711	1.39290	0.99918
3.95	51.935	1.71546	.01925	25.958	1.41427	25.977	1.41459	0.99926
4.00	54.598	1.73718	.01832	27.290	1.43600	27.308	1.43629	0.99933
4.10	60.340	1.78061	.01657	30.162	1.47946	30.178	1.47970	0.99945
4.20	66.686	1.82404	.01500	33.336	1.52291	33.351	1.52310	0.99955
4.30	73.700	1.86747	.01357	36.843	1.56636	36.857	1.56652	0.99963
4.40	81.451	1.91090	.01227	40.719	1.60980	40.732	1.60993	0.99970
4.50	90.017	1.95433	.01111	45.003	1.65324	45.014	1.65335	0.99975
4.60	99.484	1.99775	.01005	49.737	1.69668	49.747	1.69677	0.99980
4.70	109.95	2.04118	.00910	54.969	1.74012	54.978	1.74019	0.99983
4.80	121.51	2.08461	.00823	60.751	1.78355	60.759	1.78361	0.99986
4.90	134.29	2.12804	.00745	67.141	1.82699	67.149	1.82704	0.99989
5.00	148.41	2.17147	.00674	74.203	1.87042	74.210	1.87046	0.99991
5.10	164.02	2.21490	.00610	82.008	1.91389	82.014	1.91389	0.99993
5.20	181.27	2.25833	.00552	90.633	1.95729	90.639	1.95731	0.99994
5.30	200.34	2.30176	.00499	100.17	2.00074	100.17	2.00074	0.99995
5.40	221.41	2.34519	.00452	110.70	2.04415	110.71	2.04417	0.99996
5.50	244.69	2.38862	.00409	122.34	2.08758	122.35	2.08760	0.99997
5.60	270.43	2.43205	.00370	135.21	2.13101	135.22	2.13103	0.99997
5.70	298.87	2.47548	.00335	149.43	2.17444	149.44	2.17445	0.99998
5.80	330.30	2.51891	.00303	165.15	2.21787	165.15	2.21788	0.99998
5.90	365.04	2.56234	.00274	182.52	2.26130	182.52	2.26131	0.99998
6.00	403.43	2.60577	.00248	201.71	2.30473	201.72	2.30474	0.99999
6.25	518.01	2.71434	.00193	259.01	2.41331	259.01	2.41331	0.99999
6.50	665.14	2.82291	.00150	332.57	2.52188	332.57	2.52189	1.00000
6.75	854.06	2.93149	.00117	427.03	2.63046	427.03	2.63046	1.00000
7.00	1096.6	3.04006	.00091	548.32	2.73903	548.32	2.73903	1.00000
7.50	1808.0	3.25721	.00055	904.02	2.95618	904.02	2.95618	1.00000
8.00	2981.0	3.47436	.00034	1490.5	3.17333	1490.5	3.17333	1.00000
8.50	4914.8	3.69150	.00020	2457.4	3.39047	2457.4	3.39047	1.00000
9.00	8103.1	3.90865	.00012	4051.5	3.60762	4051.5	3.60762	1.00000
9.50	13360.	4.12580	.00007	6679.9	3.82477	6679.9	3.82477	1.00000
10.00	22026.	4.34294	.00005	11013.	4.04191	11013.	4.04191	1.00000

Table 4-7. Gamma and factorial functions: $\Gamma(x) = (y)!$*

x	y	0	1	2	3	4	5	6	7	8	9
1.00	0.00	1.0000	0.9994	9988	9983	9977	9971	9966	9960	9954	9949
1	1	0.9943	9938	9932	9927	9921	9916	9910	9905	9899	9894
2	2	9888	9883	9878	9872	9867	9862	9856	9851	9846	9841
3	3	9835	9830	9825	9820	9815	9810	9805	9800	9794	9789
4	4	9784	9779	9774	9769	9764	9759	9755	9750	9745	9740
5	5	9735	9730	9725	9721	9716	9711	9706	9702	9697	9692
6	6	9687	9683	9678	9673	9669	9664	9660	9655	9651	9646
7	7	9642	9637	9633	9628	9624	9619	9615	9610	9606	9602
8	8	9597	9593	9589	9584	9580	9576	9571	9567	9563	9559
9	9	9555	9550	9546	9542	9538	9534	9530	9526	9522	9518
1.10	0.10	9514	9509	9505	9501	9498	9494	9490	9486	9482	9478
1	1	9474	9470	9466	9462	9459	9455	9451	9447	9443	9440
2	2	9436	9432	9428	9425	9421	9417	9414	9410	9407	9403
3	3	9399	9396	9392	9389	9385	9382	9378	9375	9371	9368
4	4	9364	9361	9357	9354	9350	9347	9344	9340	9337	9334
5	5	9330	9327	9324	9321	9317	9314	9311	9308	9304	9301
6	6	9298	9295	9292	9289	9285	9282	9279	9276	9273	9270
7	7	9267	9264	9261	9258	9255	9252	9249	9246	9243	9240
8	8	9237	9234	9231	9229	9226	9223	9220	9217	9214	9212
9	9	9209	9206	9203	9201	9198	9195	9192	9190	9187	9184
1.20	0.20	9182	9179	9176	9174	9171	9169	9166	9163	9161	9158
1	1	9156	9153	9151	9148	9146	9143	9141	9138	9136	9133
2	2	9131	9129	9126	9124	9122	9119	9117	9114	9112	9110
3	3	9108	9105	9103	9101	9098	9096	9094	9092	9090	9087
4	4	9085	9083	9081	9079	9077	9074	9072	9070	9068	9066
5	5	9064	9062	9060	9058	9056	9054	9052	9050	9048	9046
6	6	9044	9042	9040	9038	9036	9034	9032	9031	9029	9027
7	7	9025	9023	9021	9020	9018	9016	9014	9012	9011	9009
8	8	9007	9005	9004	9002	9000	8999	8997	8995	8994	8992
9	9	8990	8989	8987	8986	8984	8982	8981	8979	8978	8976
1.30	0.30	8975	8973	8972	8970	8969	8967	8966	8964	8963	8961
1	1	8960	8959	8957	8956	8954	8953	8952	8950	8949	8948
2	2	8946	8945	8944	8943	8941	8940	8939	8937	8936	8935
3	3	8934	8933	8931	8930	8929	8928	8927	8926	8924	8923
4	4	8922	8921	8920	8919	8918	8917	8916	8915	8914	8913
5	5	8912	8911	8910	8909	8908	8907	8906	8905	8904	8903
6	6	8902	8901	8900	8899	8898	8897	8897	8896	8895	8894
7	7	8893	8892	8892	8891	8890	8889	8888	8888	8887	8886
8	8	8885	8885	8884	8883	8883	8882	8881	8880	8880	8879
9	9	8879	8878	8877	8877	8876	8875	8875	8874	8874	8873
1.40	0.40	8873	8872	8872	8871	8871	8870	8870	8869	8869	8868
1	1	8868	8867	8867	8866	8866	8865	8865	8865	8864	8864
2	2	8864	8863	8863	8863	8862	8862	8862	8861	8861	8861
3	3	8860	8860	8860	8860	8859	8859	8859	8859	8858	8858
4	4	8858	8858	8858	8858	8857	8857	8857	8857	8857	8857
5	5	8857	8857	8856	8856	8856	8856	8856	8856	8856	8856
6	6	8856	8856	8856	8856	8856	8856	8856	8856	8856	8856
7	7	8856	8856	8856	8857	8857	8857	8857	8857	8857	8857
8	8	8857	8858	8858	8858	8858	8858	8859	8859	8859	8859
9	9	8859	8860	8860	8860	8860	8861	8861	8861	8862	8862
1.50	0.50	8862	8863	8863	8863	8864	8864	8864	8865	8865	8866

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt = (x-1)! \qquad y! = \int_0^\infty t^y e^{-t}\,dt = \Gamma(1+y)$$

For higher values of argument

$$\Gamma(x) = (x-1)\Gamma(x-1) = (x-1)(x-2)\Gamma(x-2) = \cdots$$

$$x! = x(x-1)! = x(x-1)(x-2)! = \cdots$$

Example: $\Gamma(4.7) = (3.7)! = 3.7 \times 2.7 \times 1.7 \times 0.9086 = 15.43.$

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* From H. E. Etherington, "Nuclear Engineering Handbook," McGraw-Hill Book Company, New York, 1958.

Table 4-7. Gamma and factorial functions: $\Gamma(x) = y!$ (continued)

x	y	0	1	2	3	4	5	6	7	8	9
1.50	0.50	0.8862	8863	8863	8863	8864	8864	8864	8865	8865	8866
1	1	8866	8866	8867	8867	8868	8868	8869	8869	8869	8870
2	2	8870	8871	8871	8872	8872	8873	8873	8874	8875	8875
3	3	8876	8876	8877	8877	8878	8879	8879	8880	8880	8881
4	4	8882	8882	8883	8884	8884	8885	8886	8887	8887	8888
5	5	8889	8889	8890	8891	8892	8892	8893	8894	8895	8896
6	6	8896	8897	8898	8899	8900	8901	8901	8902	8903	8904
7	7	8905	8906	8907	8908	8909	8909	8910	8911	8912	8913
8	8	8914	8915	8916	8917	8918	8919	8920	8921	8922	8923
9	9	8924	8925	8926	8927	8929	8930	8931	8932	8933	8934
1.60	60	8935	8936	8937	8939	8940	8941	8942	8943	8944	8946
1	1	8947	8948	8949	8950	8952	8953	8954	8955	8957	8958
2	2	8959	8961	8962	8963	8964	8966	8967	8968	8970	8971
3	3	8972	8974	8975	8977	8978	8979	8981	8982	8984	8985
4	4	8986	8988	8989	8991	8992	8994	8995	8997	8998	9000
5	5	9001	9003	9004	9006	9007	9009	9010	9012	9014	9015
6	6	9017	9018	9020	9021	9023	9025	9026	9028	9030	9031
7	7	9033	9035	9036	9038	9040	9041	9043	9045	9047	9048
8	8	9050	9052	9054	9055	9057	9059	9061	9062	9064	9066
9	9	9068	9070	9071	9073	9075	9077	9079	9081	9083	9084
1.70	70	9086	9088	9090	9092	9094	9096	9098	9100	9102	9104
1	1	9106	9108	9110	9112	9114	9116	9118	9120	9122	9124
2	2	9126	9128	9130	9132	9134	9136	9138	9140	9142	9145
3	3	9147	9149	9151	9153	9155	9157	9160	9162	9164	9166
4	4	9168	9170	9173	9175	9177	9179	9182	9184	9186	9188
5	5	9191	9193	9195	9197	9200	9202	9204	9207	9209	9211
6	6	9214	9216	9218	9221	9223	9226	9228	9230	9233	9235
7	7	9238	9240	9242	9245	9247	9250	9252	9255	9257	9260
8	8	9262	9265	9267	9270	9272	9275	9277	9280	9283	9285
9	9	9288	9290	9293	9295	9298	9301	9303	9306	9309	9311
1.80	80	9314	9316	9319	9322	9325	9327	9330	9333	9335	9338
1	1	9341	9343	9346	9349	9352	9355	9357	9360	9363	9366
2	2	9368	9371	9374	9377	9380	9383	9385	9388	9291	9394
3	3	9397	9400	9403	9406	9408	9411	9414	9417	9420	9423
4	4	9426	9429	9432	9435	9438	9441	9444	9447	9450	9453
5	5	9456	9459	9462	9465	9468	9471	9474	9478	9481	9484
6	6	9487	9490	9493	9496	9499	9503	9506	9509	9512	9515
7	7	9518	9522	9525	9528	9531	9534	9538	9541	9544	9547
8	8	9551	9554	9557	9561	9564	9567	9570	9574	9577	9580
9	9	9584	9587	9591	9594	9597	9601	9604	9607	9611	9614
1.90	90	9618	9621	9625	9628	9631	9635	9638	9642	9645	9649
1	1	9652	9656	9659	9663	9666	9670	9673	9677	9681	9684
2	2	9688	9691	9695	9699	9702	9706	9709	9713	9717	9720
3	3	9724	9728	9731	9735	9739	9742	9746	9750	9754	9757
4	4	9761	9765	9768	9772	9776	9780	9784	9787	9791	9795
5	5	9799	9803	9806	9810	9814	9818	9822	9826	9830	9834
6	6	9837	9841	9845	9849	9853	9857	9861	9865	9869	9873
7	7	9877	9881	9885	9889	9893	9897	9901	9905	9909	9913
8	8	9917	9921	9925	9929	9933	9938	9942	9946	9950	9954
9	9	9958	9962	9966	9971	9975	9979	9983	9987	9992	9996
2.00	1.00	1.0000	0004	0008	0013	0017	0021	0026	0030	0034	0038

$\Gamma(3/2) = (1/2)! = 1/2\,\sqrt{\pi}$ $\quad\Gamma(1/2) = (-1/2)! = \sqrt{\pi}$

Minimum value of the function: $\Gamma(1.461632) = (0.461632)! = 0.885603$.

Table 4-8*a*. Binomial coefficients*

N	$C(N,0)$	$C(N,1)$	$C(N,2)$	$C(N,3)$	$C(N,4)$	$C(N,5)$	$C(N,6)$	$C(N,7)$	$C(N,8)$	$C(N,9)$	$C(N,10)$
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66
13	1	13	78	286	715	1287	1716	1716	1287	715	286
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756

Table 4-8*b*. Factorials*

N	$N!$	N	$N!$	N	$N!$	N	$N!$	N	$N!$
1	1.0000	21	5.1091(19)	41	3.3453(49)	61	5.0758(83)	81	5.7971(120)
2	2.0000	22	1.1240(21)	42	1.4050(51)	62	3.1470(85)	82	4.7536(122)
3	6.0000	23	2.5852(22)	43	6.0415(52)	63	1.9826(87)	83	3.9455(124)
4	2.4000(1)	24	6.2045(23)	44	2.6583(54)	64	1.2689(89)	84	3.3142(126)
5	1.2000(2)	25	1.5511(25)	45	1.1962(56)	65	8.2477(90)	85	2.8171(128)
6	7.2000(2)	26	4.0329(26)	46	5.5026(57)	66	5.4434(92)	86	2.4227(130)
7	5.0400(3)	27	1.0889(28)	47	2.5862(59)	67	3.6471(94)	87	2.1078(132)
8	4.0320(4)	28	3.0489(29)	48	1.2414(61)	68	2.4800(96)	88	1.8548(134)
9	3.6288(5)	29	8.8418(30)	49	6.0828(62)	69	1.7112(98)	89	1.6508(136)
10	3.6288(6)	30	2.6525(32)	50	3.0414(64)	70	1.1979(100)	90	1.4857(138)
11	3.9917(7)	31	8.2228(33)	51	1.5511(66)	71	8.5048(101)	91	1.3520(140)
12	4.7900(8)	32	2.6313(35)	52	8.0658(67)	72	6.1234(103)	92	1.2438(142)
13	6.2270(9)	33	8.6833(36)	53	4.2749(69)	73	4.4701(105)	93	1.1568(144)
14	8.7178(10)	34	2.9523(38)	54	2.3084(71)	74	3.3079(107)	94	1.0874(146)
15	1.3077(12)	35	1.0333(40)	55	1.2696(73)	75	2.4809(109)	95	1.0330(148)
16	2.0923(13)	36	3.7199(41)	56	7.1100(74)	76	1.8855(111)	96	9.9168(149)
17	3.5569(14)	37	1.3764(43)	57	4.0527(76)	77	1.4518(113)	97	9.6193(151)
18	6.4024(15)	38	5.2302(44)	58	2.3506(78)	78	1.1324(115)	98	9.4269(153)
19	1.2165(17)	39	2.0398(46)	59	1.3868(80)	79	8.9462(116)	99	9.3326(155)
20	2.4329(18)	40	8.1592(47)	60	8.3210(81)	80	7.1569(118)	100	9.3326(157)

The number in parentheses indicates the power of 10 by which the entry must be multiplied. For example, 8! = 40,320.

* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 4-9. Logarithms of factorials*

	0	1	2	3	4	5	6	7	8	9
00	0.0000	0.0000	0.3010	0.7782	1.3802	2.0792	2.8573	3.7024	4.6055	5.5598
10	6.5598	7.6012	8.6803	9.7943	10.9404	12.1165	13.3206	14.5511	15.8063	17.0851
20	18.3861	19.7083	21.0508	22.4125	23.7927	25.1906	26.6056	28.0370	29.4841	30.9465
30	32.4237	33.9150	35.4202	36.9387	38.4702	40.0142	41.5705	43.1387	44.7185	46.3096
40	47.9116	49.5244	51.1477	52.7811	54.4246	56.0778	57.7406	59.4127	61.0939	62.7841
50	64.4831	66.1906	67.9066	69.6309	71.3633	73.1037	74.8519	76.6077	78.3712	80.1420
60	81.9202	83.7055	85.4979	87.2972	89.1034	90.9163	92.7359	94.5619	96.3945	98.2333
70	100.0784	101.9297	103.7870	105.6503	107.5196	109.3946	111.2754	113.1619	115.0540	116.9516
80	118.8547	120.7632	122.6770	124.5961	126.5204	128.4498	130.3843	132.3238	134.2683	136.2177
90	138.1719	140.1310	142.0948	144.0632	146.0364	148.0141	149.9964	151.9831	153.9744	155.9700
100	157.9700	159.9743	161.9829	163.9958	166.0128	168.0340	170.0593	172.0887	174.1221	176.1595
110	178.2009	180.2462	182.2955	184.3485	186.4054	188.4661	190.5306	192.5988	194.6707	196.7462
120	198.8254	200.9082	202.9945	205.0844	207.1779	209.2748	211.3751	213.4790	215.5862	217.6967
130	219.8107	221.9280	224.0485	226.1724	228.2995	230.4298	232.5634	234.7001	236.8400	238.9830
140	241.1291	243.2783	245.4306	247.5860	249.7443	251.9057	254.0700	256.2374	258.4076	260.5808
150	262.7569	264.9359	267.1177	269.3024	271.4899	273.6803	275.8734	278.0693	280.2679	282.4693
160	284.6735	286.8803	289.0898	291.3020	293.5168	295.7343	297.9544	300.1771	302.4024	304.6303
170	306.8608	309.0938	311.3293	313.5674	315.8079	318.0509	320.2965	322.5444	324.7948	327.0477
180	329.3030	331.5606	333.8207	336.0832	338.3480	340.6152	342.8847	345.1565	347.4307	349.7071
190	351.9859	354.2669	356.5502	358.8358	361.1236	363.4136	365.7059	368.0003	370.2970	372.5959

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* From E. L. Grant, "Statistical Quality Control," 3d ed., McGraw-Hill Book Company, New York, 1964.

Table 4-9. **Logarithms of factorials** (continued)

	0	1	2	3	4	5	6	7	8	9
200	374.8969	377.2001	379.5054	381.8129	384.1226	386.4343	388.7482	391.0642	393.3822	395.7024
210	398.0246	400.3489	402.6752	405.0036	407.3340	409.6664	412.0009	414.3373	416.6758	419.0162
220	421.3587	423.7031	426.0494	428.3977	430.7480	433.1002	435.4543	437.8103	440.1682	442.5281
230	444.8898	447.2534	449.6189	451.9862	454.3555	456.7265	459.0994	461.4742	463.8508	466.2292
240	468.6094	470.9914	473.3752	475.7608	478.1482	480.5374	482.9283	485.3210	487.7154	490.1116
250	492.5096	494.9093	497.3107	499.7138	502.1186	504.5252	506.9334	509.3433	511.7549	514.1682
260	516.5832	518.9999	521.4182	523.8381	526.2597	528.6830	531.1078	533.5344	535.9625	538.3922
270	540.8236	543.2566	545.6912	548.1273	550.5651	553.0044	555.4453	557.8878	560.3318	562.7774
280	565.2246	567.6733	570.1235	572.5753	575.0287	577.4835	579.9399	582.3977	584.8571	587.3180
290	589.7804	592.2443	594.7097	597.1766	599.6449	602.1147	604.5860	607.0588	609.5330	612.0087
300	614.4858	616.9644	619.4444	621.9258	624.4087	626.8930	629.3787	631.8659	634.3544	636.8444
310	639.3357	641.8285	644.3226	646.8182	649.3151	651.8134	654.3131	656.8142	659.3166	661.8204
320	664.3255	666.8320	669.3399	671.8491	674.3596	676.8715	679.3847	681.8993	684.4152	686.9324
330	689.4509	691.9707	694.4918	697.0143	699.5380	702.0631	704.5894	707.1170	709.6460	712.1762
340	714.7076	717.2404	719.7744	722.3097	724.8463	727.3841	729.9232	732.4635	735.0051	737.5479
350	740.0920	742.6373	745.1838	747.7316	750.2806	752.8308	755.3823	757.9349	760.4888	763.0439
360	765.6002	768.1577	770.7164	773.2764	775.8375	778.3997	780.9632	783.5279	786.0937	788.6608
370	791.2290	793.7983	796.3689	798.9406	801.5135	804.0875	806.6627	809.2390	811.8165	814.3952
380	816.9749	819.5559	822.1379	824.7211	827.3055	829.8909	832.4775	835.0652	837.6540	840.2440
390	842.8351	845.4272	848.0205	850.6149	853.2104	855.8070	858.4047	861.0035	863.6034	866.2044

Table 4-9. Logarithms of factorials (continued)

	0	1	2	3	4	5	6	7	8	9
400	868.8064	871.4096	874.0138	876.6191	879.2255	881.8329	884.4415	887.0510	889.6617	892.2734
410	894.8862	897.5001	900.1150	902.7309	905.3479	907.9660	910.5850	913.2052	915.8264	918.4486
420	921.0718	923.6961	926.3214	928.9478	931.5751	934.2035	936.8329	939.4633	942.0948	944.7272
430	947.3607	949.9952	952.6307	955.2672	957.9047	960.5431	963.1826	965.8231	968.4646	971.1071
440	973.7505	976.3949	979.0404	981.6868	984.3342	986.9825	989.6318	992.2822	994.9334	997.5857
450	1000.2389	1002.8931	1005.5482	1008.2043	1010.8614	1013.5194	1016.1783	1018.8383	1021.4991	1024.1609
460	1026.8237	1029.4874	1032.1520	1034.8176	1037.4841	1040.1516	1042.8200	1045.4893	1048.1595	1050.8307
470	1053.5028	1056.1758	1058.8498	1061.5246	1064.2004	1066.8771	1069.5547	1072.2332	1074.9127	1077.5930
480	1080.2742	1082.9564	1085.6394	1088.3234	1091.0082	1093.6940	1096.3806	1099.0681	1101.7565	1104.4458
490	1107.1360	1109.8271	1112.5191	1115.2119	1117.9057	1120.6003	1123.2958	1125.9921	1128.6893	1131.3874
500	1134.0864	1136.7862	1139.4869	1142.1885	1144.8909	1147.5942	1150.2984	1153.0034	1155.7093	1158.4160
510	1161.1236	1163.8320	1166.5412	1169.2514	1171.9623	1174.6741	1177.3868	1180.1003	1182.8146	1185.5298
520	1188.2458	1190.9626	1193.6803	1196.3988	1199.1181	1201.8383	1204.5593	1207.2811	1210.0037	1212.7272
530	1215.4514	1218.1765	1220.9024	1223.6292	1226.3567	1229.0851	1231.8142	1234.5442	1237.2750	1240.0066
540	1242.7390	1245.4722	1248.2062	1250.9410	1253.6766	1256.4130	1259.1501	1261.8881	1264.6269	1267.3665
550	1270.1069	1272.8480	1275.5899	1278.3327	1281.0762	1283.8205	1286.5655	1289.3114	1292.0580	1294.8054
560	1297.5536	1300.3026	1303.0523	1305.8028	1308.5541	1311.3062	1314.0590	1316.8126	1319.5669	1322.3220
570	1325.0779	1327.8345	1330.5919	1333.3501	1336.1090	1338.8687	1341.6291	1344.3903	1347.1522	1349.9149
580	1352.6783	1355.4425	1358.2074	1360.9731	1363.7395	1366.5066	1369.2745	1372.0432	1374.8126	1377.5827
590	1380.3535	1383.1251	1385.8974	1388.6705	1391.4443	1394.2188	1396.9940	1399.7700	1402.5467	1405.3241

Table 4-9. Logarithms of factorials (continued)

	0	1	2	3	4	5	6	7	8	9
600	1408.1023	1410.8812	1413.6608	1416.4411	1419.2221	1422.0039	1424.7863	1427.5695	1430.3534	1433.1380
610	1435.9234	1438.7094	1441.4962	1444.2836	1447.0718	1449.8607	1452.6503	1455.4405	1458.2315	1461.0232
620	1463.8156	1466.6087	1469.4025	1472.1970	1474.9922	1477.7880	1480.5846	1483.3819	1486.1798	1488.9785
630	1491.7778	1494.5779	1497.3786	1500.1800	1502.9821	1505.7849	1508.5883	1511.3924	1514.1973	1517.0028
640	1519.8090	1522.6158	1525.4233	1528.2316	1531.0404	1533.8500	1536.6602	1539.4711	1542.2827	1545.0950
650	1547.9079	1550.7215	1553.5357	1556.3506	1559.1662	1561.9824	1564.7993	1567.6169	1570.4351	1573.2540
660	1576.0736	1578.8938	1581.7146	1584.5361	1587.3583	1590.1811	1593.0046	1595.8287	1598.6535	1601.4789
670	1604.3050	1607.1317	1609.9591	1612.7871	1615.6158	1618.4451	1621.2750	1624.1056	1626.9368	1629.7687
680	1632.6012	1635.4344	1638.2681	1641.1026	1643.9376	1646.7733	1649.6096	1652.4466	1655.2842	1658.1224
690	1660.9612	1663.8007	1666.6408	1669.4816	1672.3229	1675.1649	1678.0075	1680.8508	1683.6946	1686.5391
700	1689.3842	1692.2299	1695.0762	1697.9232	1700.7708	1703.6190	1706.4678	1709.3172	1712.1672	1715.0179
710	1717.8691	1720.7210	1723.5735	1726.4266	1729.2803	1732.1346	1734.9895	1737.8450	1740.7011	1743.5578
720	1746.4152	1749.2731	1752.1316	1754.9908	1757.8505	1760.7109	1763.5718	1766.4333	1769.2955	1772.1582
730	1775.0215	1777.8854	1780.7499	1783.6150	1786.4807	1789.3470	1792.2139	1795.0814	1797.9494	1800.8181
740	1803.6873	1806.5571	1809.4275	1812.2985	1815.1701	1818.0423	1820.9150	1823.7883	1826.6622	1829.5367
750	1832.4118	1835.2874	1838.1636	1841.0404	1843.9178	1846.7957	1849.6742	1852.5533	1855.4330	1858.3133
760	1861.1941	1864.0755	1866.9574	1869.8399	1872.7230	1875.6067	1878.4909	1881.3757	1884.2611	1887.1470
770	1890.0335	1892.9205	1895.8082	1898.6963	1901.5851	1904.4744	1907.3642	1910.2547	1913.1456	1916.0372
780	1918.9293	1921.8219	1924.7151	1927.6089	1930.5032	1933.3981	1936.2935	1939.1895	1942.0860	1944.9831
790	1947.8807	1950.7789	1953.6776	1956.5769	1959.4767	1962.3771	1965.2780	1968.1794	1971.0814	1973.9840

Table 4-9. Logarithms of factorials (continued)

	0	1	2	3	4	5	6	7	8	9
800	1976.8871	1979.7907	1982.6949	1985.5996	1988.5049	1991.4107	1994.3170	1997.2239	2000.1313	2003.0392
810	2005.9477	2008.8567	2011.7663	2014.6764	2017.5870	2020.4982	2023.4099	2026.3221	2029.2348	2032.1481
820	2035.0619	2037.9763	2040.8911	2043.8065	2046.7225	2049.6389	2052.5559	2055.4734	2058.3914	2061.3100
830	2064.2291	2067.1487	2070.0688	2072.9894	2075.9106	2078.8323	2081.7545	2084.6772	2087.6005	2090.5242
840	2093.4485	2096.3733	2099.2986	2102.2244	2105.1508	2108.0776	2111.0050	2113.9329	2116.8613	2119.7902
850	2122.7196	2125.6495	2128.5800	2131.5109	2134.4424	2137.3744	2140.3068	2143.2398	2146.1733	2149.1073
860	2152.0418	2154.9768	2157.9123	2160.8483	2163.7848	2166.7218	2169.6594	2172.5974	2175.5359	2178.4749
870	2181.4144	2184.3545	2187.2950	2190.2360	2193.1775	2196.1195	2199.0620	2202.0050	2204.9485	2207.8925
880	2210.8370	2213.7820	2216.7274	2219.6734	2222.6198	2225.5668	2228.5142	2231.4621	2234.4106	2237.3595
890	2240.3088	2243.2587	2246.2091	2249.1599	2252.1113	2255.0631	2258.0154	2260.9682	2263.9215	2266.8752
900	2269.8295	2272.7842	2275.7394	2278.6951	2281.6513	2284.6079	2287.5650	2290.5226	2293.4807	2296.4393
910	2299.3983	2302.3579	2305.3179	2308.2783	2311.2393	2314.2007	2317.1626	2320.1250	2323.0878	2326.0511
920	2329.0149	2331.9792	2334.9439	2337.9091	2340.8748	2343.8409	2346.8075	2349.7746	2352.7421	2355.7102
930	2358.6786	2361.6476	2364.6170	2367.5869	2370.5572	2373.5281	2376.4993	2379.4711	2382.4433	2385.4159
940	2388.3891	2391.3627	2394.3367	2397.3112	2400.2862	2403.2616	2406.2375	2409.2139	2412.1907	2415.1679
950	2418.1457	2421.1238	2424.1025	2427.0816	2430.0611	2433.0411	2436.0216	2439.0025	2441.9839	2444.9657
960	2447.9479	2450.9307	2453.9138	2456.8975	2459.8815	2462.8661	2465.8511	2468.8365	2471.8224	2474.8087
970	2477.7954	2480.7827	2483.7703	2486.7584	2489.7470	2492.7360	2495.7255	2498.7154	2501.7057	2504.6965
980	2507.6877	2510.6794	2513.6715	2516.6640	2519.6570	2522.6505	2525.6443	2528.6387	2531.6334	2534.6286
990	2537.6242	2540.6203	2543.6168	2546.6138	2549.6112	2552.6090	2555.6073	2558.6059	2561.6051	2564.6046
1,000	2567.6046	2570.6051	2573.6059	2576.6072	2579.6090	2582.6111	2585.6137	2588.6168	2591.6202	2594.6241

section

5

COMMON PROBABILITY DISTRIBUTIONS

Tables 5-1 to 5-5 present various aspects of the *normal distribution* with mean μ and variance σ^2, whose probability density is

$$f\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Tabulated data are expressed in terms of the *standardized normal distribution* with mean zero and variance one, so that

$$f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}$$

To use the tables, convert to standard scores

$$z = \frac{x-\mu}{\sigma}$$

Table 5-1, intended for quick reference, employs coarse values of the standard scores z. Since the normal distribution is symmetrical, the ordinate of $-z$ equals the ordinate of z, and the area lying to the left of $-z$ equals the area lying to the right of z, and vice versa. Consequently, only positive values of the standardized deviate are listed in Table 5-1.

Table 5-2 gives the values of the ordinate f of the normal-distribution density and the area A between $-z$ and $+z$ as functions of the standard scores z and also as functions of $x = \mu + z\sigma$. Values of $1 - A$ are also tabulated; the area to the right of z or to the left of $-z$ can be determined through division of $1 - A$ by 2.

Table 5-3 lists values of the cumulative normal distribution

$$F(x) = \int_{-\infty}^{\frac{x-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}}\, e^{-u^2/2}\, du$$

or simply the area to the left of z. Table 5-3 also yields the area lying between two standard scores z and $z' < z$ by simple subtraction of the tabulated entry for z' from the entry for z.

Table 5-4 lists standardized-normal-deviate scores and ordinates as a function of the area or proportion. The area is the cumulative proportion or probability, i.e., the area to the left of the standard score. Only positive values of z are listed, corresponding to areas larger than one-half. For cumulative probabilities smaller than one-half, use column C in the table and assign a negative sign to the standardized-deviate score. This table is extremely useful in a number of psychometric applications. The determination of standardized-deviate scores as a function of proportions arises in a number of models used in psychophysics and psychological scaling. The interested reader should consult refs. 5 and 11. Of particular interest are the applications to paired-comparisons data and the normal-ogive model for mental tests.

Table 5-5 provides a number of functions of p, q, z, and f, where p and $q = p - 1$ are proportions and z and f are normal-distribution standard scores and ordinates corresponding to p (or q) when p is regarded as a cumulative probability. This table also has numerous uses in classical psychophysical procedures and psychological scaling techniques. In addition, the value $\sqrt{pq}$ is particularly useful in the calculation of point-biserial coefficients of correlation, and a point-biserial correlation may be converted to a biserial coefficient by multiplication with $\sqrt{pq}/f$ (refs. 4 and 8).

The *binomial distribution* is the probability distribution of X "successes" in N independent success-failure trials, where the probability of a "success" in a single trial is p and as such finds wide statistical application. In particular, note that X/N is the statistical relative frequency, in a sample of size N, of an event having the "true" probability p.

Table 5-6 lists exact binomial probabilities for various values of p through one-half and values of N from 2 through 10. For larger values of p, replace p by $1 - p$ and X by $N - X$ to find binomial probabilities in the table.

For $N > 10$, tables of the cumulative standardized normal distribution (Table 5-3) can be used to approximate binomial probabilities $P(X,N)$ with the aid of the formula

$$P(X,N) \approx F\left(\frac{X + \frac{1}{2} - \mu}{\sigma}\right) - F\left(\frac{X - \frac{1}{2} - \mu}{\sigma}\right) \qquad (X = 0,1,2, \ldots ,N)$$

where $\mu = Np$ and $\sigma^2 = Np(1 - p)$.

The Poisson distribution furnishes an even better approximation to the binomial when p is either very small or very large (see below). Exact cumulative binomial probabilities may also be computed by use of the incomplete beta function, also discussed below.

A number of statistical applications revolve around the binomial distribution with $p = \frac{1}{2}$. Of special interest is the sign test for comparing two related samples of observations (Sec. 7).

Table 5-7 lists values of the *cumulative Poisson distribution* function

$$F_{\text{Poisson}}(X) = \sum_{k=0}^{X} e^{-\xi} \frac{\xi^k}{k!} \qquad (X = 0,1,2, \ldots)$$

Poisson probabilities $P(X)$ can be obtained from

$$P(X) = F_{\text{Poisson}}(X) - F_{\text{Poisson}}(X - 1)$$

In addition to its many scientific uses, the Poisson distribution provides an excellent approximation to the binomial distribution when the parameter p of the binomial distribution is small and N is relatively large. When this approximation is used the parameter ξ of the Poisson distribution is equated to the product Np. For large values of ξ, the normal distribution provides a reasonable approximation to the Poisson distribution. Since both the mean and variance of a Poisson distributed variable are equal to ξ, the normal approximation is obtained from

$$P(X) \approx F\left(\frac{X + \frac{1}{2} - \xi}{\xi}\right) - F\left(\frac{X - \frac{1}{2} - \xi}{\xi}\right) \qquad (X = 0,1,2, \ldots)$$

The *gamma distribution* is a continuous probability distribution defined by its probability density

$$f(x) = \frac{x^{\alpha}e^{-x/\beta}}{\beta^{\alpha+1}\Gamma(\alpha + 1)} \qquad (x \geq 0,\ \alpha > -1,\ \beta > 0)$$

$f(x) = 0$ for negative x. Table 5-8 lists values of x/β corresponding to given values of the cumulative distribution function

$$F(x) = \int_{-\infty}^{X} f(\lambda)\, d\lambda$$

The gamma distribution may also be used to evaluate cumulative probabilities of the Poisson distribution, and vice versa, by the relationship

$$F_{\xi}(X = \alpha) = \text{Prob}\ [X \leq \alpha] = 1 - \text{Prob}\ [x \leq \xi] = 1 - G_{\alpha}(x = \xi)$$

where X is the Poisson-distributed variate with parameter ξ, and x is a gamma-distributed variate with $\beta = 1$. Since X is Poisson distributed, α must be an integer. $F_{\xi}(X = \alpha)$ denotes the cumulative Poisson distribution function with parameter ξ evaluated at the point α, and $G_{\alpha}(x = \xi)$ denotes the cumulative gamma distribution function with parameter α (and $\beta = 1$) evaluated at the point ξ.

The gamma distribution also furnishes an additional means of approximating binomial probabilities that is useful for values of N that are not large enough to yield good approximations by the normal distribution. For the details of this approximation and a discussion of the relation between Poisson and gamma distributions, see ref. 12.

As the parameter α increases, the gamma distribution approaches a normal distribution with mean $\beta(\alpha + 1)$ and variance $\beta^2(\alpha + 1)$. One obtains a more accurate normal approximation to a gamma distribution by regarding $2\sqrt{x/\beta}$ as approximately normal with mean $\sqrt{4\alpha + 3}$ and variance one; an even more accurate approximation results from $[x/\beta(\alpha + 1)]^{1/3}$, which is asymptotically normal with mean $(9\alpha + 8)/(9\alpha + 9)$ and variance $1/9(\alpha + 1)$ (ref. 1).

The *beta distribution* describes a continuous random variable defined over the closed interval $[a,b]$ with the density function

$$f(u) = \frac{(u - a)^{\alpha-1}(b - u)^{\beta-1}}{(b - a)^{\alpha+\beta-1}B(\alpha,\beta)}$$

where $B(\alpha,\beta)$ is the beta function of α and β; both α and β must be nonnegative. This density function can take on a wide variety of shapes including J shapes, U shapes, and various kinds of unimodal skewed or symmetrical shapes. The upper and lower limits a and b are relatively unimportant, for, by the simple change of variable

$$x = \frac{u - a}{b - a} \qquad \text{or} \qquad u = (b - a)x + a$$

one can express u in terms of a new variable x which is beta-distributed over the closed interval [0,1], with the probability density

$$f(x) = \frac{x^{\alpha-1}(1 - x)^{\beta-1}}{B(\alpha,\beta)}$$

The corresponding cumulative distribution function

$$F(x) = \int_{-\infty}^{X} f(\lambda)\, d\lambda = I_x(\alpha,\beta)$$

is called an *incomplete beta-function ratio.*

Table 5-9 contains fractiles corresponding to the cumulative probabilities 0.005, 0.01, 0.025, 0.05, and 0.10. The relation

$$I_x(\alpha,\beta) = 1 - I_{(1-x)}(\beta,\alpha)$$

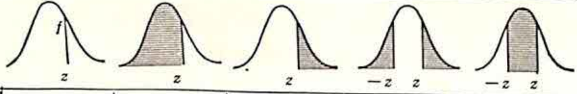
provides a means for determining the fractiles corresponding to 0.90, 0.95, 0.975, 0.99, and 0.995 as well.

The relation

$$\sum_{k=X}^{N} \binom{N}{k} p^k(1 - p)^{N-k} = I_p(X, N - X + 1)$$

also permits determination of right-tail (and also left-tail) fractiles of the binomial distribution with the aid of Table 5-9. The beta distribution is also related to the F distribution usually used in the analysis of variance (Sec. 6) and describes the exact sampling distribution of a squared multiple correlation based on normally distributed variates when the theoretical or population value is zero (ref. 1).

Table 5-1. Standardized normal distribution: Quick-reference data*



z	f(z)	F(z)	R(z)	2R(z)	W(z)
.0	.3989	.5000	.5000	1.0000	0
.1	.3970	.5398	.4602	.9203	.0797
.2	.3910	.5793	.4207	.8415	.1585
.3	.3814	.6179	.3821	.7642	.2358
.4	.3683	.6554	.3446	.6892	.3108
.5	.3521	.6915	.3085	.6171	.3829
.6	.3332	.7257	.2743	.5485	.4515
.7	.3123	.7580	.2420	.4839	.5161
.8	.2897	.7881	.2119	.4237	.5763
.9	.2661	.8159	.1841	.3681	.6319
1.0	.2420	.8413	.1587	.3173	.6827
1.1	.2179	.8643	.1357	.2713	.7287
1.2	.1942	.8849	.1151	.2301	.7699
1.3	.1714	.9032	.0968	.1936	.8064
1.4	.1497	.9192	.0808	.1615	.8385
1.5	.1295	.9332	.0668	.1336	.8664
1.6	.1109	.9452	.0548	.1096	.8904
1.7	.0940	.9554	.0446	.0891	.9109
1.8	.0790	.9641	.0359	.0719	.9281
1.9	.0656	.9713	.0287	.0574	.9426
2.0	.0540	.9772	.0228	.0455	.9545
2.1	.0440	.9821	.0179	.0357	.9643
2.2	.0355	.9861	.0139	.0278	.9722
2.3	.0283	.9893	.0107	.0214	.9786
2.4	.0224	.9918	.0082	.0164	.9836
2.5	.0175	.9938	.0062	.0124	.9876
2.6	.0136	.9953	.0047	.0093	.9907
2.7	.0104	.9965	.0035	.0069	.9931
2.8	.0079	.9974	.0026	.0051	.9949
2.9	.0060	.9981	.0019	.0037	.9963
3.0	.0044	.9987	.0013	.0027	.9973
Fractiles					
1.2816	.1755	.9000	.1000	.2000	.8000
1.6449	.1031	.9500	.0500	.1000	.9000
1.9600	.0584	.9750	.0250	.0500	.9500
2.0537	.0484	.9800	.0200	.0400	.9600
2.3263	.0267	.9900	.0100	.0200	.9800
2.5758	.0145	.9950	.0050	.0100	.9900

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* From G. A. Wadsworth and J. G. Bryan, "Introduction to Probability and Random Variables," McGraw-Hill Book Company, New York, 1960.

Table 5-2. Normal distribution*

Ordinates f at $\pm z$, and areas A between $-z$ and $+z$, of the normal distribution.

z	x	f	A	$1 - A$	z	x	f	A	$1 - A$
0	μ	.399	.0000	1.0000	± 1.50	$\mu \pm 1.50\sigma$	.1295	.8664	.1336
$\pm .05$	$\mu \pm .05\sigma$	.398	.0399	.9601	± 1.55	$\mu \pm 1.55\sigma$	.1200	.8789	.1211
$\pm .10$	$\mu \pm .10\sigma$	.397	.0797	.9203	± 1.60	$\mu \pm 1.60\sigma$	.1109	.8904	.1096
$\pm .15$	$\mu \pm .15\sigma$	.394	.1192	.8808	± 1.65	$\mu \pm 1.65\sigma$	.1023	.9011	.0989
$\pm .20$	$\mu \pm .20\sigma$	.391	.1585	.8415	± 1.70	$\mu \pm 1.70\sigma$	.0940	.9109	.0891
$\pm .25$	$\mu \pm .25\sigma$	.387	.1974	.8026	± 1.75	$\mu \pm 1.75\sigma$	.0863	.9199	.0801
$\pm .30$	$\mu \pm .30\sigma$	.381	.2358	.7642	± 1.80	$\mu \pm 1.80\sigma$	.0790	.9281	.0719
$\pm .35$	$\mu \pm .35\sigma$	.375	.2737	.7263	± 1.85	$\mu \pm 1.85\sigma$	.0721	.9357	.0643
$\pm .40$	$\mu \pm .40\sigma$	.368	.3108	.6892	± 1.90	$\mu \pm 1.90\sigma$	.0656	.9426	.0574
$\pm .45$	$\mu \pm .45\sigma$	.361	.3473	.6527	± 1.95	$\mu \pm 1.95\sigma$	.0596	.9488	.0512
$\pm .50$	$\mu \pm .50\sigma$	.352	.3829	.6171	± 2.00	$\mu \pm 2.00\sigma$	.0540	.9545	.0455
$\pm .55$	$\mu \pm .55\sigma$	.343	.4177	.5823	± 2.05	$\mu \pm 2.05\sigma$	.0488	.9596	.0404
$\pm .60$	$\mu \pm .60\sigma$	.333	.4515	.5485	± 2.10	$\mu \pm 2.10\sigma$	.0440	.9643	.0357
$\pm .65$	$\mu \pm .65\sigma$	.323	.4843	.5157	± 2.15	$\mu \pm 2.15\sigma$	.0396	.9684	.0316
$\pm .70$	$\mu \pm .70\sigma$	.312	.5161	.4839	± 2.20	$\mu \pm 2.20\sigma$	.0355	.9722	.0278
$\pm .75$	$\mu \pm .75\sigma$	.301	.5467	.4533	± 2.25	$\mu \pm 2.25\sigma$	.0317	.9756	.0244
$\pm .80$	$\mu \pm .80\sigma$	.290	.5763	.4237	± 2.30	$\mu \pm 2.30\sigma$	.0283	.9786	.0214
$\pm .85$	$\mu \pm .85\sigma$	.278	.6047	.3953	± 2.35	$\mu + 2.35\sigma$	.0252	.9812	.0188
$\pm .90$	$\mu \pm .90\sigma$	.266	.6319	.3681	± 2.40	$\mu \pm 2.40\sigma$	.0224	.9836	.0164
$\pm .95$	$\mu \pm .95\sigma$	.254	.6579	.3421	± 2.45	$\mu \pm 2.45\sigma$	.0198	.9857	.0143
± 1.00	$\mu \pm 1.00\sigma$	.242	.6827	.3173	± 2.50	$\mu \pm 2.50\sigma$	.0175	.9876	.0124
± 1.05	$\mu \pm 1.05\sigma$	.230	.7063	.2937	± 2.55	$\mu \pm 2.55\sigma$	.0154	.9892	.0108
± 1.10	$\mu \pm 1.10\sigma$	.218	.7287	.2713	± 2.60	$\mu \pm 2.60\sigma$	.0136	.9907	.0093
± 1.15	$\mu \pm 1.15\sigma$	.206	.7499	.2501	± 2.65	$\mu \pm 2.65\sigma$	.0119	.9920	.0080
± 1.20	$\mu \pm 1.20\sigma$	.194	.7699	.2301	± 2.70	$\mu \pm 2.70\sigma$	.0104	.9931	.0069
± 1.25	$\mu \pm 1.25\sigma$	.183	.7887	.2113	± 2.75	$\mu \pm 2.75\sigma$	.0091	.9940	.0060
± 1.30	$\mu \pm 1.30\sigma$	.171	.8064	.1936	± 2.80	$\mu \pm 2.80\sigma$	.0079	.9949	.0051
± 1.35	$\mu \pm 1.35\sigma$	.160	.8230	.1770	± 2.85	$\mu \pm 2.85\sigma$	.0069	.9956	.0044
± 1.40	$\mu \pm 1.40\sigma$	.150	.8385	.1615	± 2.90	$\mu \pm 2.90\sigma$	.0060	.9963	.0037
± 1.45	$\mu \pm 1.45\sigma$	.139	.8529	.1471	± 2.95	$\mu \pm 2.95\sigma$	.0051	.9968	.0032
± 1.50	$\mu \pm 1.50\sigma$	.130	.8664	.1336	± 3.00	$\mu \pm 3.00\sigma$	.0044	.9973	.0027
					± 4.00	$\mu \pm 4.00\sigma$	.0001	.99994	.00006
					± 5.00	$\mu \pm 5.00\sigma$	.000001	.9999994	.0000006
$\pm .000$	μ	.3989	.0000	1.0000	± 1.036	$\mu \pm 1.036\sigma$	.2331	.7000	.3000
$\pm .126$	$\mu \pm .126\sigma$	.3958	.1000	.9000	± 1.282	$\mu \pm 1.282\sigma$	.1755	.8000	.2000
$\pm .253$	$\mu \pm .253\sigma$	.3863	.2000	.8000	± 1.645	$\mu \pm 1.645\sigma$	.1031	.9000	.1000
$\pm .385$	$\mu \pm .385\sigma$	.3704	.3000	.7000	± 1.960	$\mu \pm 1.960\sigma$	.0584	.9500	.0500
$\pm .524$	$\mu \pm .524\sigma$	.3477	.4000	.6000	± 2.576	$\mu \pm 2.576\sigma$	.0145	.9900	.0100
$\pm .674$	$\mu \pm .674\sigma$	.3178	.5000	.5000	± 3.291	$\mu \pm 3.291\sigma$	.0018	.9990	.0010
$\pm .842$	$\mu \pm .842\sigma$	.2800	.6000	.4000	± 3.891	$\mu \pm 3.891\sigma$	.0002	.9999	.0001

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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1962.

Table 5-4. **Standard scores (or deviates) and ordinates corresponding to divisions of the area under the normal curve into a larger proportion (B) and a smaller proportion (C); also the value $\sqrt{BC}$***

B *The larger area*	z *Standard score*	f *Ordinate*	$\sqrt{BC}$	C *The smaller area*
.500	.0000	.3989	.5000	.500
.505	.0125	.3989	.5000	.495
.510	.0251	.3988	.4999	.490
.515	.0376	.3987	.4998	.485
.520	.0502	.3984	.4996	.480
.525	.0627	.3982	.4994	.475
.530	.0753	.3978	.4991	.470
.535	.0878	.3974	.4988	.465
.540	.1004	.3969	.4984	.460
.545	.1130	.3964	.4980	.455
.550	.1257	.3958	.4975	.450
.555	.1383	.3951	.4970	.445
.560	.1510	.3944	.4964	.440
.565	.1637	.3936	.4958	.435
.570	.1764	.3928	.4951	.430
.575	.1891	.3919	.4943	.425
.580	.2019	.3909	.4936	.420
.585	.2147	.3899	.4927	.415
.590	.2275	.3887	.4918	.410
.595	.2404	.3876	.4909	.405
.600	.2533	.3863	.4899	.400
.605	.2663	.3850	.4889	.395
.610	.2793	.3837	.4877	.390
.615	.2924	.3822	.4867	.385
.620	.3055	.3808	.4854	.380
.625	.3186	.3792	.4841	.375
.630	.3319	.3776	.4828	.370
.635	.3451	.3759	.4814	.365
.640	.3585	.3741	.4800	.360
.645	.3719	.3723	.4785	.355
.650	.3853	.3704	.4770	.350
.655	.3989	.3684	.4754	.345
.660	.4125	.3664	.4737	.340
.665	.4261	.3643	.4720	.335
.670	.4399	.3621	.4702	.330
.675	.4538	.3599	.4684	.325
.680	.4677	.3576	.4665	.320
.685	.4817	.3552	.4645	.315
.690	.4959	.3528	.4625	.310
.695	.5101	.3503	.4604	.305
.700	.5244	.3477	.4583	.300
.705	.5388	.3450	.4560	.295
.710	.5534	.3423	.4538	.290
.715	.5681	.3395	.4514	.285
.720	.5828	.3366	.4490	.280

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* From J. P. Guilford, "Fundamental Statistics in Psychology and Education," 4th ed., McGraw-Hill Book Company, New York, 1965.

Table 5-4. **Standard scores (or deviates) and ordinates corresponding to divisions of the area under the normal curve into a larger proportion (B) and a smaller proportion (C); also the value $\sqrt{BC}$ (continued)**

B *The larger area*	z *Standard score*	f *Ordinate*	$\sqrt{BC}$	C *The smaller area*
.725	.5978	.3337	.4465	.275
.730	.6128	.3306	.4440	.270
.735	.6280	.3275	.4413	.265
.740	.6433	.3244	.4386	.260
.745	.6588	.3211	.4359	.255
.750	.6745	.3178	.4330	.250
.755	.6903	.3144	.4301	.245
.760	.7063	.3109	.4271	.240
.765	.7225	.3073	.4240	.235
.770	.7388	.3036	.4208	.230
.775	.7554	.2999	.4176	.225
.780	.7722	.2961	.4142	.220
.785	.7892	.2922	.4108	.215
.790	.8064	.2882	.4073	.210
.795	.8239	.2841	.4037	.205
.800	.8416	.2800	.4000	.200
.805	.8596	.2757	.3962	.195
.810	.8779	.2714	.3923	.190
.815	.8965	.2669	.3883	.185
.820	.9154	.2624	.3842	.180
.825	.9346	.2578	.3800	.175
.830	.9542	.2531	.3756	.170
.835	.9741	.2482	.3712	.165
.840	.9945	.2433	.3666	.160
.845	1.0152	.2383	.3619	.155
.850	1.0364	.2332	.3571	.150
.855	1.0581	.2279	.3521	.145
.860	1.0803	.2226	.3470	.140
.865	1.1031	.2171	.3417	.135
.870	1.1264	.2115	.3363	.130
.875	1.1503	.2059	.3307	.125
.880	1.1750	.2000	.3250	.120
.885	1.2004	.1941	.3190	.115
.890	1.2265	.1880	.3129	.110
.895	1.2536	.1818	.3066	.105
.900	1.2816	.1755	.3000	.100
.905	1.3106	.1690	.2932	.095
.910	1.3408	.1624	.2862	.090
.915	1.3722	.1556	.2789	.085
.920	1.4051	.1487	.2713	.080
.925	1.4395	.1416	.2634	.075
.930	1.4757	.1343	.2551	.070
.935	1.5141	.1268	.2465	.065
.940	1.5548	.1191	.2375	.060
.945	1.5982	.1112	.2280	.055

Table 5-4. **Standard scores (or deviates) and ordinates corresponding to divisions of the area under the normal curve into a larger proportion (B) and a smaller proportion (C); also the value $\sqrt{BC}$ (continued)**

B *The larger area*	z *Standard score*	f *Ordinate*	$\sqrt{BC}$	C *The smaller area*
.950	1.6449	.1031	.2179	.050
.955	1.6954	.0948	.2073	.045
.960	1.7507	.0862	.1960	.040
.965	1.8119	.0773	.1838	.035
.970	1.8808	.0680	.1706	.030
.975	1.9600	.0584	.1561	.025
.980	2.0537	.0484	.1400	.020
.985	2.1701	.0379	.1226	.015
.990	2.3263	.0267	.0995	.010
.995	2.5758	.0145	.0705	.005
.996	2.6521	.0118	.0631	.004
.997	2.7478	.0091	.0547	.003
.998	2.8782	.0063	.0447	.002
.999	3.0902	.0034	.0316	.001
.9995	3.2905	.0018	.0224	.0005

Table 5-5. **Functions of p, q, z, and f, where p and q are proportions ($p + q = 1.00$) and z and f are deviate scores and ordinates for the normal distribution curve***†

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	
p (or q)	pq	$\sqrt{pq}$	pq/f	$\sqrt{pq}/f$	p/f	f/p	zf/p	f	zf/q	f/q	q/f	$\sqrt{p/q}$	$\sqrt{q/p}$	q (or p)
.99	.0099	.0995−	.3715	3.733	37.15−	.02692	−.06262	.02665	6.2002	2.665	.3752	9.950	.1005	.01
.98	.0196	.1400	.4048	2.892	20.24	.04941	−.1015	.04842	4.9719	2.421	.4131	7.000	.1429	.02
.97	.0291	.1706	.4277	2.507	14.26	.07015	−.1319	.06804	4.2657	2.268	.4409	5.686	.1759	.03
.96	.0384	.1960	.4456	2.274	11.14	.08976	−.1571	.08617	3.7717	2.154	.4642	4.899	.2041	.04
.95	.0475	.2179	.4605	2.113	9.211	.1086	−.1786	.1031	3.3928	2.063	.4848	4.359	.2294	.05
.94	.0564	.2375−	.4735	1.994	7.891	.1267	−.1970	.1191	3.0868	1.985	.5037	3.958	.2526	.06
.93	.0651	.2551	.4848	1.900	6.926	.1444	−.2131	.1343	2.8307	1.918	.5213	3.645	.2743	.07
.92	.0736	.2713	.4951	1.825	6.188	.1616	−.2271	.1487	2.6110	1.858	.5381	3.391	.2949	.08
.91	.0819	.2862	.5043	1.762	5.604	.1785	−.2393	.1624	2.4191	1.804	.5542	3.180	.3145	.09
.90	.0900	.3000	.5128	1.709	5.128	.1950	−.2499	.1755	2.2491	1.755	.5698	3.000	.3333	.10
.89	.0979	.3129	.5206	1.664	4.733	.2113	−.2591	.1880	2.0966	1.709	.5850	2.844	.3516	.11
.88	.1056	.3250	.5279	1.625	4.399	.2273	−.2671	.2000	1.9587	1.667	.5999	2.708	.3693	.12
.87	.1131	.3363	.5346	1.590	4.112	.2432	−.2739	.2115	1.8330	1.627	.6145	2.587	.3865	.13
.86	.1204	.3470	.5409	1.559	3.864	.2588	−.2796	.2226	1.7175	1.590	.6290	2.478	.4035	.14
.85	.1275	.3571	.5468	1.532	3.646	.2743	−.2843	.2332	1.6110	1.554	.6433	2.380	.4201	.15
.84	.1344	.3666	.5524	1.507	3.452	.2896	−.2880	.2433	1.5123	1.521	.6576	2.291	.4365	.16
.83	.1411	.3756	.5576	1.484	3.280	.3049	−.2909	.2531	1.4203	1.489	.6718	2.210	.4525	.17
.82	.1476	.3842	.5625	1.464	3.125	.3200	−.2929	.2624	1.3344	1.458	.6860	2.134	.4685	.18
.81	.1539	.3923	.5671	1.446	2.985	.3350	−.2941	.2714	1.2538	1.428	.7002	2.065	.4844	.19
.80	.1600	.4000	.5715	1.429	2.858	.3500	−.2946	.2800	1.1781	1.400	.7144	2.000	.5000	.20
.79	.1659	.4073	.5756	1.413	2.741	.3648	−.2942	.2882	1.1067	1.372	.7287	1.940	.5156	.21
.78	.1716	.4142	.5796	1.399	2.634	.3796	−.2931	.2961	1.0393	1.346	.7430	1.883	.5311	.22
.77	.1771	.4208	.5832	1.386	2.536	.3943	−.2913	.3036	.9754	1.320	.7575	1.830	.5465	.23
.76	.1824	.4271	.5867	1.374	2.445	.4090	−.2889	.3109	.9149	1.295	.7720	1.780	.5620	.24
.75	.1875	.4330	.5900	1.363	2.360	.4237	−.2858	.3178	.8573	1.271	.7867	1.732	.5774	.25

* When p is less than .50, interchange p and q, as the headings of the first and last columns indicate.

† From J. P. Guilford, "Psychometric Methods," 2d ed., McGraw-Hill Book Company, New York, 1954.

Table 5-5. **Functions of p, q, z, and f where p and q are proportions ($p + q = 1.00$) and z and f are deviate scores and ordinates for the normal distribution curve (continued)**

	A	B	C	D	E	F	G	H	I	J	K	L	M	
p (or q)	pq	$\sqrt{pq}$	pq/f	$\sqrt{pq}/f$	p/f	f/p	zf/p	f	zf/q	f/q	q/f	$\sqrt{p/q}$	$\sqrt{q/p}$	q (or p)
.74	.1924	.4386	.5931	1.352	2.281	.4384	−.2820	.3244	.8026	1.248	.8016	1.687	.5928	.26
.73	.1971	.4440	.5961	1.343	2.208	.4529	−.2775	.3306	.7504	1.225	.8166	1.644	.6082	.27
.72	.2016	.4490	.5989	1.334	2.139	.4675	−.2725	.3366	.7006	1.202	.8318	1.604	.6236	.28
.71	.2059	.4538	.6015	1.326	2.074	.4822	−.2668	.3423	.6532	1.180	.8472	1.565	.6391	.29
.70	.2100	.4583	.6040	1.318	2.013	.4967	−.2605	.3477	.6078	1.159	.8628	1.528	.6547	.30
.69	.2139	.4625—	.6063	1.311	1.956	.5113	−.2535	.3528	.5643	1.138	.8787	1.492	.6703	.31
.68	.2176	.4665—	.6085	1.304	1.902	.5259	−.2460	.3576	.5227	1.118	.8949	1.458	.6860	.32
.67	.2211	.4702	.6106	1.298	1.850	.5405	−.2378	.3621	.4828	1.097	.9112	1.425	.7018	.33
.66	.2244	.4737	.6124	1.293	1.801	.5552	−.2290	.3664	.4445	1.078	.9279	1.393	.7178	.34
.65	.2275	.4770	.6142	1.288	1.755	.5698	−.2196	.3704	.4078	1.058	.9449	1.363	.7338	.35
.64	.2304	.4800	.6158	1.283	1.711	.5845	−.2095	.3741	.3725	1.039	.9623	1.333	.7500	.36
.63	.2331	.4828	.6174	1.279	1.669	.5993	−.1989	.3776	.3387	1.020	.9800	1.305	.7663	.37
.62	.2356	.4854	.6188	1.275	1.628	.6141	−.1876	.3808	.3061	1.002	.9980	1.277	.7829	.38
.61	.2379	.4877	.6200	1.271	1.590	.6290	−.1757	.3837	.2748	.9938	1.016	1.251	.7996	.39
.60	.2400	.4899	.6212	1.268	1.553	.6439	−.1631	.3863	.2447	.9659	1.035	1.225	.8165	.40
.59	.2419	.4918	.6223	1.265	1.518	.6589	−.1499	.3888	.2158	.9482	1.055	1.200	.8336	.41
.58	.2436	.4936	.6232	1.263	1.484	.6739	−.1361	.3909	.1879	.9307	1.074	1.175	.8510	.42
.57	.2451	.4951	.6240	1.260	1.451	.6891	−.1215	.3928	.1611	.9134	1.095	1.151	.8686	.43
.56	.2464	.4964	.6247	1.259	1.420	.7043	−.1063	.3944	.1353	.8964	1.116	1.128	.8864	.44
.55	.2475	.4975—	.6253	1.257	1.390	.7196	−.09043	.3958	.1105	.8796	1.137	1.106	.9045	.45
.54	.2484	.4984	.6258	1.256	1.360	.7351	−.07382	.3969	.0867	.8629	1.159	1.083	.9229	.46
.53	.2491	.4991	.6262	1.255	1.332	.7506	−.05650	.3978	.0637	.8464	1.181	1.062	.9417	.47
.52	.2496	.4996	.6264	1.254	1.305	.7662	−.03843	.3984	.0416	.8301	1.205	1.041	.9608	.48
.51	.2499	.4999	.6266	1.253	1.279	.7820	−.01960	.3988	.0204	.8139	1.229	1.020	.9802	.49
.50	.2500	.5000	.6267	1.253	1.253	.7979	−.00000	.3989	.0000	.7979	1.253	1.000	1.0000	.50

Table 5-6. Binomial probabilities, $P(X,N) = \binom{N}{X} p^X(1 - p)^{N-X}$ for $N \leq 10$ and for various values of p*

N	X \ p	.01	.05	.10	.15	.20	.25	.30	⅓	.35	.40	.45	.50
2	0	.9801	.9025	.8100	.7225	.6400	.5625	.4900	.4444	.4225	.3600	.3025	.2500
	1	.0198	.0950	.1800	.2550	.3200	.3750	.4200	.4444	.4550	.4800	.4950	.5000
	2	.0001	.0025	.0100	.0225	.0400	.0625	.0900	.1111	.1225	.1600	.2025	.2500
3	0	.9703	.8574	.7290	.6141	.5120	.4219	.3430	.2963	.2746	.2160	.1664	.1250
	1	.0294	.1354	.2430	.3251	.3840	.4219	.4410	.4444	.4436	.4320	.4084	.3750
	2	.0003	.0071	.0270	.0574	.0960	.1406	.1890	.2222	.2389	.2880	.3341	.3750
	3	.0000	.0001	.0010	.0034	.0080	.0156	.0270	.0370	.0429	.0640	.0911	.1250
4	0	.9606	.8145	.6561	.5220	.4096	.3164	.2401	.1975	.1785	.1296	.0915	.0625
	1	.0388	.1715	.2916	.3685	.4096	.4219	.4116	.3951	.3845	.3456	.2995	.2500
	2	.0006	.0135	.0486	.0975	.1536	.2109	.2646	.2963	.3105	.3456	.3675	.3750
	3	.0000	.0005	.0036	.0115	.0256	.0469	.0756	.0988	.1115	.1536	.2005	.2500
	4	.0000	.0000	.0001	.0005	.0016	.0039	.0081	.0123	.0150	.0256	.0410	.0625
5	0	.9510	.7738	.5905	.4437	.3277	.2373	.1681	.1317	.1160	.0778	.0503	.0312
	1	.0480	.2036	.3280	.3915	.4096	.3955	.3602	.3292	.3124	.2592	.2059	.1562
	2	.0010	.0214	.0729	.1382	.2048	.2637	.3087	.3292	.3364	.3456	.3369	.3125
	3	.0000	.0011	.0081	.0244	.0512	.0879	.1323	.1646	.1811	.2304	.2757	.3125
	4	.0000	.0000	.0004	.0022	.0064	.0146	.0284	.0412	.0488	.0768	.1128	.1562
	5	.0000	.0000	.0000	.0001	.0003	.0010	.0024	.0041	.0053	.0102	.0185	.0312
6	0	.9415	.7351	.5314	.3771	.2621	.1780	.1176	.0878	.0754	.0467	.0277	.0156
	1	.0571	.2321	.3543	.3993	.3932	.3560	.3025	.2634	.2437	.1866	.1359	.0938
	2	.0014	.0305	.0984	.1762	.2458	.2966	.3241	.3292	.3280	.3110	.2780	.2344
	3	.0000	.0021	.0146	.0415	.0819	.1318	.1852	.2195	.2355	.2765	.3032	.3125
	4	.0000	.0001	.0012	.0055	.0154	.0330	.0595	.0823	.0951	.1382	.1861	.2344
	5	.0000	.0000	.0001	.0004	.0015	.0044	.0102	.0165	.0205	.0369	.0609	.0938
	6	.0000	.0000	.0000	.0000	.0001	.0002	.0007	.0014	.0018	.0041	.0083	.0156
7	0	.9321	.6983	.4783	.3206	.2097	.1335	.0824	.0585	.0490	.0280	.0152	.0078
	1	.0659	.2573	.3720	.3960	.3670	.3115	.2471	.2048	.1848	.1306	.0872	.0547
	2	.0020	.0406	.1240	.2097	.2753	.3115	.3177	.3073	.2985	.2613	.2140	.1641
	3	.0000	.0036	.0230	.0617	.1147	.1730	.2269	.2561	.2679	.2903	.2918	.2734
	4	.0000	.0002	.0026	.0109	.0287	.0577	.0972	.1280	.1442	.1935	.2388	.2734
	5	.0000	.0000	.0002	.0012	.0043	.0115	.0250	.0384	.0466	.0774	.1172	.1641
	6	.0000	.0000	.0000	.0001	.0004	.0013	.0036	.0064	.0084	.0172	.0320	.0547
	7	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0006	.0016	.0037	.0078
8	0	.9227	.6634	.4305	.2725	.1678	.1001	.0576	.0390	.0319	.0168	.0084	.0039
	1	.0746	.2793	.3826	.3847	.3355	.2670	.1977	.1561	.1373	.0896	.0548	.0312
	2	.0026	.0515	.1488	.2376	.2936	.3115	.2965	.2731	.2587	.2090	.1569	.1094
	3	.0001	.0054	.0331	.0839	.1468	.2076	.2541	.2731	.2786	.2787	.2568	.2188
	4	.0000	.0004	.0046	.0185	.0459	.0865	.1361	.1707	.1875	.2322	.2627	.2734
	5	.0000	.0000	.0004	.0026	.0092	.0231	.0467	.0683	.0808	.1239	.1719	.2188
	6	.0000	.0000	.0000	.0002	.0011	.0038	.0100	.0171	.0217	.0413	.0703	.1094
	7	.0000	.0000	.0000	.0000	.0001	.0004	.0012	.0024	.0033	.0079	.0164	.0312
	8	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0002	.0007	.0017	.0039
9	0	.9135	.6302	.3874	.2316	.1342	.0751	.0404	.0260	.0207	.0101	.0046	.0020
	1	.0830	.2985	.3874	.3679	.3020	.2253	.1556	.1171	.1004	.0605	.0339	.0176
	2	.0034	.0629	.1722	.2597	.3020	.3003	.2668	.2341	.2162	.1612	.1110	.0703
	3	.0001	.0077	.0446	.1069	.1762	.2336	.2668	.2731	.2716	.2508	.2119	.1641
	4	.0000	.0006	.0074	.0283	.0661	.1168	.1715	.2048	.2194	.2508	.2600	.2461
	5	.0000	.0000	.0008	.0050	.0165	.0389	.0735	.1024	.1181	.1672	.2128	.2461
	6	.0000	.0000	.0001	.0006	.0028	.0087	.0210	.0341	.0424	.0743	.1160	.1641
	7	.0000	.0000	.0000	.0000	.0003	.0012	.0039	.0073	.0098	.0212	.0407	.0703
	8	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0009	.0013	.0035	.0083	.0176
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0003	.0008	.0020
10	0	.9044	.5987	.3487	.1969	.1074	.0563	.0282	.0173	.0135	.0060	.0025	.0010
	1	.0914	.3151	.3874	.3474	.2684	.1877	.1211	.0867	.0725	.0403	.0207	.0098
	2	.0042	.0746	.1937	.2759	.3020	.2816	.2335	.1951	.1757	.1209	.0763	.0439
	3	.0001	.0105	.0574	.1298	.2013	.2503	.2668	.2601	.2522	.2150	.1665	.1172
	4	.0000	.0010	.0112	.0401	.0881	.1460	.2001	.2276	.2377	.2508	.2384	.2051
	5	.0000	.0001	.0015	.0085	.0264	.0584	.1029	.1366	.1536	.2007	.2340	.2461
	6	.0000	.0000	.0001	.0012	.0055	.0162	.0368	.0569	.0689	.1115	.1596	.2051
	7	.0000	.0000	.0000	.0001	.0008	.0031	.0090	.0163	.0212	.0425	.0746	.1172
	8	.0000	.0000	.0000	.0000	.0001	.0004	.0014	.0030	.0043	.0106	.0229	.0439
	9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0005	.0016	.0042	.0098
	10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0010

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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1962.

Table 5-7. Cumulative Poisson distribution*

1,000 × probability of X or less occurrences of event that has average number of occurrences equal to $\xi = Np$.

X / $\xi = Np$	0	1	2	3	4	5	6	7	8	9
0.02	980	1,000								
0.04	961	999	1,000							
0.06	942	998	1,000							
0.08	923	997	1,000							
0.10	905	995	1,000							
0.15	861	990	999	1,000						
0.20	819	982	999	1,000						
0.25	779	974	998	1,000						
0.30	741	963	996	1,000						
0.35	705	951	994	1,000						
0.40	670	938	992	999	1,000					
0.45	638	925	989	999	1,000					
0.50	607	910	986	998	1,000					
0.55	577	894	982	998	1,000					
0.60	549	878	977	997	1,000					
0.65	522	861	972	996	999	1,000				
0.70	497	844	966	994	999	1,000				
0.75	472	827	959	993	999	1,000				
0.80	449	809	953	991	999	1,000				
0.85	427	791	945	989	998	1,000				
0.90	407	772	937	987	998	1,000				
0.95	387	754	929	984	997	1,000				
1.00	368	736	920	981	996	999	1,000			
1.1	333	699	900	974	995	999	1,000			
1.2	301	663	879	966	992	998	1,000			
1.3	273	627	857	957	989	998	1,000			
1.4	247	592	833	946	986	997	999	1,000		
1.5	223	558	809	934	981	996	999	1,000		
1.6	202	525	783	921	976	994	999	1,000		
1.7	183	493	757	907	970	992	998	1,000		
1.8	165	463	731	891	964	990	997	999	1,000	
1.9	150	434	704	875	956	987	997	999	1,000	
2.0	135	406	677	857	947	983	995	999	1,000	

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* From E. L. Grant, "Statistical Quality Control," 3d ed., McGraw-Hill Book Company, New York, 1964.

Table 5-7. Cumulative Poisson distribution (continued)

$\xi = Np$ \ X	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2.2	111	355	623	819	928	975	993	998	1,000								
2.4	091	308	570	779	904	964	988	997	999	1,000							
2.6	074	267	518	736	877	951	983	995	999	1,000							
2.8	061	231	469	692	848	935	976	992	998	999	1,000						
3.0	050	199	423	647	815	916	966	988	996	999	1,000						
3.2	041	171	380	603	781	895	955	983	994	998	1,000						
3.4	033	147	340	558	744	871	942	977	992	997	999	1,000					
3.6	027	126	303	515	706	844	927	969	988	996	999	1,000					
3.8	022	107	269	473	668	816	909	960	984	994	998	999	1,000				
4.0	018	092	238	433	629	785	889	949	979	992	997	999	1,000				
4.2	015	078	210	395	590	753	867	936	972	989	996	999	1,000				
4.4	012	066	185	359	551	720	844	921	964	985	994	998	999	1,000			
4.6	010	056	163	326	513	686	818	905	955	980	992	997	999	1,000			
4.8	008	048	143	294	476	651	791	887	944	975	990	996	999	1,000			
5.0	007	040	125	265	440	616	762	867	932	968	986	995	998	999	1,000		
5.2	006	034	109	238	406	581	732	845	918	960	982	993	997	999	1,000		
5.4	005	029	095	213	373	546	702	822	903	951	977	990	996	999	1,000		
5.6	004	024	082	191	342	512	670	797	886	941	972	988	995	998	999	1,000	
5.8	003	021	072	170	313	478	638	771	867	929	965	984	993	997	999	1,000	
6.0	002	017	062	151	285	446	606	744	847	916	957	980	991	996	999	999	1,000

Table 5-7. Cumulative Poisson distribution (continued)

$\xi = Np$ \ X	0	1	2	3	4	5	6	7	8	9
6.2	002	015	054	134	259	414	574	716	826	902
6.4	002	012	046	119	235	384	542	687	803	886
6.6	001	010	040	105	213	355	511	658	780	869
6.8	001	009	034	093	192	327	480	628	755	850
7.0	001	007	030	082	173	301	450	599	729	830
7.2	001	006	025	072	156	276	420	569	703	810
7.4	001	005	022	063	140	253	392	539	676	788
7.6	001	004	019	055	125	231	365	510	648	765
7.8	000	004	016	048	112	210	338	481	620	741
8.0	000	003	014	042	100	191	313	453	593	717
8.5	000	002	009	030	074	150	256	386	523	653
9.0	000	001	006	021	055	116	207	324	456	587
9.5	000	001	004	015	040	089	165	269	392	522
10.0	000	000	003	010	029	067	130	220	333	458

	10	11	12	13	14	15	16	17	18	19
6.2	949	975	989	995	998	999	1,000			
6.4	939	969	986	994	997	999	1,000			
6.6	927	963	982	992	997	999	999	1,000		
6.8	915	955	978	990	996	998	999	1,000		
7.0	901	947	973	987	994	998	999	1,000		
7.2	887	937	967	984	993	997	999	999	1,000	
7.4	871	926	961	980	991	996	998	999	1,000	
7.6	854	915	954	976	989	995	998	999	1,000	
7.8	835	902	945	971	986	993	997	999	1,000	
8.0	816	888	936	966	983	992	996	998	999	1,000
8.5	763	849	909	949	973	986	993	997	999	999
9.0	706	803	876	926	959	978	989	995	998	999
9.5	645	752	836	898	940	967	982	991	996	998
10.0	583	697	792	864	917	951	973	986	993	997

	20	21	22
8.5	1,000		
9.0	1,000		
9.5	999	1,000	
10.0	998	999	1,000

Table 5-7. Cumulative Poisson distribution (continued)

X $\xi = Np$	0	1	2	3	4	5	6	7	8	9
10.5	000	000	002	007	021	050	102	179	279	397
11.0	000	000	001	005	015	038	079	143	232	341
11.5	000	000	001	003	011	028	060	114	191	289
12.0	000	000	001	002	008	020	046	090	155	242
12.5	000	000	000	002	005	015	035	070	125	201
13.0	000	000	000	001	004	011	026	054	100	166
13.5	000	000	000	001	003	008	019	041	079	135
14.0	000	000	000	000	002	006	014	032	062	109
14.5	000	000	000	000	001	004	010	024	048	088
15.0	000	000	000	000	001	003	008	018	037	070

	10	11	12	13	14	15	16	17	18	19
10.5	521	639	742	825	888	932	960	978	988	994
11.0	460	579	689	781	854	907	944	968	982	991
11.5	402	520	633	733	815	878	924	954	974	986
12.0	347	462	576	682	772	844	899	937	963	979
12.5	297	406	519	628	725	806	869	916	948	969
13.0	252	353	463	573	675	764	835	890	930	957
13.5	211	304	409	518	623	718	798	861	908	942
14.0	176	260	358	464	570	669	756	827	883	923
14.5	145	220	311	413	518	619	711	790	853	901
15.0	118	185	268	363	466	568	664	749	819	875

	20	21	22	23	24	25	26	27	28	29
10.5	997	999	999	1,000						
11.0	995	998	999	1,000						
11.5	992	996	998	999	1,000					
12.0	988	994	997	999	999	1,000				
12.5	983	991	995	998	999	999	1,000			
13.0	975	986	992	996	998	999	1,000			
13.5	965	980	989	994	997	998	999	1,000		
14.0	952	971	983	991	995	997	999	999	1,000	
14.5	936	960	976	986	992	996	998	999	999	1,000
15.0	917	947	967	981	989	994	997	998	999	1,000

Table 5-7. Cumulative Poisson distribution (continued)

X $\xi = Np$	4	5	6	7	8	9	10	11	12	13
16	000	001	004	010	022	043	077	127	193	275
17	000	001	002	005	013	026	049	085	135	201
18	000	000	001	003	007	015	030	055	092	143
19	000	000	001	002	004	009	018	035	061	098
20	000	000	000	001	002	005	011	021	039	066
21	000	000	000	000	001	003	006	013	025	043
22	000	000	000	000	001	002	004	008	015	028
23	000	000	000	000	000	001	002	004	009	017
24	000	000	000	000	000	000	001	003	005	011
25	000	000	000	000	000	000	001	001	003	006
	14	15	16	17	18	19	20	21	22	23
16	368	467	566	659	742	812	868	911	942	963
17	281	371	468	564	655	736	805	861	905	937
18	208	287	375	469	562	651	731	799	855	899
19	150	215	292	378	469	561	647	725	793	849
20	105	157	221	297	381	470	559	644	721	787
21	072	111	163	227	302	384	471	558	640	716
22	048	077	117	169	232	306	387	472	556	637
23	031	052	082	123	175	238	310	389	472	555
24	020	034	056	087	128	180	243	314	392	473
25	012	022	038	060	092	134	185	247	318	394
	24	25	26	27	28	29	30	31	32	33
16	978	987	993	996	998	999	999	1,000		
17	959	975	985	991	995	997	999	999	1,000	
18	932	955	972	983	990	994	997	998	999	1,000
19	893	927	951	969	980	988	993	996	998	999
20	843	888	922	948	966	978	987	992	995	997
21	782	838	883	917	944	963	976	985	991	994
22	712	777	832	877	913	940	959	973	983	989
23	635	708	772	827	873	908	936	956	971	981
24	554	632	704	768	823	868	904	932	953	969
25	473	553	629	700	763	818	863	900	929	950
	34	35	36	37	38	39	40	41	42	43
19	999	1,000								
20	999	999	1,000							
21	997	998	999	999	1,000					
22	994	996	998	999	999	1,000				
23	988	993	996	997	999	999	1,000			
24	979	987	992	995	997	998	999	999	1,000	
25	966	978	985	991	994	997	998	999	999	1,000

Table 5-8*a*. Selected percentage points of the gamma distribution: Values of x/β corresponding to stated values of $F(x)$*

α \ F	.01	.05	.10	.25	.50	.75	.90	.95	.99
−.5	.0000785	.00197	.00790	.0508	.227	.662	1.353	1.921	3.317
0	.01005	.0513	.105	.288	.693	1.386	2.303	2.996	4.605
.5	.0574	.176	.292	.606	1.183	2.054	3.126	3.907	5.672
1.0	.149	.355	.532	.961	1.678	2.693	3.890	4.744	6.638
1.5	.277	.573	.805	1.337	2.176	3.313	4.618	5.535	7.543
2.0	.436	.818	1.102	1.727	2.674	3.920	5.322	6.296	8.406
2.5	.620	1.084	1.417	2.127	3.173	4.519	6.008	7.034	9.238
3.0	.823	1.366	1.745	2.535	3.672	5.109	6.681	7.754	10.045
3.5	1.044	1.663	2.084	2.949	4.171	5.694	7.342	8.460	10.833
4.0	1.279	1.970	2.433	3.369	4.671	6.274	7.994	9.154	11.605
4.5	1.527	2.287	2.789	3.792	5.170	6.850	8.638	9.838	12.362
5.0	1.785	2.613	3.152	4.219	5.670	7.423	9.275	10.513	13.108
5.5	2.053	2.946	3.521	4.650	6.170	7.992	9.906	11.181	13.844
6.0	2.330	3.285	3.895	5.083	6.670	8.558	10.532	11.842	14.571
6.5	2.615	3.630	4.273	5.518	7.169	9.123	11.154	12.498	15.289
7.0	2.906	3.981	4.656	5.956	7.669	9.684	11.771	13.148	16.000
7.5	3.204	4.336	5.043	6.396	8.169	10.244	12.384	13.794	16.704
8.0	3.507	4.695	5.432	6.838	8.669	10.802	12.995	14.435	17.403
8.5	3.816	5.058	5.825	7.281	9.169	11.359	13.602	15.072	18.095
9.0	4.130	5.425	6.221	7.726	9.669	11.914	14.206	15.705	18.783
9.5	4.449	5.796	6.620	8.172	10.169	12.467	14.808	16.335	19.466
10.0	4.771	6.169	7.021	8.620	10.668	13.020	15.407	16.962	20.145
11.0	5.428	6.924	7.829	9.519	11.668	14.121	16.598	18.208	21.490
12.0	6.099	7.690	8.646	10.422	12.668	15.217	17.782	19.443	22.821
13.0	6.782	8.464	9.470	11.329	13.668	16.310	18.958	20.669	24.139
14.0	7.477	9.246	10.300	12.239	14.668	17.400	20.128	21.886	25.446
15.0	8.180	10.035	11.135	13.152	15.668	18.487	21.293	23.098	26.744
20.0	11.825	14.072	15.382	17.755	20.668	23.883	27.045	29.062	33.104
25.0	15.623	18.218	19.717	22.404	25.667	29.234	32.711	34.916	39.308
30.0	19.532	22.444	24.113	27.085	30.667	34.552	38.315	40.691	45.401

* Adapted from Catherine M. Thompson, Tables of Percentage Points of the Chi-square Distribution, *Biometrika*, vol. 32, pt. II, p. 187, October, 1941. Reprinted from G. A. Wadsworth and J. G. Bryan, "Introduction to Probability and Random Variables," McGraw-Hill Book Company, New York, 1960.

Table 5-9. Fractiles of the beta distribution (continued)

$I_x(\alpha,\beta) = 1 - I_{1-x}(\beta,\alpha) = 0.025$

	β										
α	1	2	3	4	5	6	10	15	20	30	60
1	.0250	.0126	.0084	.0063	.0051	.0042	.0025	.0017	.0013	.0008	.0004
2	.1581	.0943	.0676	.0527	.0433	.0367	.0228	.0155	.0117	.0079	.0040
3	.2924	.1941	.1466	.1181	.0990	.0852	.0549	.0380	.0291	.0198	.0101
4	.3976	.2836	.2228	.1840	.1570	.1370	.0909	.0641	.0495	.0340	.0176
5	.4782	.3588	.2904	.2449	.2120	.1871	.1276	.0915	.0713	.0495	.0259
6	.5407	.4213	.3491	.2993	.2624	.2338	.1634	.1189	.0936	.0656	.0346
7	.5904	.4735	.3999	.3476	.3079	.2767	.1975	.1459	.1157	.0819	.0437
8	.6306	.5175	.4439	.3903	.3489	.3158	.2298	.1720	.1375	.0983	.0530
9	.6637	.5550	.4822	.4281	.3857	.3514	.2602	.1971	.1588	.1144	.0623
10	.6915	.5872	.5159	.4619	.4190	.3838	.2886	.2211	.1794	.1304	.0717
11	.7151	.6152	.5455	.4920	.4490	.4134	.3153	.2440	.1993	.1460	.0811
12	.7354	.6397	.5719	.5191	.4762	.4404	.3402	.2659	.2185	.1613	.0905
13	.7530	.6613	.5954	.5435	.5010	.4652	.3636	.2867	.2370	.1762	.0998
14	.7684	.6805	.6165	.5657	.5236	.4880	.3854	.3065	.2548	.1908	.1090
15	.7820	.6977	.6356	.5858	.5444	.5090	.4059	.3253	.2718	.2049	.1181
20	.8316	.7618	.7084	.6641	.6262	.5930	.4917	.4070	.3478	.2700	.1620
30	.8843	.8330	.7919	.7567	.7255	.6974	.6067	.5242	.4624	.3750	.2403
60	.9404	.9120	.8883	.8671	.8476	.8295	.7668	.7030	.6502	.5666	.4111
∞	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$I_x(\alpha,\beta) = 1 - I_{1-x}(\beta,\alpha) = 0.05$

	β										
α	1	2	3	4	5	6	10	15	20	30	60
1	.0500	.0253	.0170	.0127	.0102	.0085	.0051	.0034	.0026	.0017	.0009
2	.2236	.1354	.0976	.0764	.0628	.0534	.0333	.0227	.0172	.0116	.0059
3	.3684	.2486	.1893	.1532	.1288	.1111	.0719	.0499	.0382	.0260	.0133
4	.4729	.3426	.2713	.2253	.1929	.1688	.1127	.0797	.0617	.0425	.0220
5	.5493	.4182	.3413	.2892	.2514	.2224	.1527	.1099	.0859	.0598	.0313
6	.6070	.4793	.4003	.3449	.3035	.2712	.1909	.1396	.1101	.0774	.0410
7	.6518	.5293	.4504	.3934	.3498	.3152	.2267	.1682	.1338	.0950	.0508
8	.6877	.5709	.4931	.4356	.3909	.3548	.2601	.1956	.1568	.1124	.0608
9	.7169	.6058	.5299	.4727	.4274	.3904	.2912	.2216	.1791	.1295	.0708
10	.7411	.6356	.5619	.5054	.4600	.4226	.3201	.2464	.2005	.1462	.0808
11	.7616	.6613	.5899	.5343	.4892	.4516	.3469	.2698	.2211	.1625	.0907
12	.7791	.6837	.6146	.5602	.5156	.4781	.3719	.2921	.2408	.1784	.1005
13	.7942	.7033	.6366	.5834	.5394	.5022	.3952	.3131	.2597	.1938	.1102
14	.8074	.7206	.6562	.6044	.5611	.5242	.4168	.3331	.2778	.2088	.1198
15	.8190	.7360	.6738	.6233	.5809	.5444	.4371	.3520	.2951	.2233	.1293
20	.8609	.7933	.7405	.6964	.6582	.6246	.5210	.4332	.3714	.2894	.1745
30	.9050	.8559	.8161	.7815	.7507	.7228	.6318	.5481	.4848	.3946	.2542
60	.9513	.9246	.9019	.8815	.8627	.8450	.7834	.7202	.6674	.5833	.4252
∞	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$I_x(\alpha,\beta) = 1 - I_{1-x}(\beta,\alpha) = 0.1$

	β										
α	1	2	3	4	5	6	10	15	20	30	60
1	.1000	.0513	.0345	.0260	.0209	.0174	.0105	.0070	.0053	.0035	.0018
2	.3162	.1958	.1426	.1122	.0926	.0788	.0495	.0337	.0256	.0173	.0088
3	.4642	.3205	.2466	.2009	.1696	.1468	.0957	.0667	.0512	.0349	.0179
4	.5623	.4161	.3332	.2786	.2397	.2104	.1416	.1006	.0781	.0539	.0280
5	.6310	.4897	.4038	.3446	.3010	.2673	.1851	.1339	.1050	.0733	.0385
6	.6813	.5474	.4618	.4006	.3542	.3177	.2256	.1659	.1312	.0926	.0492
7	.7197	.5938	.5099	.4483	.4005	.3623	.2629	.1962	.1566	.1116	.0600
8	.7499	.6316	.5504	.4892	.4410	.4018	.2973	.2248	.1809	.1302	.0708
9	.7743	.6632	.5848	.5247	.4766	.4369	.3288	.2518	.2042	.1483	.0815
10	.7943	.6898	.6145	.5557	.5080	.4683	.3579	.2772	.2264	.1658	.0921
11	.8111	.7125	.6402	.5830	.5360	.4965	.3848	.3011	.2476	.1828	.1026
12	.8254	.7322	.6628	.6072	.5611	.5219	.4095	.3236	.2678	.1993	.1129
13	.8377	.7493	.6827	.6288	.5836	.5450	.4325	.3448	.2870	.2152	.1231
14	.8483	.7644	.7004	.6481	.6040	.5660	.4538	.3648	.3053	.2305	.1331
15	.8577	.7778	.7163	.6656	.6225	.5851	.4736	.3837	.3228	.2454	.1430
20	.8912	.8271	.7758	.7322	.6941	.6603	.5548	.4639	.3991	.3124	.1896
30	.9261	.8802	.8421	.8086	.7785	.7510	.6603	.5754	.5107	.4175	.2706
60	.9624	.9377	.9164	.8970	.8790	.8620	.8019	.7395	.6869	.6024	.4416
∞	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

section

6

CLASSICAL STATISTICAL TESTS

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This section of tables consists of tables for various "classical" tests of significance and related tests. Most of these tests require strong assumptions about the nature of the population distribution. The most frequent applications are described in any of a number of standard texts on statistical applications.

Table 6-1 is used with *Fisher's exact test for comparing two proportions* on the basis of independent samples drawn from two different populations. Suppose the data are arranged in the following way:

Sample	$-$	$+$
1	A	B
2	C	D

The letters A, B, C, and D denote frequencies of the properties "$-$" and "$+$" in the two samples. The exact probability of observing a given set of frequencies, under the hypothesis that the theoretical proportion of $+$'s is the same in both populations, is given by the distribution of D (or A, or B, or C) if the marginal totals are regarded as fixed. The distribution of D is the *hypergeometric distribution.* The significance levels given in Table 6-1 err on the conservative side and correspond to a *one-tailed* critical region. For a *two-tailed test,* the significance levels are *doubled.*

Table 6-2 shows *confidence belts for proportions* based on samples of size N drawn from populations in which the true proportion is p. The area outside of a given confidence interval is a two-tailed (or one-tailed) critical region for rejecting the hypothesis that a sample proportion occurred by random sampling from a population in which the true proportion is the value of p corresponding to the confidence interval, with significance level equal to one minus the confidence coefficient (or one-half of one minus the confidence coefficient).

Table 6-3 lists *fractiles of student's t distribution* for various values of the parameter df (degrees of freedom). The t statistic is widely applied for testing hypotheses about means of normally distributed random variables and has a number of other uses (e.g., testing hypotheses about regression coefficients) as well. The entries in Table 6-3 are values of t corresponding to the various cumulative probabilities listed at the top of each column. Since the distribution of t is symmetrical, values of t corresponding to cumulative probabilities of 0.30, 0.20, etc., may be obtained by attaching negative signs to the entries in the column labeled 0.70, 0.30, and so on. A one-tailed critical region is given, depending on the nature of the alternative hypothesis, by

$$t_{\text{OBS}} \geq t_{1-\alpha}(df)$$

or

$$t_{\text{OBS}} \leq -t_{1-\alpha}(df)$$

where t_{OBS} is an observed value of t, α is the desired level of significance, and $t_{1-\alpha}(df)$ is the $1 - \alpha$ fractile of the distribution of t with df degrees of freedom. Two-tailed critical regions are obtained from

$$|t_{\mathrm{OBS}}| \geq t_{1-\alpha/2}(df)$$

or essentially by doubling one minus the percentage values listed at the top of Table 6-3.

Table 6-4 is a table of the *power of the t test* against various alternatives. In both tables, the quantity β is the probability of a Type II error, $1 - \beta$ is the *power*, and α is the *significance level* or probability of a Type I error. The quantity d' used in Table 6-4*c* is the so-called *noncentrality parameter of the noncentral t distribution* and, in most applications, is one of the two designated functions of hypothetical means and population standard deviation. Table 6-4*b* gives, for various values of degrees of freedom and α, the value of d required for the power listed at the top of each column. Table 6-4*c* shows, for various values of α and d', the sample size required for the power listed at the top of each column. One can use Table 6-4*b* with the *t test for matched or repeated measures* simply by regarding the difference score as the variable of interest.

If one of k treatments in an experiment represents a control condition, it is frequently desired to compare each of $k - 1$ remaining means with the control-group mean, with a significance level of α for the aggregate of the $k - 1$ tests. Since each test uses the same control-group mean and a common estimate of the sampling variance of the difference between means, the tests are not independent. The t statistic for the difference between condition i and the control condition is

$$t = \frac{M_i - M_c}{\sqrt{2MS_{\mathrm{error}}/N}}$$

where M_i and M_c denote the treatment and control means, respectively, N is the number of observations each mean is based on, and MS_{error} denotes the pooled within-group variance (ref. 2). The parameters for evaluating the distribution of the $(k - 1)t$ statistics are k and df, the number of degrees of freedom for MS_{error}. Table 6-5 contains *critical values of t for two-tailed tests* corresponding to various values of the significance level α and the two parameters k and df. The hypothesis of "no difference" is rejected if the absolute value of the computed t for a given comparison exceeds the appropriate tabled value.

Table 6-6 shows *fractiles of the distribution of the F ratio*, i.e., values of F corresponding to given cumulative probabilities. The distribution of F has two parameters labeled n and m; n is the number of degrees of freedom for the denominator of the F ratio, and m is the number of degrees of freedom for the numerator. Some other percentage points of F may be determined from the percentage points of the incomplete β function (Table 5-9) by means of the relationship

$$\text{Prob }(F_{m,n} > F_0) = I_{x_0}(m/2, n/2)$$

with

$$x_0 = \frac{mF_0}{n + mF_0}$$

Values of F corresponding to $G = 0.10, 0.05, 0.025$, etc., may be determined from

$$F_{m,n}(1 - \alpha) = \frac{1}{F_{n,m}(\alpha)}$$

The F distribution finds its greatest use in *analysis of variance* where one ordinarily tests the equality of several means ($\mu_1 = \mu_2 = \cdot \cdot \cdot \mu_k$) against the alternative that the means are not all the same. A particular alternative can be characterized by the quantity

$$\phi^2 = \frac{\Sigma(\mu_i - \bar{\mu})^2/k}{\sigma^2/N}$$

where $\bar{\mu}$ is the average of the hypothetical alternative means, σ^2 is the theoretical value of the variance within groups or mean square error, and N is the number of observations from each population.

Table 6-7 shows graphs of the *power of the F test* against various alternatives characterized by ϕ. Ordinates give the value of $1 - \beta$ for each abscissa ϕ, where β is the probability of a Type II error.

A simple but adequate *test of the homogeneity of k within-group variances* in an analysis-of-variance experiment employs the test statistic

$$F_{\max} = \frac{\text{largest of } k \text{ treatment variances}}{\text{smallest of } k \text{ treatment variances}}$$

All k variances must be based on the same sample size.

Table 6-8 lists *critical values of the* $F_{\max}$ *statistic* for different values of k and $N - 1$. If an observed $F_{\max}$ exceeds the tabulated value associated with the α level of significance, the hypothesis of homogeneity of variance is rejected.

Cochran's test similarly uses the statistic

$$C = \frac{\text{the largest of } k \text{ treatment variances}}{\text{the sum of } k \text{ treatment variances}}$$

The parameters of the sampling distribution of C, when the hypothesis of homogeneity is true, are k and $N - 1$, the degrees of freedom for each of the variances.

Table 6-9 lists *critical values of C for the levels of significance* $\alpha = 0.05$ *and* $\alpha = 0.01$. The hypothesis of homogeneity of variance is rejected if the observed value of C exceeds the appropriate tabulated value.

Although the hypothesis that several means are equal is usually tested with an F ratio, the *studentized range* statistic defined by

$$q = \frac{M_{\max} - M_{\min}}{\sqrt{NMS_{\text{error}}}}$$

where $M_{\max}$ and $M_{\min}$ denote the largest and smallest sample means, respectively, N is the number of observations each mean is based on, and MS_{error} denotes the within-groups variance, can also be used to test the hypothesis $\mu_1 = \mu_2 = \cdots = \mu_r$. The sampling distribution of q is approximated by the distribution of the "studentized range" with parameters r = number of treatments and df = degrees of freedom for the mean square error.

Table 6-10 contains *critical values of the studentized range* for two levels of significance, $\alpha = 0.05$ and $\alpha = 0.01$, and various values of r and df. The hypothesis of equality of means is rejected if the observed value of q exceeds the tabulated value.

Table 6-11 lists *fractiles of the chi-square distribution* for various values of the parameter df. That is, the entries in the table are values of chi-square such that the cumulative probability or area to the left of a specified value is equivalent to the per cent indicated at the top of the corresponding column. The significance level for a statistic distributed as chi-square is then one minus the cumulative probability. The chi-square distribution is simply that of a gamma-distributed variate with $\beta = 2$ and $\alpha = (n/2 - 1)$, in the notation used in conjunction with Table 5-8. The quantity n must be integer-valued and corresponds to the degrees-of-freedom (df) parameter in chi-square applications. Because of this, Table 5-8 can be utilized to determine values of chi-square for percentiles not included in Table 6-11, and vice versa. The connection between the gamma distribution and the chi-square distribution also permits use of the normal approximations discussed in connection with the gamma distribution for large values of df.

The chi-square distribution finds wide usage, principally as an asymptotic distribution used to approximate the exact sampling distribution of many statistics. Perhaps the most important and extensive use of chi-square statistics is in connection with enumeration statistics, that is, in comparing observed and expected frequencies in (1) testing *hypotheses about the mathematical form of a single distribution*, (2) testing *the hypothesis that two or more variates or attributes are identically distributed*, and (3) testing *the hypothesis that variates or attributes are statistically independent*.

Still other uses depend on the fact that sums of squares of normally distributed random variables are distributed as chi-square when multiplied by an appropriate constant. The sample variance of a normally distributed variate, when divided by the theoretical variance, is distributed like "chi-square/df," i.e., like the ratio of a chi-square variate to its parameter df.

Table 6-12 shows *fractiles of chi-square/df* for various values of df. This table is particularly useful for constructing confidence intervals for variances and can also be used in place of a chi-square distribution table.

Table 6-1. Fisher's exact test for proportions*

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 3$	$C + D = 3$	3	0	—	—	—
$A + B = 4$	$C + D = 4$	4	0	0	—	—
	$C + D = 3$	4	0	—	—	—
$A + B = 5$	$C + D = 5$	5	1	1	0	0
		4	0	0	—	—
	$C + D = 4$	5	1	0	0	—
		4	0	—	—	—
	$C + D = 3$	5	0	0	—	—
	$C + D = 2$	5	0	—	—	—
$A + B = 6$	$C + D = 6$	6	2	1	1	0
		5	1	0	0	—
		4	0	—	—	—
	$C + D = 5$	6	1	0	0	0
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 4$	6	1	0	0	0
		5	0	0	—	—
	$C + D = 3$	6	0	0	—	—
		5	0	—	—	—
	$C + D = 2$	6	0	—	—	—
$A + B = 7$	$C + D = 7$	7	3	2	1	1
		6	1	1	0	0
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 6$	7	2	2	1	1
		6	1	0	0	0
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 5$	7	2	1	0	0
		6	1	0	0	—
		5	0	—	—	—
	$C + D = 4$	7	1	1	0	0
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 3$	7	0	0	0	—
		6	0	—	—	—
	$C + D = 2$	7	0	—	—	—

* Adapted from D. J. Finney, The Fisher-Yates Test of Significance in 2 × 2 Contingency Tables, *Biometrika*, vol. 35, pp. 149–154, 1948, with the kind permission of the author and the publisher. Reprinted from Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 8$	$C + D = 8$	8	4	3	2	2
		7	2	2	1	0
		6	1	1	0	0
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 7$	8	3	2	2	1
		7	2	1	1	0
		6	1	0	0	—
		5	0	0	—	—
	$C + D = 6$	8	2	2	1	1
		7	1	1	0	0
		6	0	0	0	—
		5	0	—	—	—
	$C + D = 5$	8	2	1	1	0
		7	1	0	0	0
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 4$	8	1	1	0	0
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 3$	8	0	0	0	—
		7	0	0	—	—
	$C + D = 2$	8	0	0	—	—
$A + B = 9$	$C + D = 9$	9	5	4	3	3
		8	3	3	2	1
		7	2	1	1	0
		6	1	1	0	0
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 8$	9	4	3	3	2
		8	3	2	1	1
		7	2	1	0	0
		6	1	0	0	—
		5	0	0	—	—
	$C + D = 7$	9	3	3	2	2
		8	2	2	1	0
		7	1	1	0	0
		6	0	0	—	—
		5	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

Table 6-1. **Fisher's exact test for proportions (continued)**

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 9$	$C + D = 6$	9	3	2	1	1
		8	2	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 5$	9	2	1	1	1
		8	1	1	0	0
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 4$	9	1	1	0	0
		8	0	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 3$	9	1	0	0	0
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 2$	9	0	0	—	—
$A + B = 10$	$C + D = 10$	10	6	5	4	3
		9	4	3	3	2
		8	3	2	1	1
		7	2	1	1	0
		6	1	0	0	—
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 9$	10	5	4	3	3
		9	4	3	2	2
		8	2	2	1	1
		7	1	1	0	0
		6	1	0	0	—
		5	0	0	—	—
	$C + D = 8$	10	4	4	3	2
		9	3	2	2	1
		8	2	1	1	0
		7	1	1	0	0
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 7$	10	3	3	2	2
		9	2	2	1	1
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 10$	$C + D = 6$	10	3	2	2	1
		9	2	1	1	0
		8	1	1	0	0
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 5$	10	2	2	1	1
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 4$	10	1	1	0	0
		9	1	0	0	0
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 3$	10	1	0	0	0
		9	0	0	—	—
		8	0	—	—	—
	$C + D = 2$	10	0	0	—	—
		9	0	—	—	—
$A + B = 11$	$C + D = 11$	11	7	6	5	4
		10	5	4	3	3
		9	4	3	2	2
		8	3	2	1	1
		7	2	1	0	0
		6	1	0	0	—
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 10$	11	6	5	4	4
		10	4	4	3	2
		9	3	3	2	1
		8	2	2	1	0
		7	1	1	0	0
		6	1	0	0	—
		5	0	—	—	—
	$C + D = 9$	11	5	4	4	3
		10	4	3	2	2
		9	3	2	1	1
		8	2	1	1	0
		7	1	1	0	0
		6	0	0	—	—
		5	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 11$	$C + D = 8$	11	4	4	3	3
		10	3	3	2	1
		9	2	2	1	1
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 7$	11	4	3	2	2
		10	3	2	1	1
		9	2	1	1	0
		8	1	1	0	0
		7	0	0	—	—
		6	0	0	—	—
	$C + D = 6$	11	3	2	2	1
		10	2	1	1	0
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 5$	11	2	2	1	1
		10	1	1	0	0
		9	1	0	0	0
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 4$	11	1	1	1	0
		10	1	0	0	0
		9	0	0	—	—
		8	0	—	—	—
	$C + D = 3$	11	1	0	0	0
		10	0	0	—	—
		9	0	—	—	—
	$C + D = 2$	11	0	0	—	—
		10	0	—	—	—
$A + B = 12$	$C + D = 12$	12	8	7	6	5
		11	6	5	4	4
		10	5	4	3	2
		9	4	3	2	1
		8	3	2	1	1
		7	2	1	0	0
		6	1	0	0	—
		5	0	0	—	—
		4	0	—	—	—

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 12$	$C + D = 11$	12	7	6	5	5
		11	5	5	4	3
		10	4	3	2	2
		9	3	2	2	1
		8	2	1	1	0
		7	1	1	0	0
		6	1	0	0	—
		5	0	0	—	—
	$C + D = 10$	12	6	5	5	4
		11	5	4	3	3
		10	4	3	2	2
		9	3	2	1	1
		8	2	1	0	0
		7	1	0	0	0
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 9$	12	5	5	4	3
		11	4	3	3	2
		10	3	2	2	1
		9	2	2	1	0
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 8$	12	5	4	3	3
		11	3	3	2	2
		10	2	2	1	1
		9	2	1	1	0
		8	1	1	0	0
		7	0	0	—	—
		6	0	0	—	—
	$C + D = 7$	12	4	3	3	2
		11	3	2	2	1
		10	2	1	1	0
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 12$	$C + D = 6$	12	3	3	2	2
		11	2	2	1	1
		10	1	1	0	0
		9	1	0	0	0
		8	0	0	—	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 5$	12	2	2	1	1
		11	1	1	1	0
		10	1	0	0	0
		9	0	0	0	—
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 4$	12	2	1	1	0
		11	1	0	0	0
		10	0	0	0	—
		9	0	0	—	—
		8	0	—	—	—
	$C + D = 3$	12	1	0	0	0
		11	0	0	0	—
		10	0	0	—	—
		9	0	—	—	—
	$C + D = 2$	12	0	0	—	—
		11	0	—	—	—
$A + B = 13$	$C + D = 13$	13	9	8	7	6
		12	7	6	5	4
		11	6	5	4	3
		10	4	4	3	2
		9	3	3	2	1
		8	2	2	1	0
		7	2	1	0	0
		6	1	0	0	—
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 12$	13	8	7	6	5
		12	6	5	5	4
		11	5	4	3	3
		10	4	3	2	2
		9	3	2	1	1
		8	2	1	1	0
		7	1	1	0	0
		6	1	0	0	—
		5	0	0	—	—

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 13$	$C + D = 11$	13	7	6	5	5
		12	6	5	4	3
		11	4	4	3	2
		10	3	3	2	1
		9	3	2	1	1
		8	2	1	0	0
		7	1	0	0	0
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 10$	13	6	6	5	4
		12	5	4	3	3
		11	4	3	2	2
		10	3	2	1	1
		9	2	1	1	0
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 9$	13	5	5	4	4
		12	4	4	3	2
		11	3	3	2	1
		10	2	2	1	1
		9	2	1	0	0
		8	1	1	0	0
		7	0	0	—	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 8$	13	5	4	3	3
		12	4	3	2	2
		11	3	2	1	1
		10	2	1	1	0
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 7$	13	4	3	3	2
		12	3	2	2	1
		11	2	2	1	1
		10	1	1	0	0
		9	1	0	0	0
		8	0	0	—	—
		7	0	0	—	—
		6	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 13$	$C + D = 6$	13	3	3	2	2
		12	2	2	1	1
		11	2	1	1	0
		10	1	1	0	0
		9	1	0	0	—
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 5$	13	2	2	1	1
		12	2	1	1	0
		11	1	1	0	0
		10	1	0	0	—
		9	0	0	—	—
		8	0	—	—	—
	$C + D = 4$	13	2	1	1	0
		12	1	1	0	0
		11	0	0	0	—
		10	0	0	—	—
		9	0	—	—	—
	$C + D = 3$	13	1	1	0	0
		12	0	0	0	—
		11	0	0	—	—
		10	0	—	—	—
	$C + D = 2$	13	0	0	0	—
		12	0	—	—	—
$A + B = 14$	$C + D = 14$	14	10	9	8	7
		13	8	7	6	5
		12	6	6	5	4
		11	5	4	3	3
		10	4	3	2	2
		9	3	2	2	1
		8	2	2	1	0
		7	1	1	0	0
		6	1	0	0	—
		5	0	0	—	—
		4	0	—	—	—

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 14$	$C + D = 13$	14	9	8	7	6
		13	7	6	5	5
		12	6	5	4	3
		11	5	4	3	2
		10	4	3	2	2
		9	3	2	1	1
		8	2	1	1	0
		7	1	1	0	0
		6	1	0	—	—
		5	0	0	—	—
	$C + D = 12$	14	8	7	6	6
		13	6	6	5	4
		12	5	4	4	3
		11	4	3	3	2
		10	3	3	2	1
		9	2	2	1	1
		8	2	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 11$	14	7	6	6	5
		13	6	5	4	4
		12	5	4	3	3
		11	4	3	2	2
		10	3	2	1	1
		9	2	1	1	0
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 10$	14	6	6	5	4
		13	5	4	4	3
		12	4	3	3	2
		11	3	3	2	1
		10	2	2	1	1
		9	2	1	0	0
		8	1	1	0	0
		7	0	0	0	—
		6	0	0	—	—
		5	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 14$	$C + D = 9$	14	6	5	4	4
		13	4	4	3	3
		12	3	3	2	2
		11	3	2	1	1
		10	2	1	1	0
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 8$	14	5	4	4	3
		13	4	3	2	2
		12	3	2	2	1
		11	2	2	1	1
		10	2	1	0	0
		9	1	0	0	0
		8	0	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 7$	14	4	3	3	2
		13	3	2	2	1
		12	2	2	1	1
		11	2	1	1	0
		10	1	1	0	0
		9	1	0	0	—
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 6$	14	3	3	2	2
		13	2	2	1	1
		12	2	1	1	0
		11	1	1	0	0
		10	1	0	0	—
		9	0	0	—	—
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 5$	14	2	2	1	1
		13	2	1	1	0
		12	1	1	0	0
		11	1	0	0	0
		10	0	0	—	—
		9	0	0	—	—
		8	0	—	—	—

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 14$	$C + D = 4$	14	2	1	1	1
		13	1	1	0	0
		12	1	0	0	0
		11	0	0	—	—
		10	0	0	—	—
		9	0	—	—	—
	$C + D = 3$	14	1	1	0	0
		13	0	0	0	—
		12	0	0	—	—
		11	0	—	—	—
	$C + D = 2$	14	0	0	0	—
		13	0	0	—	—
		12	0	—	—	—
$A + B = 15$	$C + D = 15$	15	11	10	9	8
		14	9	8	7	6
		13	7	6	5	5
		12	6	5	4	4
		11	5	4	3	3
		10	4	3	2	2
		9	3	2	1	1
		8	2	1	1	0
		7	1	1	0	0
		6	1	0	0	—
		5	0	0	—	—
		4	0	—	—	—
	$C + D = 14$	15	10	9	8	7
		14	8	7	6	6
		13	7	6	5	4
		12	6	5	4	3
		11	5	4	3	2
		10	4	3	2	1
		9	3	2	1	1
		8	2	1	1	0
		7	1	1	0	0
		6	1	0	—	—
		5	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 15$	$C + D = 13$	15	9	8	7	7
		14	7	7	6	5
		13	6	5	4	4
		12	5	4	3	3
		11	4	3	2	2
		10	3	2	2	1
		9	2	2	1	0
		8	2	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 12$	15	8	7	7	6
		14	7	6	5	4
		13	6	5	4	3
		12	5	4	3	2
		11	4	3	2	2
		10	3	2	1	1
		9	2	1	1	0
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 11$	15	7	7	6	5
		14	6	5	4	4
		13	5	4	3	3
		12	4	3	2	2
		11	3	2	2	1
		10	2	2	1	1
		9	2	1	0	0
		8	1	1	0	0
		7	1	0	0	—
		6	0	0	—	—
		5	0	—	—	—
	$C + D = 10$	15	6	6	5	5
		14	5	5	4	3
		13	4	4	3	2
		12	3	3	2	2
		11	3	2	1	1
		10	2	1	1	0
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—

Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 15$	$C + D = 9$	15	6	5	4	4
		14	5	4	3	3
		13	4	3	2	2
		12	3	2	2	1
		11	2	2	1	1
		10	2	1	0	0
		9	1	1	0	0
		8	1	0	0	—
		7	0	0	—	—
		6	0	—	—	—
	$C + D = 8$	15	5	4	4	3
		14	4	3	3	2
		13	3	2	2	1
		12	2	2	1	1
		11	2	1	1	0
		10	1	1	0	0
		9	1	0	0	—
		8	0	0	—	—
		7	0	—	—	—
		6	0	—	—	—
	$C + D = 7$	15	4	4	3	3
		14	3	3	2	2
		13	2	2	1	1
		12	2	1	1	0
		11	1	1	0	0
		10	1	0	0	0
		9	0	0	—	—
		8	0	0	—	—
		7	0	—	—	—
	$C + D = 6$	15	3	3	2	2
		14	2	2	1	1
		13	2	1	1	0
		12	1	1	0	0
		11	1	0	0	0
		10	0	0	0	—
		9	0	0	—	—
		8	0	—	—	—
	$C + D = 5$	15	2	2	2	1
		14	2	1	1	1
		13	1	1	0	0
		12	1	0	0	0
		11	0	0	0	—
		10	0	0	—	—
		9	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

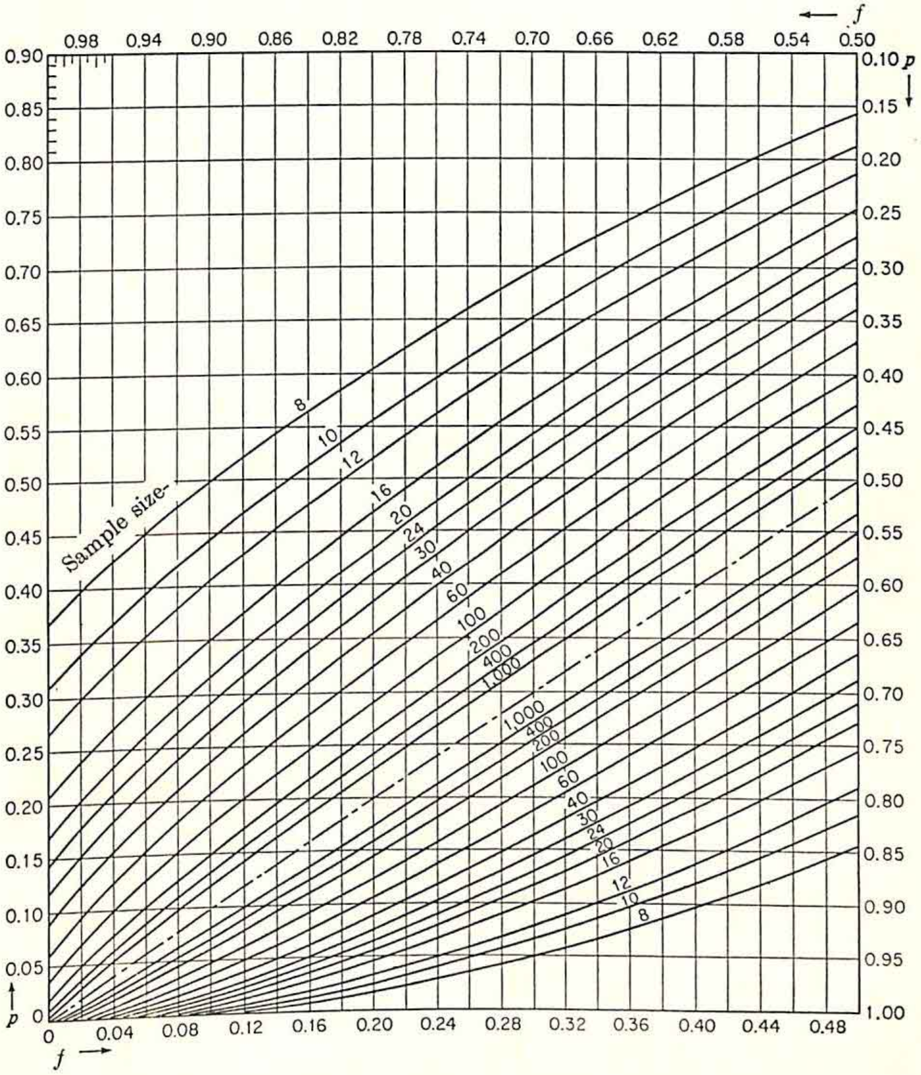
Table 6-1. Fisher's exact test for proportions (continued)

Totals in right margin		B (or A)†	Level of significance			
			.05	.025	.01	.005
$A + B = 15$	$C + D = 4$	15	2	1	1	1
		14	1	1	0	0
		13	1	0	0	0
		12	0	0	0	—
		11	0	0	—	—
		10	0	—	—	—
	$C + D = 3$	15	1	1	0	0
		14	0	0	0	0
		13	0	0	—	—
		12	0	0	—	—
		11	0	—	—	—
	$C + D = 2$	15	0	0	0	—
		14	0	0	—	—
		13	0	—	—	—

† When B is entered in the middle column, the significance levels are for D. When A is used in place of B, the significance levels are for C.

TABLE 6-2. CONFIDENCE INTERVALS FOR PROBABILITIES p ESTIMATED FROM RANDOM-SAMPLE STATISTICAL RELATIVE FREQUENCIES f*

Table 6-2*a*. Confidence coefficient, $1 - \alpha = 0.95$



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* Figures reprinted by permission from E. S. Pearson and H. O. Hartley, (eds.), "Biometrica Tables for Statisticians," 2d ed., vol. 1, pp. 204–205, Cambridge University Press, New York, 1958. Reprinted from Leon Levine, "Methods for Solving Engineering Problems," McGraw-Hill Book Company, New York, 1964.

Table 6-2b. Confidence coefficient, $1 - \alpha = 0.99$

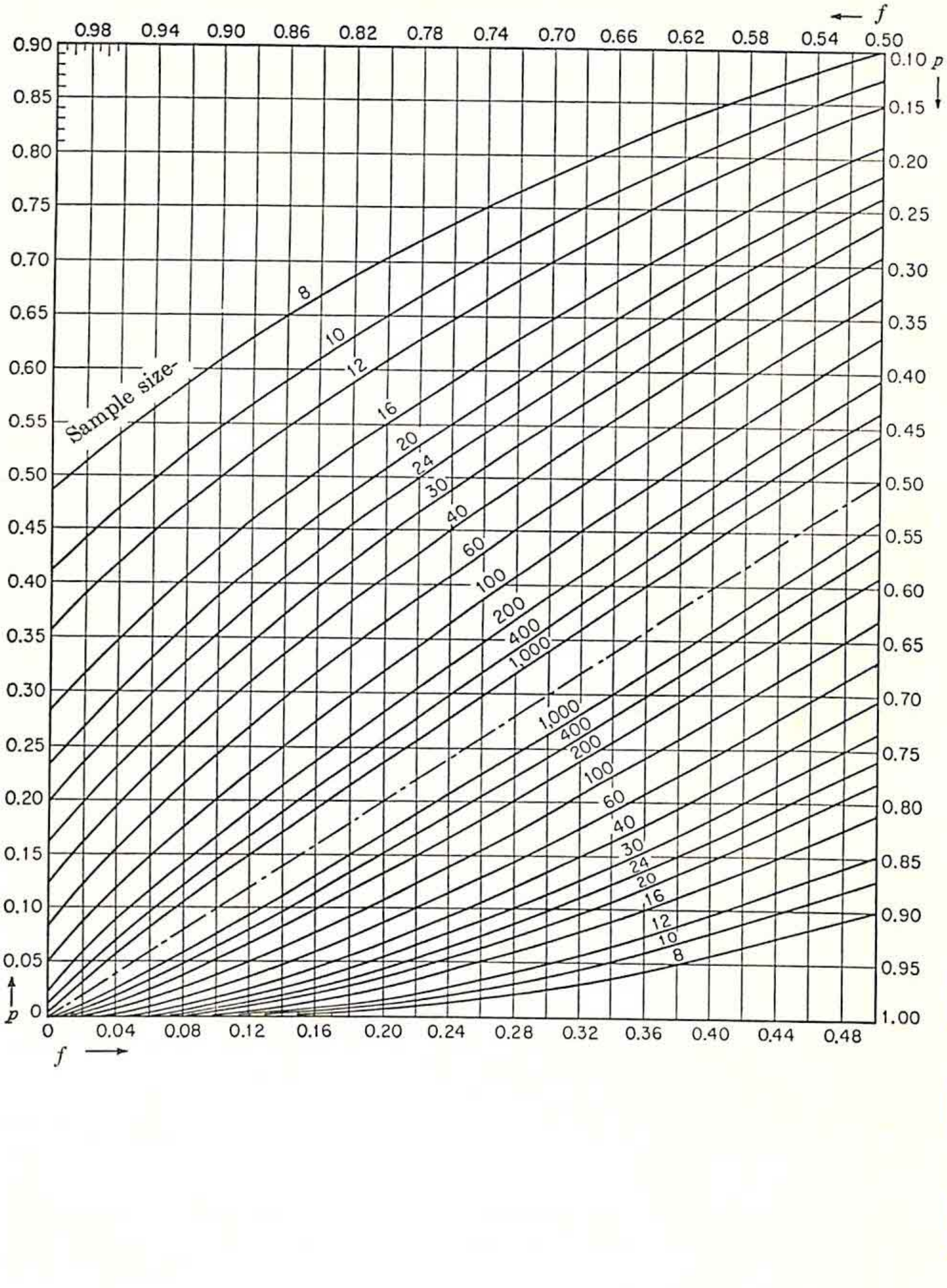


Table 6-3. Student's *t* distribution*

df	Percentile point						
	70	80	90	95	97.5	99	99.5
1	.73	1.38	3.08	6.31	12.71	31.82	63.66
2	.62	1.06	1.89	2.92	4.30	6.96	9.92
3	.58	.98	1.64	2.35	3.18	4.54	5.84
4	.57	.94	1.53	2.13	2.78	3.75	4.60
5	.56	.92	1.48	2.01	2.57	3.36	4.03
6	.55	.91	1.44	1.94	2.45	3.14	3.71
7	.55	.90	1.42	1.90	2.36	3.00	3.50
8	.55	.89	1.40	1.86	2.31	2.90	3.36
9	.54	.88	1.38	1.83	2.26	2.82	3.25
10	.54	.88	1.37	1.81	2.23	2.76	3.17
11	.54	.88	1.36	1.80	2.20	2.72	3.11
12	.54	.87	1.36	1.78	2.18	2.68	3.06
13	.54	.87	1.35	1.77	2.16	2.65	3.01
14	.54	.87	1.34	1.76	2.14	2.62	2.98
15	.54	.87	1.34	1.75	2.13	2.60	2.95
16	.54	.86	1.34	1.75	2.12	2.58	2.92
17	.53	.86	1.33	1.74	2.11	2.57	2.90
18	.53	.86	1.33	1.73	2.10	2.55	2.88
19	.53	.86	1.33	1.73	2.09	2.54	2.86
20	.53	.86	1.32	1.72	2.09	2.53	2.84
21	.53	.86	1.32	1.72	2.08	2.52	2.83
22	.53	.86	1.32	1.72	2.07	2.51	2.82
23	.53	.86	1.32	1.71	2.07	2.50	2.81
24	.53	.86	1.32	1.71	2.06	2.49	2.80
25	.53	.86	1.32	1.71	2.06	2.48	2.79
26	.53	.86	1.32	1.71	2.06	2.48	2.78
27	.53	.86	1.31	1.70	2.05	2.47	2.77
28	.53	.86	1.31	1.70	2.05	2.47	2.76
29	.53	.85	1.31	1.70	2.04	2.46	2.76
30	.53	.85	1.31	1.70	2.04	2.46	2.75
40	.53	.85	1.30	1.68	2.02	2.42	2.70
50	.53	.85	1.30	1.67	2.01	2.40	2.68
60	.53	.85	1.30	1.67	2.00	2.39	2.66
80	.53	.85	1.29	1.66	1.99	2.37	2.64
100	.53	.84	1.29	1.66	1.98	2.36	2.63
200	.52	.84	1.29	1.65	1.97	2.34	2.60
500	.52	.84	1.28	1.65	1.96	2.33	2.59
∞	.52	.84	1.28	1.64	1.96	2.33	2.58

* From B. J. Winer, "Statistical Principles in Experimental Design," McGraw-Hill Book Company, New York, 1962.

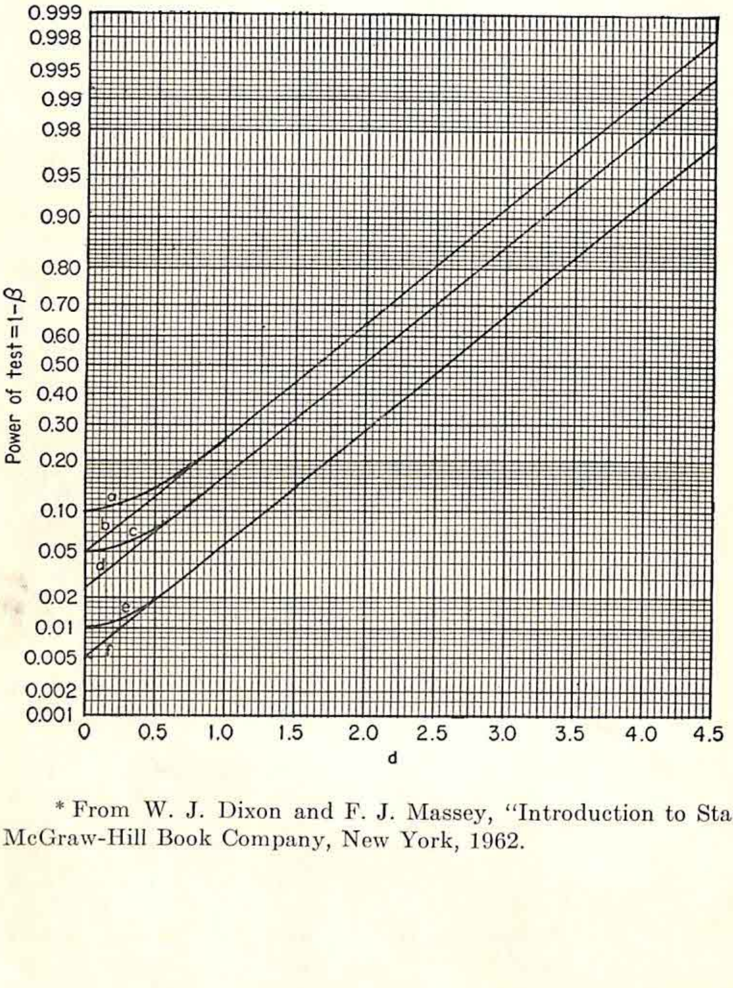
TABLE 6-4. POWER FUNCTION OF THE t TEST*

Table 6-4*a*. Power curves for one- and two-sided tests when σ is known

For one-sided tests $d = \dfrac{\mu - \mu_0}{\sigma/\sqrt{N}}$ or $d = \dfrac{(\mu_1 - \mu_2)}{\sigma\sqrt{\dfrac{1}{N_1} + \dfrac{1}{N_2}}}$

For two-sided tests $d = \dfrac{|\mu - \mu_0|}{\sigma/\sqrt{N}}$ or $d = \dfrac{|\mu_1 - \mu_2|}{\sigma\sqrt{\dfrac{1}{N_1} + \dfrac{1}{N_2}}}$

Curve (*a*) is for two-sided $\alpha = .10$ tests, curve (*b*) is for one-sided $\alpha = .05$ tests, curve (*c*) is for two-sided $\alpha = .05$ tests, curve (*d*) is for one-sided $\alpha = .025$ tests, curve (*e*) is for two-sided $\alpha = .01$ tests, and curve (*f*) is for one-sided $\alpha = .005$ tests.



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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1962.

Table 6-4b. Values of $d = \dfrac{\mu - \mu_0}{\sigma/\sqrt{N}}$ for a single sample or $d = \dfrac{\mu_1 - \mu_2}{\sigma\sqrt{1/N_1 + 1/N_2}}$ for two samples such that a one-sided test at the α level of significance has power $1 - \beta$

The values in the tables are approximate values of $|d|$ for two-sided tests at level of significance 2α.

$\alpha = .005$, one-sided test ($\alpha = .01$, two-sided test)

df \ $1-\beta$	.1	.2	.3	.4	.5	.6	.7	.8	.9	.95	.975	.99	.995	df
4	1.99	2.73	3.29	3.78	4.25	4.73	5.25	5.88	6.77	7.52	8.18	8.98	9.51	4
5	1.83	2.48	2.97	3.39	3.79	4.20	4.64	5.17	5.91	6.53	7.08	7.73	8.18	5
6	1.72	2.33	2.78	3.16	3.53	3.90	4.30	4.77	5.43	5.99	6.47	7.05	7.43	6
7	1.65	2.23	2.65	3.01	3.36	3.71	4.08	4.52	5.13	5.65	6.09	6.62	6.98	7
8	1.60	2.16	2.56	2.91	3.24	3.57	3.92	4.34	4.92	5.41	5.84	6.33	6.67	8
9	1.56	2.10	2.50	2.83	3.15	3.47	3.81	4.21	4.77	5.24	5.65	6.12	6.45	9
10	1.53	2.06	2.45	2.78	3.09	3.40	3.73	4.12	4.67	5.12	5.51	5.98	6.29	10
12	1.49	2.00	2.37	2.69	2.99	3.28	3.60	3.98	4.50	4.94	5.31	5.75	6.05	12
16	1.44	1.93	2.28	2.59	2.87	3.16	3.46	3.82	4.32	4.73	5.09	5.50	5.79	16
24	1.39	1.86	2.20	2.49	2.77	3.04	3.33	3.68	4.15	4.54	4.89	5.28	5.55	24
36	1.36	1.82	2.15	2.44	2.70	2.97	3.25	3.59	4.05	4.43	4.76	5.15	5.41	36
∞	1.294	1.734	2.052	2.323	2.576	2.829	3.100	3.418	3.858	4.221	4.536	4.902	5.152	∞
$\alpha = .0125$, one-sided test ($\alpha = .025$, two-sided test)														
4	1.32	1.96	2.43	2.87	3.24	3.64	4.07	4.58	5.31	5.93	6.47	7.10	7.53	4
5	1.25	1.83	2.26	2.63	2.98	3.34	3.72	4.18	4.82	5.35	5.82	6.37	6.76	5
6	1.19	1.75	2.15	2.50	2.83	3.16	3.52	3.94	4.53	5.03	5.46	5.96	6.30	6
7	1.15	1.69	2.08	2.41	2.73	3.05	3.39	3.79	4.35	4.82	5.22	5.70	6.02	7
8	1.13	1.65	2.03	2.35	2.66	2.96	3.29	3.68	4.22	4.67	5.06	5.52	5.84	8
9	1.11	1.62	1.99	2.32	2.60	2.90	3.22	3.60	4.13	4.56	4.94	5.39	5.69	9
10	1.09	1.59	1.96	2.27	2.56	2.86	3.18	3.54	4.06	4.48	4.86	5.29	5.59	10
12	1.07	1.56	1.91	2.21	2.50	2.79	3.11	3.45	3.95	4.37	4.73	5.14	5.43	12
16	1.05	1.52	1.87	2.17	2.44	2.72	3.01	3.36	3.84	4.24	4.58	4.98	5.26	16
24	1.01	1.48	1.81	2.10	2.37	2.63	2.92	3.26	3.72	4.11	4.44	4.83	5.09	24
36	.99	1.45	1.78	2.06	2.32	2.59	2.87	3.19	3.66	4.03	4.36	4.74	5.00	36
∞	.959	1.399	1.717	1.988	2.241	2.494	2.765	3.083	3.523	3.886	4.201	4.567	4.817	∞
$\alpha = .025$, one-sided test ($\alpha = .05$, two-sided test)														
4	.87	1.44	1.86	2.23	2.58	2.93	3.31	3.76	4.40	4.93	5.40	5.94	6.32	4
5	.82	1.37	1.77	2.11	2.43	2.75	3.10	3.51	4.09	4.58	5.00	5.48	5.82	5
6	.80	1.32	1.70	2.03	2.34	2.64	2.98	3.37	3.91	4.37	4.76	5.23	5.54	6
7	.78	1.29	1.66	1.98	2.27	2.57	2.89	3.27	3.80	4.23	4.61	5.06	5.36	7
8	.77	1.27	1.63	1.94	2.23	2.52	2.83	3.20	3.71	4.14	4.51	4.94	5.23	8
9	.76	1.25	1.60	1.91	2.20	2.48	2.79	3.15	3.65	4.07	4.43	4.85	5.14	9
10	.75	1.23	1.59	1.89	2.17	2.45	2.76	3.11	3.60	4.01	4.37	4.78	5.07	10
12	.74	1.21	1.56	1.86	2.13	2.41	2.71	3.05	3.54	3.94	4.28	4.69	4.96	12
16	.72	1.19	1.53	1.82	2.09	2.36	2.65	2.98	3.46	3.84	4.18	4.58	4.84	16
24	.71	1.16	1.50	1.78	2.04	2.31	2.59	2.92	3.38	3.76	4.09	4.47	4.72	24
36	.70	1.15	1.48	1.75	2.01	2.27	2.55	2.88	3.33	3.71	4.03	4.41	4.66	36
∞	.678	1.118	1.436	1.707	1.960	2.213	2.484	2.802	3.242	3.605	3.920	4.286	4.536	∞
$\alpha = .05$, one-sided test ($\alpha = .10$, two-sided test)														
4	.43	.96	1.34	1.68	1.99	2.30	2.64	3.04	3.60	4.07	4.48	4.95	5.29	4
5	.42	.92	1.29	1.61	1.91	2.21	2.51	2.90	3.43	3.87	4.25	4.70	5.00	5
6	.40	.90	1.26	1.57	1.86	2.15	2.46	2.82	3.33	3.75	4.12	4.55	4.84	6
7	.40	.89	1.24	1.54	1.82	2.11	2.41	2.77	3.26	3.68	4.03	4.45	4.73	7
8	.39	.88	1.22	1.52	1.80	2.08	2.38	2.73	3.21	3.62	3.97	4.37	4.66	8
9	.39	.87	1.21	1.50	1.78	2.06	2.35	2.70	3.18	3.57	3.92	4.32	4.60	9
10	.39	.86	1.20	1.49	1.77	2.04	2.33	2.67	3.15	3.54	3.88	4.28	4.56	10
12	.38	.85	1.19	1.48	1.75	2.02	2.30	2.64	3.11	3.50	3.82	4.23	4.49	12
16	.38	.84	1.17	1.45	1.72	1.98	2.27	2.60	3.06	3.44	3.77	4.16	4.42	16
24	.37	.83	1.15	1.43	1.69	1.95	2.23	2.56	3.01	3.39	3.71	4.09	4.35	24
36	.37	.82	1.14	1.42	1.68	1.94	2.21	2.54	2.98	3.36	3.68	4.05	4.31	36
∞	.363	.803	1.121	1.392	1.645	1.898	2.169	2.487	2.927	3.290	3.605	3.971	4.221	∞

Table 6-4c. Sample size needed to attain power $1 - \beta$ against $d' = (\mu - \mu_0)/\sigma$ in the one-sample case or $d' = (\mu_1 - \mu_2)/\sigma$ in the two-sample case

Each sample in the two-sample case should be of the given size.

α	d'	One-sided, single-sample test							One-sided, two-sample test							d'
		$1 - \beta$														
		.50	.60	.70	.80	.90	.95	.99	.50	.60	.70	.80	.90	.95	.99	
.005	.1	669	805	966	1173	1493	1785	2403	1327	1602	1922	2337	2977	3567	4806	.1
	.2	169	204	244	296	377	450	605	333	403	484	588	749	894	1206	.2
	.4	42	53	64	77	97	115	154	87	101	124	150	189	226	304	.4
	.6	22	26	31	36	45	53	71	37	44	55	65	85	100	138	.6
	.8	14	16	19	22	27	32	41	23	27	32	39	49	56	85	.8
	1.0	10	12	13	14	19	22	28	15	18	21	26	32	38	49	1.0
	1.2	8	9	10	12	14	16	21	11	13	16	18	23	27	36	1.2
	1.4	7	8	9	10	12	13	16	9	10	12	14	18	20	27	1.4
	1.6	6	7	8	8	10	11	13	7	9	10	11	14	16	21	1.6
	1.8	6	6	7	8	9	9	11	6	7	8	9	11	13	17	1.8
	2.0	4	6	6	7	8	8	10	6	6	7	8	10	11	14	2.0
	3.0	3	4	5	5	6	6	7	4	4	5	5	6	6	8	3.0
.0125	.1	506	625	768	954	1245	1514	2090	1004	1244	1529	1901	2482	3020	4180	.1
	.2	127	158	194	240	312	380	524	255	316	388	482	629	766	1057	.2
	.4	34	42	52	63	81	98	135	64	81	97	120	157	191	265	.4
	.6	17	19	24	30	37	45	61	30	37	44	54	71	84	117	.6
	.8	11	13	15	18	23	27	36	18	21	26	32	40	49	69	.8
	1.0	8	9	11	13	16	18	24	12	14	17	21	27	32	44	1.0
	1.2	7	7	8	10	12	14	18	9	11	13	15	19	23	31	1.2
	1.4	6	6	7	8	10	11	14	7	8	10	12	15	17	23	1.4
	1.6	5	6	6	7	8	9	11	6	7	8	9	12	14	18	1.6
	1.8		5	6	6	7	8	10	5	6	7	8	10	11	15	1.8
	2.0			5	6	6	7	9	4	5	6	7	8	9	12	2.0
	3.0							6		3	4	4	5	5	7	3.0
.025	.1	386	492	619	788	1054	1302	1840	771	982	1237	1574	2106	2603	3680	.1
	.2	98	124	156	201	265	327	459	193	245	310	395	527	650	922	.2
	.4	26	33	41	52	68	85	117	49	62	80	100	133	164	231	.4
	.6	13	16	20	24	32	39	53	23	29	36	45	60	77	104	.6
	.8	9	10	12	14	18	23	31	14	17	21	26	34	42	59	.8
	1.0	6	7	9	10	13	16	21	9	11	14	17	22	28	38	1.0
	1.2	6	6	7	8	10	12	15	7	8	10	12	16	19	27	1.2
	1.4	5	5	6	7	8	9	12	5	7	8	10	12	15	20	1.4
	1.6			5	6	7	8	10	5	5	6	8	10	12	16	1.6
	1.8				5	6	7	8	4	4	5	6	8	10	13	1.8
	2.0					5	6	7	4	4	5	6	7	8	11	2.0
	3.0											4	4	5	6	3.0
.05	.1	272	362	473	620	858	1084	1580	543	722	943	1235	1715	2166	3160	.1
	.2	69	92	119	156	215	272	396	138	182	237	312	430	543	793	.2
	.4	19	24	31	41	55	70	101	35	46	59	79	109	137	199	.4
	.6	9	12	15	19	26	32	46	16	21	28	35	48	62	89	.6
	.8	6	8	9	12	15	19	27	10	12	16	20	28	35	50	.8
	1.0	5	6	7	8	11	13	18	7	8	11	13	18	23	33	1.0
	1.2		5	5	6	8	10	13	5	6	8	10	13	16	23	1.2
	1.4				5	6	8	10	4	5	6	7	10	12	17	1.4
	1.6					6	6	8	4	4	5	6	8	10	14	1.6
	1.8					5	6	7		4	4	5	7	8	11	1.8
	2.0						5	6			4	4	6	7	9	2.0
	3.0												3	4	5	3.0

Table 6-5. Distribution of t statistic when comparing treatment means with a control*

df for MS_{error}	$1 - \alpha$	k = number of means (including control)								
		2	3	4	5	6	7	8	9	10
5	.95	2.02	2.44	2.68	2.85	2.98	3.08	3.16	3.24	3.30
	.975	2.57	3.03	3.39	3.66	3.88	4.06	4.22	4.36	4.49
	.99	3.37	3.90	4.21	4.43	4.60	4.73	4.85	4.94	5.03
	.995	4.03	4.63	5.09	5.44	5.73	5.97	6.18	6.36	6.53
6	.95	1.94	2.34	2.56	2.71	2.83	2.92	3.00	3.07	3.12
	.975	2.45	2.86	3.18	3.41	3.60	3.75	3.88	4.00	4.11
	.99	3.14	3.61	3.88	4.07	4.21	4.33	4.43	4.51	4.59
	.995	3.71	4.22	4.60	4.88	5.11	5.30	5.47	5.61	5.74
7	.95	1.89	2.27	2.48	2.62	2.73	2.82	2.89	2.95	3.01
	.975	2.36	2.75	3.04	3.24	3.41	3.54	3.66	3.76	3.86
	.99	3.00	3.42	3.66	3.83	3.96	4.07	4.15	4.23	4.30
	.995	3.50	3.95	4.28	4.52	4.17	4.87	5.01	5.13	5.24
8	.95	1.86	2.22	2.42	2.55	2.66	2.74	2.81	2.87	2.92
	.975	2.31	2.67	2.94	3.13	3.28	3.40	3.51	3.60	3.68
	.99	2.90	3.29	3.51	3.67	3.79	3.88	3.96	4.03	4.09
	.995	3.36	3.77	4.06	4.27	4.44	4.58	4.70	4.81	4.90
9	.95	1.83	2.18	2.37	2.50	2.60	2.68	2.75	2.81	2.86
	.975	2.26	2.61	2.86	3.04	3.18	3.29	3.39	3.48	3.55
	.99	2.82	3.19	3.40	3.55	3.66	3.75	3.82	3.89	3.94
	.995	3.25	3.63	3.90	4.09	4.24	4.37	4.48	4.57	4.65
10	.95	1.81	2.15	2.34	2.47	2.56	2.64	2.70	2.76	2.81
	.975	2.23	2.57	2.81	2.97	3.11	3.21	3.31	3.39	3.46
	.99	2.76	3.11	3.31	3.45	3.56	3.64	3.71	3.78	3.83
	.995	3.17	3.53	3.78	3.95	4.10	4.21	4.31	4.40	4.47
11	.95	1.80	2.13	2.31	2.44	2.53	2.60	2.67	2.72	2.77
	.975	2.20	2.53	2.76	2.92	3.05	3.15	3.24	3.31	3.38
	.99	2.72	3.06	3.25	3.38	3.48	3.56	3.63	3.69	3.74
	.995	3.11	3.45	3.68	3.85	3.98	4.09	4.18	4.26	4.33
12	.95	1.78	2.11	2.29	2.41	2.50	2.58	2.64	2.69	2.74
	.975	2.18	2.50	2.72	2.88	3.00	3.10	3.18	3.25	3.32
	.99	2.68	3.01	3.19	3.32	3.42	3.50	3.56	3.62	3.67
	.995	3.05	3.39	3.61	3.76	3.89	3.99	4.08	4.15	4.22
13	.95	1.77	2.09	2.27	2.39	2.48	2.55	2.61	2.66	2.71
	.975	2.16	2.48	2.69	2.84	2.96	3.06	3.14	3.21	3.27
	.99	2.65	2.97	3.15	3.27	3.37	3.44	3.51	3.56	3.61
	.995	3.01	3.33	3.54	3.69	3.81	3.91	3.99	4.06	4.13
14	.95	1.76	2.08	2.25	2.37	2.46	2.53	2.59	2.64	2.69
	.975	2.14	2.46	2.67	2.81	2.93	3.02	3.10	3.17	3.23
	.99	2.62	2.94	3.11	3.23	3.32	3.40	3.46	3.51	3.56
	.995	2.98	3.29	3.49	3.64	3.75	3.84	3.92	3.99	4.05

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Table 6-5. Distribution of t statistic when comparing treatment means with a control (continued)

df for MS_{error}	$1-\alpha$	k = number of means (including control)								
		2	3	4	5	6	7	8	9	10
16	.95	1.75	2.06	2.23	2.34	2.43	2.50	2.56	2.61	2.65
	.975	2.12	2.42	2.63	2.77	2.88	2.96	3.04	3.10	3.16
	.99	2.58	2.88	3.05	3.17	3.26	3.33	3.39	3.44	3.48
	.995	2.92	3.22	3.41	3.55	3.65	3.74	3.82	3.88	3.93
18	.95	1.73	2.04	2.21	2.32	2.41	2.48	2.53	2.58	2.62
	.975	2.10	2.40	2.59	2.73	2.84	2.92	2.99	3.05	3.11
	.99	2.55	2.84	3.01	3.12	3.21	3.27	3.33	3.38	3.42
	.995	2.88	3.17	3.35	3.48	3.58	3.67	3.74	3.80	3.85
20	.95	1.72	2.03	2.19	2.30	2.39	2.46	2.51	2.56	2.60
	.975	2.09	2.38	2.57	2.70	2.81	2.89	2.96	3.02	3.07
	.99	2.53	2.81	2.97	3.08	3.17	3.23	3.29	3.34	3.38
	995	2.85	3.13	3.31	3.43	3.53	3.61	3.67	3.73	3.78
24	.95	1.71	2.01	2.17	2.28	2.36	2.43	2.48	2.53	2.57
	.975	2.06	2.35	2.53	2.66	2.76	2.84	2.91	2.96	3.01
	.99	2.49	2.77	2.92	3.03	3.11	3.17	3.22	3.27	3.31
	.995	2.80	3.07	3.24	3.36	3.45	3.52	3.58	3.64	3.69
30	.95	1.70	1.99	2.15	2.25	2.33	2.40	2.45	2.50	2.54
	.975	2.04	2.32	2.50	2.62	2.72	2.79	2.86	2.91	2.96
	.99	2.46	2.72	2.87	2.97	3.05	3.11	3.16	3.21	3.24
	.995	2.75	3.01	3.17	3.28	3.37	3.44	3.50	3.55	3.59
40	.95	1.68	1.97	2.13	2.23	2.31	2.37	2.42	2.47	2.51
	.975	2.02	2.29	2.47	2.58	2.67	2.75	2.81	2.86	2.90
	.99	2.42	2.68	2.82	2.92	2.99	3.05	3.10	3.14	3.18
	.995	2.70	2.95	3.10	3.21	3.29	3.36	3.41	3.46	3.50
60	.95	1.67	1.95	2.10	2.21	2.28	2.35	2.39	2.44	2.48
	.975	2.00	2.27	2.43	2.55	2.63	2.70	2.76	2.81	2.85
	.99	2.39	2.64	2.78	2.87	2.94	3.00	3.04	3.08	3.12
	.995	2.66	2.90	3.04	3.14	3.22	3.28	3.33	3.38	3.42
120	.95	1.66	1.93	2.08	2.18	2.26	2.32	2.37	2.41	2.45
	.975	1.98	2.24	2.40	2.51	2.59	2.66	2.71	2.76	2.80
	.99	2.36	2.60	2.73	2.82	2.89	2.94	2.99	3.03	3.06
	.995	2.62	2.84	2.98	3.08	3.15	3.21	3.25	3.30	3.33
∞	.95	1.64	1.92	2.06	2.16	2.23	2.29	2.34	2.38	2.42
	.975	1.96	2.21	2.37	2.47	2.55	2.62	2.67	2.71	2.75
	.99	2.33	2.56	2.68	2.77	2.84	2.89	2.93	2.97	3.00
	.995	2.58	2.79	2.92	3.01	3.08	3.14	3.18	3.22	3.25

Table 6-6. Cumulative F distribution*

m degrees of freedom in numerator; n in denominator

$$G(F) = \int_0^F \frac{[(m+n-2)/2]!\,m^{m/2}n^{n/2}x^{m/2}(n+mx)^{-(m+n)/2}\,dx}{[(m-2)/2]!\,[(n-2)/2]!}$$

G	n m	1	2	3	4	5	6	7	8	9	10	12	15	20	30	60	120	∞
.90	1	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.7	61.2	61.7	62.3	62.8	63.1	63.3
.95		161	200	216	225	230	234	237	239	241	242	244	246	248	250	252	253	254
.975		648	800	864	900	922	937	948	957	963	969	977	985	993	1000	1010	1010	1020
.99		4,050	5,000	5,400	5,620	5,760	5,860	5,930	5,980	6,020	6,060	6,110	6,160	6,210	6,260	6,310	6,340	6,370
.995		16,200	20,000	21,600	22,500	23,100	23,400	23,700	23,900	24,100	24,200	24,400	24,600	24,800	25,000	25,200	25,400	25,500
.90	2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.46	9.47	9.48	9.49
.95		18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
.975		38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5
.99		98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5
.995		199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199	199
.90	3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.15	5.14	5.13
.95		10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.62	8.57	8.55	8.53
.975		17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.3	14.3	14.2	14.1	14.0	13.9	13.9
.99		34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.5	26.3	26.2	26.1
.995		55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	43.7	43.4	43.1	42.8	42.5	42.1	42.0	41.8
.90	4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.93	3.92	3.90	3.87	3.84	3.82	3.79	3.78	3.76
.95		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.75	5.69	5.66	5.63
.975		12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.46	8.36	8.31	8.26
.99		21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.8	13.7	13.6	13.5
.995		31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	21.0	20.7	20.4	20.2	19.9	19.6	19.5	19.3
.90	5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.17	3.14	3.12	3.11
.95		6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.50	4.43	4.40	4.37
.975		10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.23	6.12	6.07	6.02
.99		16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.38	9.20	9.11	9.02
.995		22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	13.6	13.4	13.1	12.9	12.7	12.4	12.3	12.1
.90	6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.80	2.76	2.74	2.72
.95		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.81	3.74	3.70	3.67
.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.07	4.96	4.90	4.85
.99		13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.23	7.06	6.97	6.88
.995		18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4	10.2	10.0	9.81	9.59	9.36	9.12	9.00	8.88
.90	7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.56	2.51	2.49	2.47
.95		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.38	3.30	3.27	3.23
.975		8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.36	4.25	4.20	4.14
.99		12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	5.99	5.82	5.74	5.65
.995		16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51	8.38	8.18	7.97	7.75	7.53	7.31	7.19	7.08
.90	8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.38	2.34	2.31	2.29
.95		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.08	3.01	2.97	2.93
.975		7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.89	3.78	3.73	3.67
.99		11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.20	5.03	4.95	4.86
.995		14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	7.21	7.01	6.81	6.61	6.40	6.18	6.06	5.95

Table 6-6. Cumulative F distribution* (continued)

.90	9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.25	2.21	2.18	2.16
.95		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.86	2.79	2.75	2.71
.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.56	3.45	3.39	3.33
.99		10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.65	4.48	4.40	4.31
.995		13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	6.42	6.23	6.03	5.83	5.62	5.41	5.30	5.19
.90	10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.15	2.11	2.08	2.06
.95		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.84	2.77	2.70	2.62	2.58	2.54
.975		6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.31	3.20	3.14	3.08
.99		10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.25	4.08	4.00	3.91
.995		12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	5.85	5.66	5.47	5.27	5.07	4.86	4.75	4.64
.90	12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.01	1.96	1.93	1.90
.95		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.47	2.38	2.34	2.30
.975		6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	2.96	2.85	2.79	2.72
.99		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.70	3.54	3.45	3.36
.995		11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	5.09	4.91	4.72	4.53	4.33	4.12	4.01	3.90
.90	15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.87	1.82	1.79	1.76
.95		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.25	2.16	2.11	2.07
.975		6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.64	2.52	2.46	2.40
.99		8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.21	3.05	2.96	2.87
.995		10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	4.42	4.25	4.07	3.88	3.69	3.48	3.37	3.26
.90	20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.74	1.68	1.64	1.61
.95		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.04	1.95	1.90	1.84
.975		5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.35	2.22	2.16	2.09
.99		8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.78	2.61	2.52	2.42
.995		9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	3.85	3.68	3.50	3.32	3.12	2.92	2.81	2.69
.90	30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.61	1.54	1.50	1.46
.95		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.84	1.74	1.68	1.62
.975		5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.07	1.94	1.87	1.79
.99		7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.39	2.21	2.11	2.01
.995		9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	3.34	3.18	3.01	2.82	2.63	2.42	2.30	2.18
.90	60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.48	1.40	1.35	1.29
.95		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.65	1.53	1.47	1.39
.975		5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.82	1.67	1.58	1.48
.99		7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.03	1.84	1.73	1.60
.995		8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	2.90	2.74	2.57	2.39	2.19	1.96	1.83	1.69
.90	120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.54	1.48	1.41	1.32	1.26	1.19
.95		3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.55	1.43	1.35	1.25
.975		5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.69	1.53	1.43	1.31
.99		6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.86	1.66	1.53	1.38
.995		8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	2.71	2.54	2.37	2.19	1.98	1.75	1.61	1.43
.90	∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.34	1.24	1.17	1.00
.95		3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.46	1.32	1.22	1.00
.975		5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.57	1.39	1.27	1.00
.99		6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.70	1.47	1.32	1.00
.995		7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	2.52	2.36	2.19	2.00	1.79	1.53	1.36	1.00

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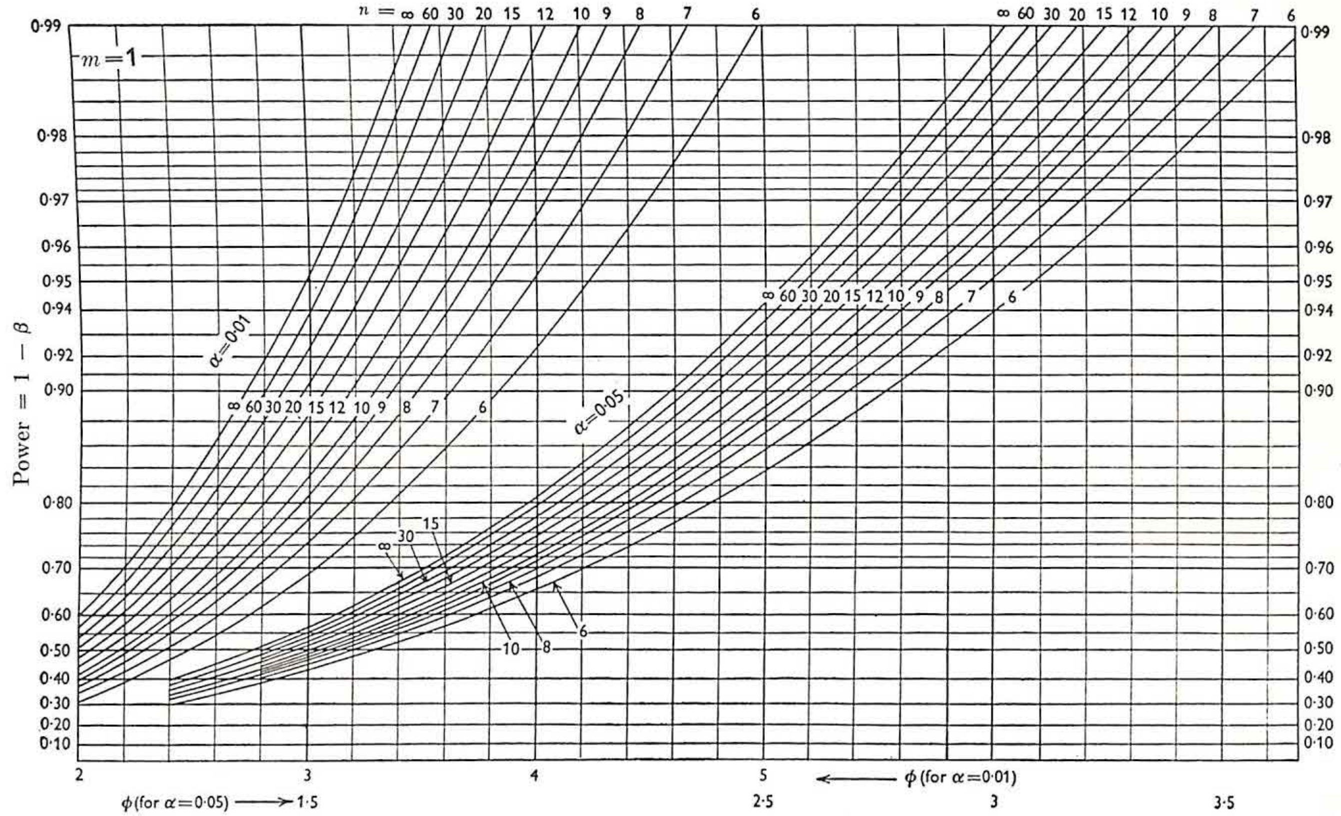
* This table is abridged from Maxine Merrington and Catherine M. Thompson, Tables of Percentage Points of the Inverted Beta Distribution, *Biometrika*, vol. 33, 1943. It is here published with the kind permission of the authors and the editor of *Biometrika*. Reprinted from A. M. Mood and F. A. Graybill, "Introduction to the Theory of Statistics," 2d ed., McGraw-Hill Book Company, New York, 1963.

Table 6-7. Power of the F test*



ϕ is the variance ratio; m and n are the sample sizes in enumerator and denominator, respectively.

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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1962.

Table 6-7. Power of the F test (continued)

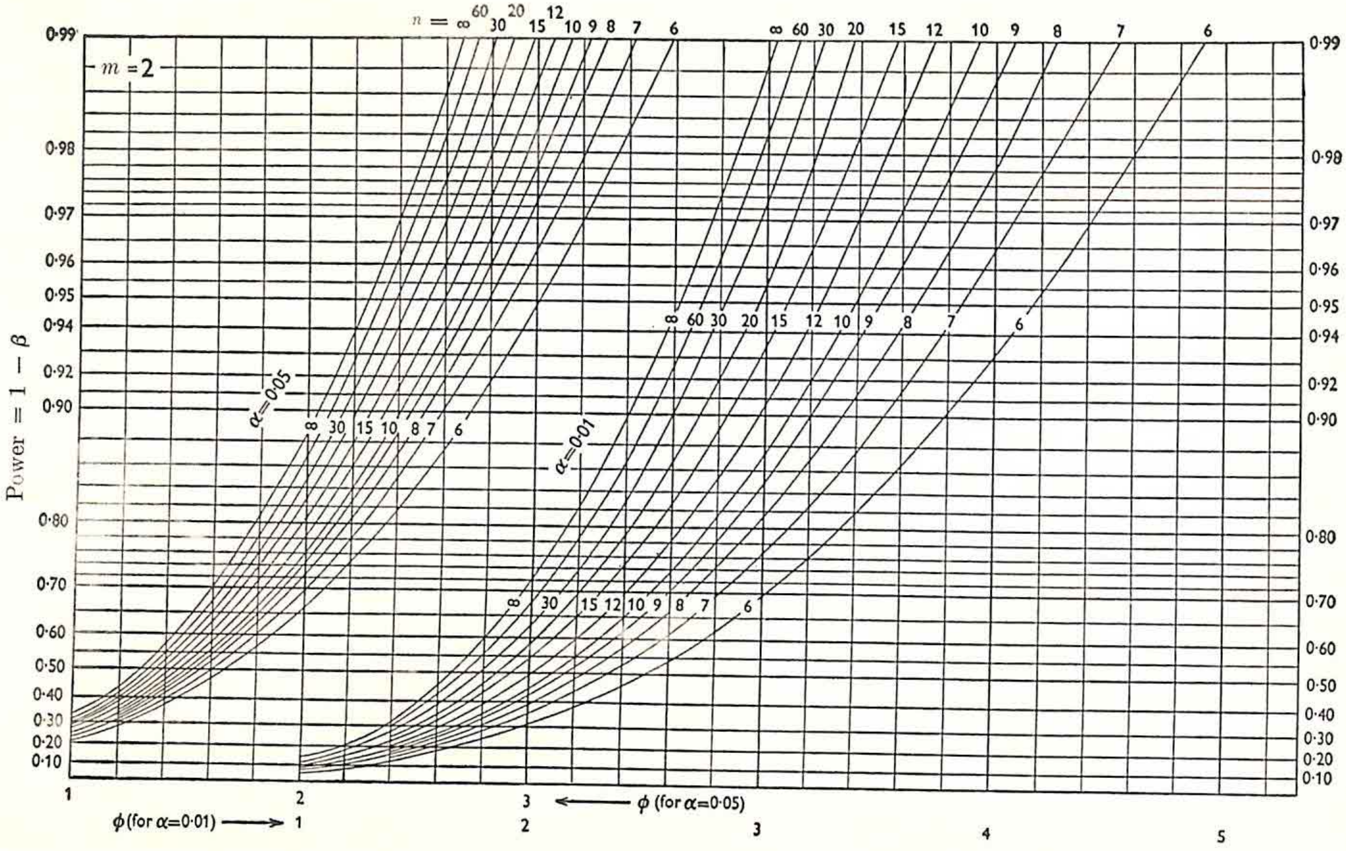

Table 6-7. Power of the *F* test (continued)

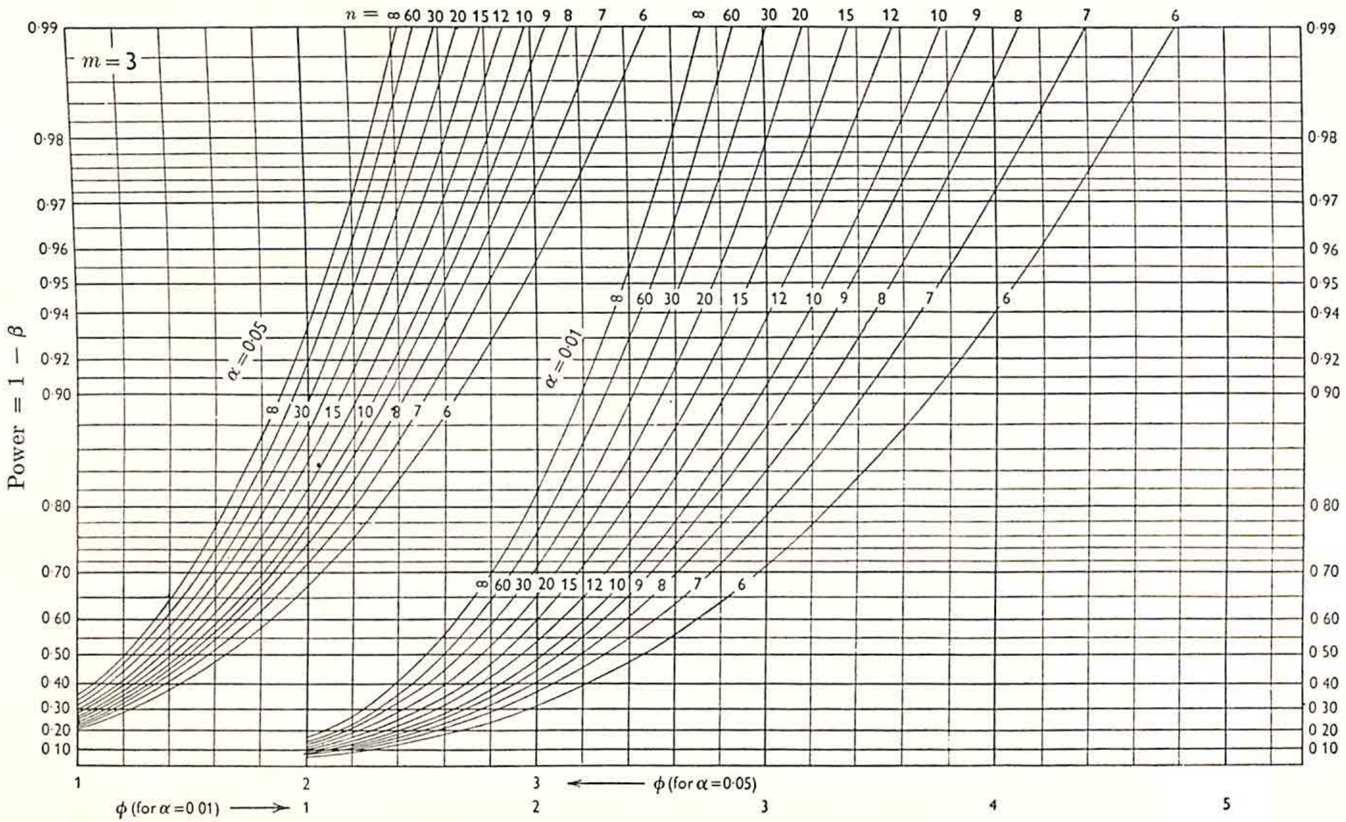

Table 6-7. Power of the *F* test (continued)

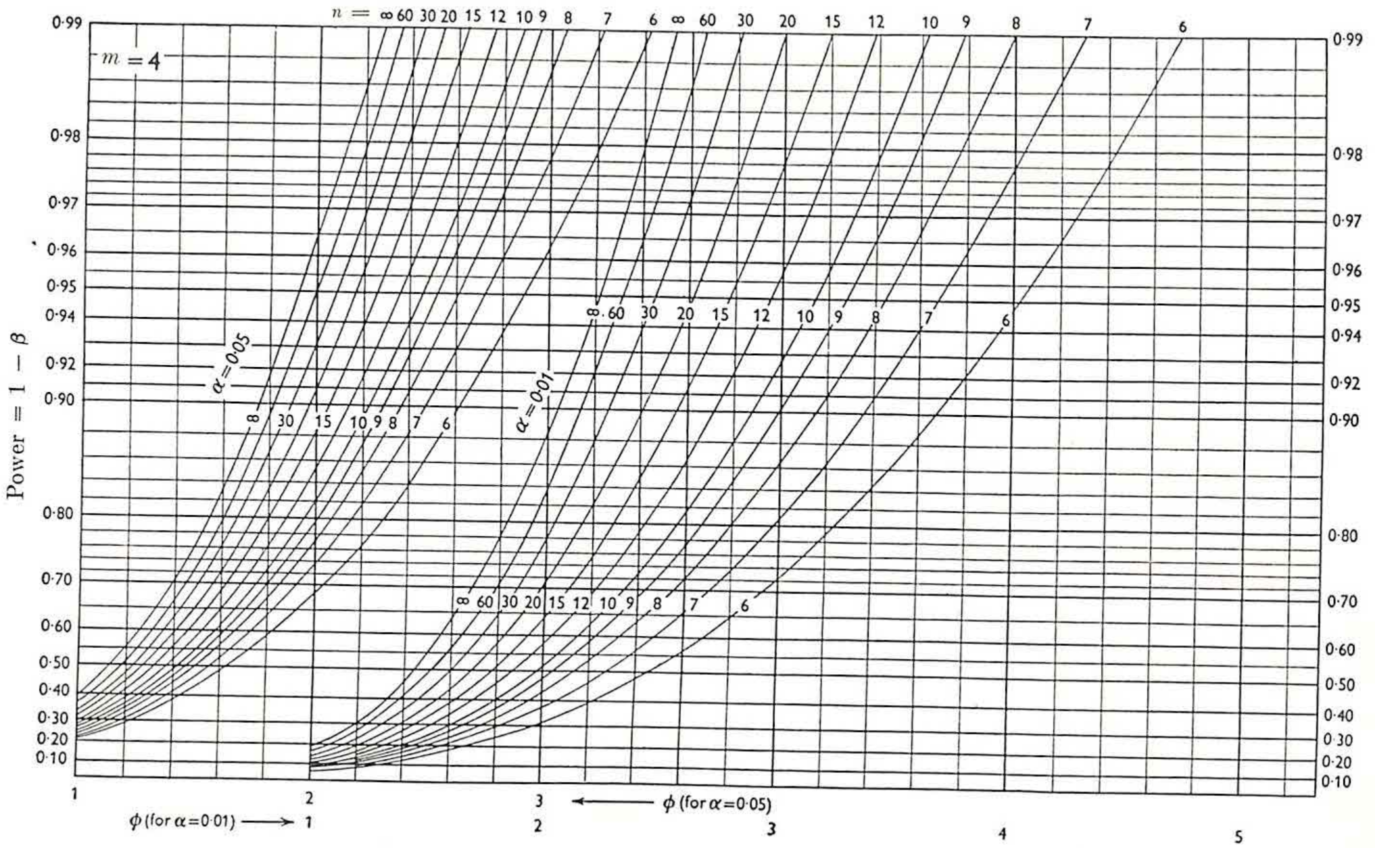

Table 6-7. Power of the F test (continued)

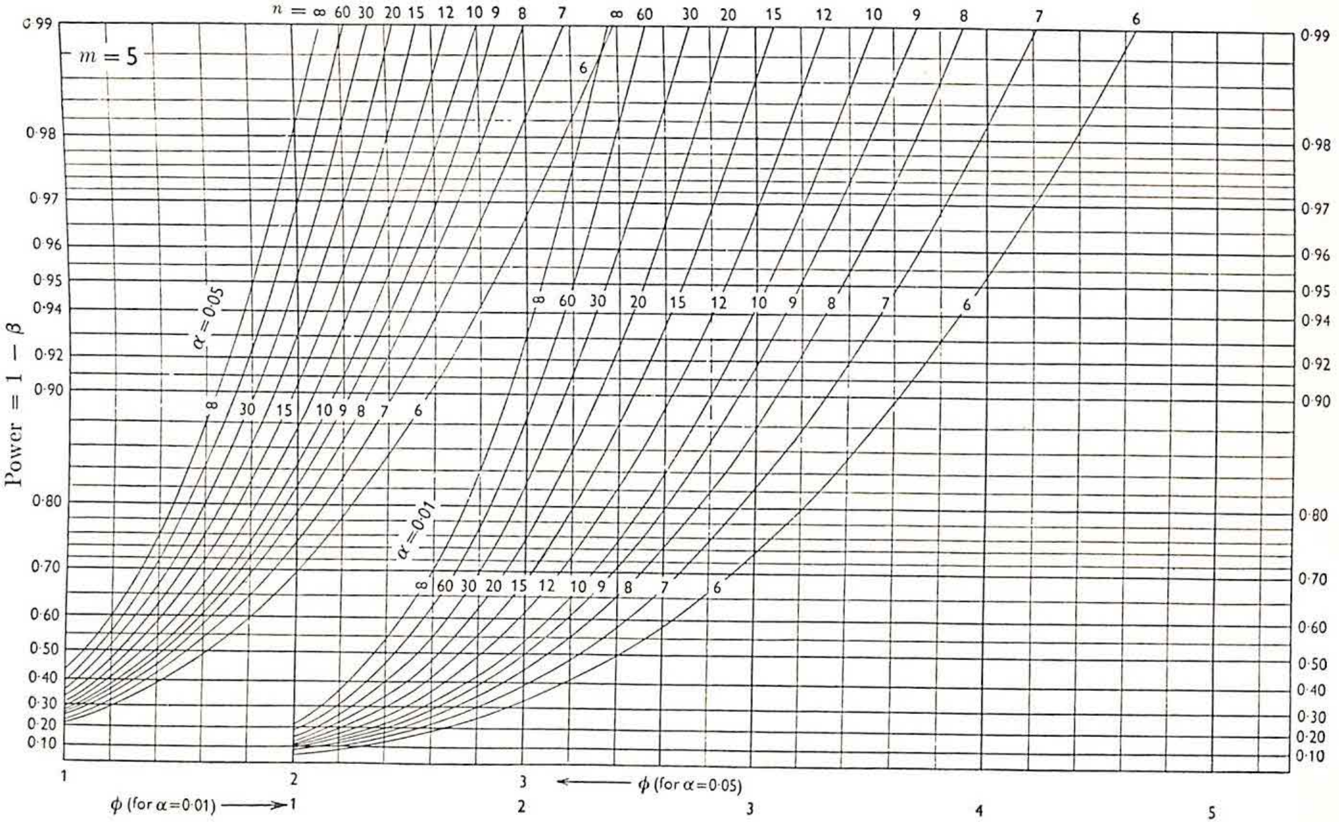

Table 6-7. Power of the F test (continued)

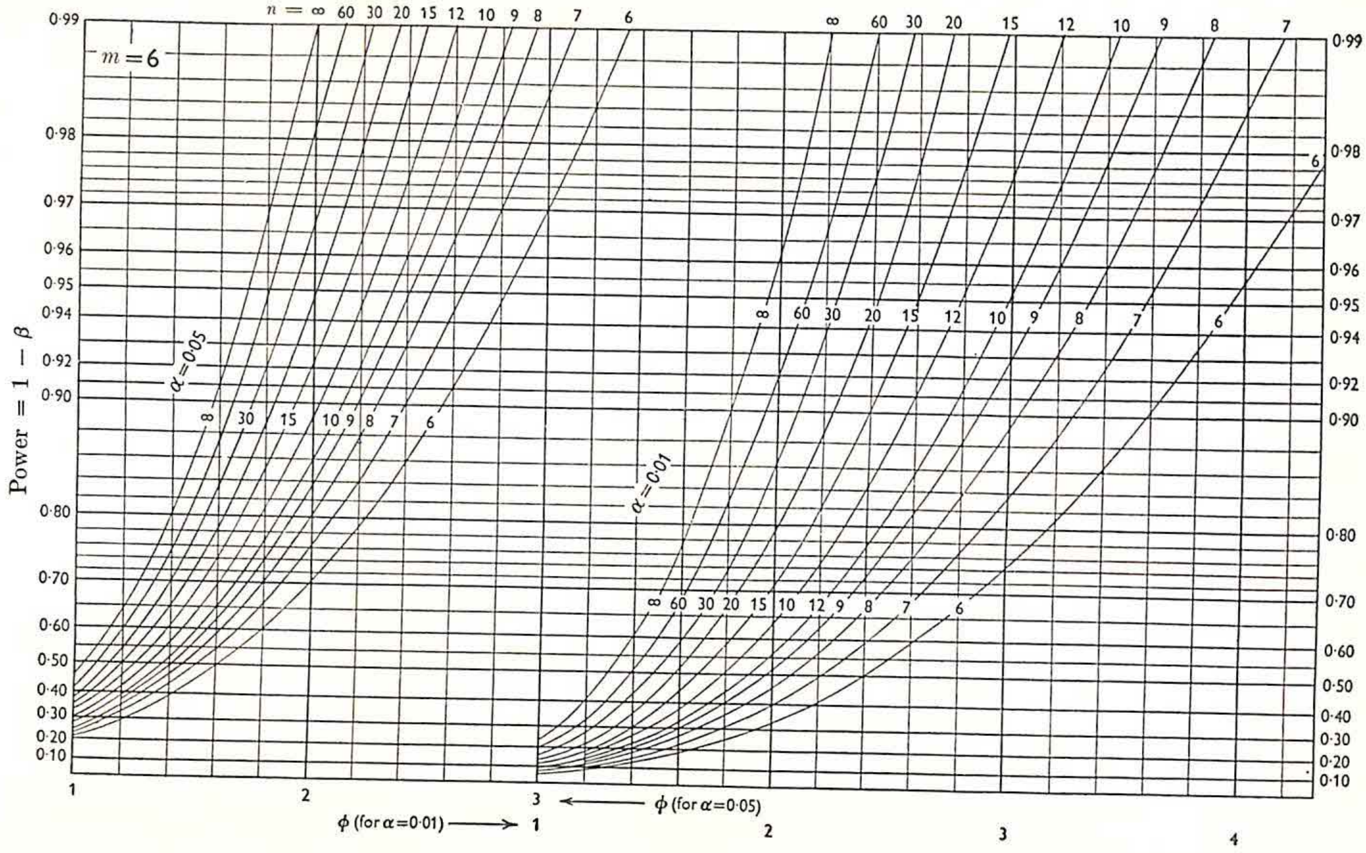

Table 6-7. Power of the F test (continued)

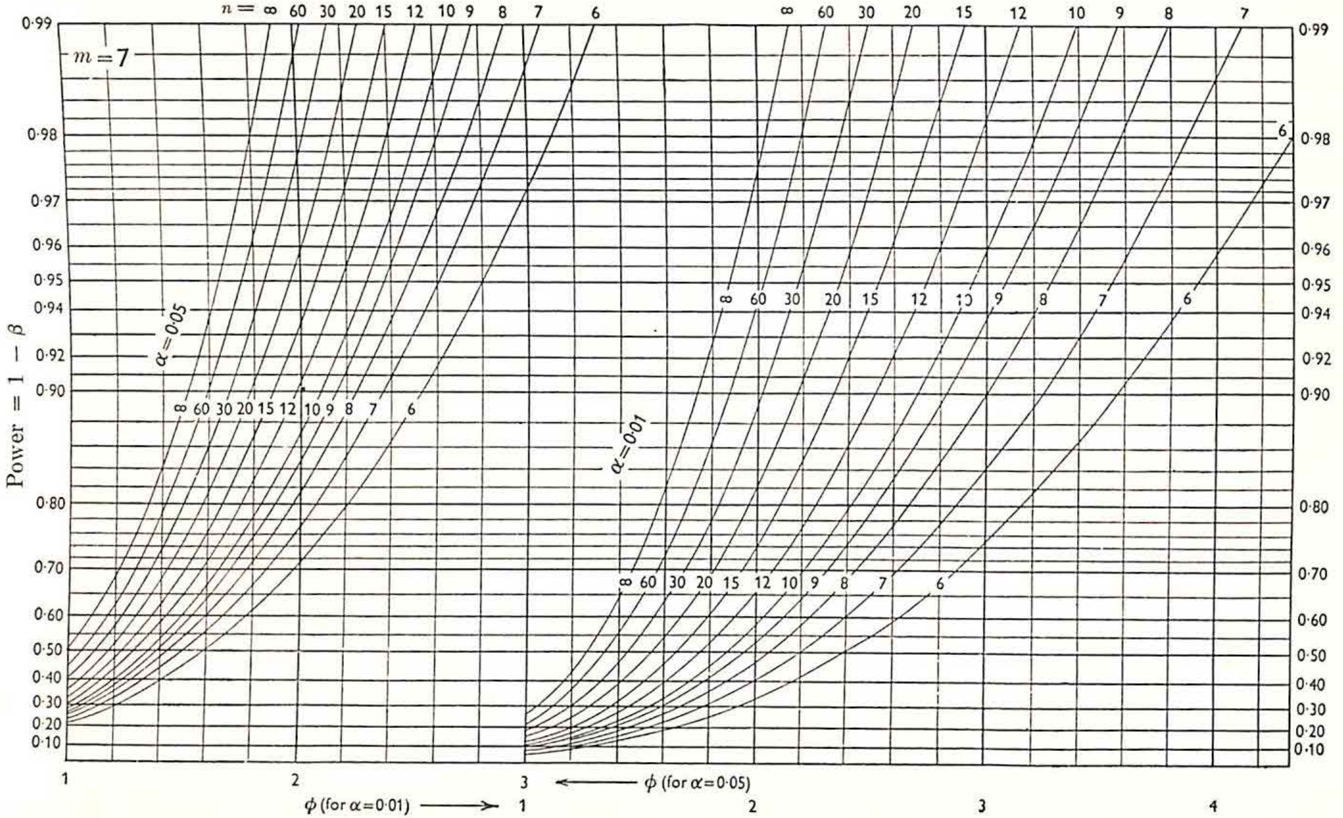

Table 6-7. Power of the F test (continued)

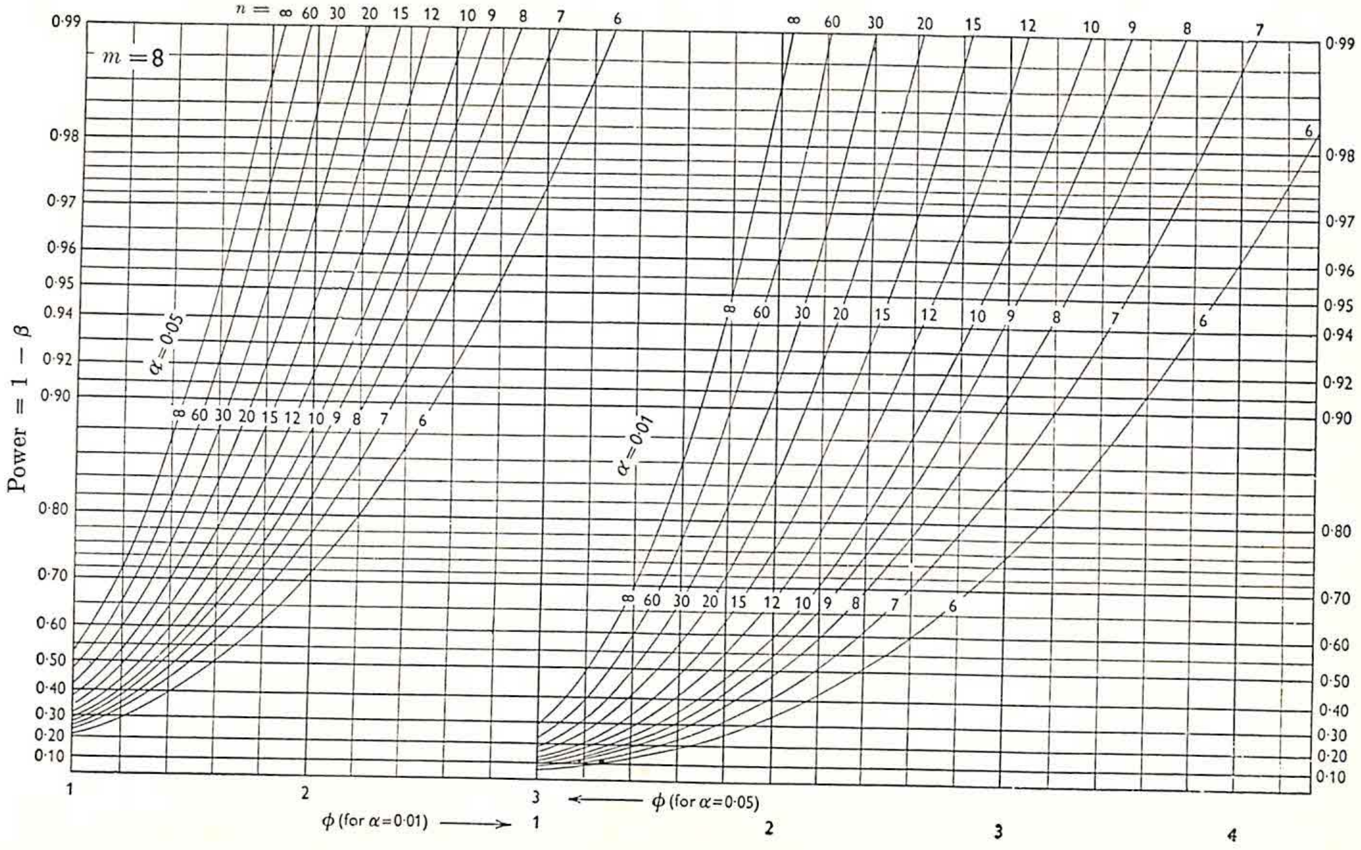

Table 6-8. Distribution of $F_{\max}$ statistic*

df for s_x^2	$1 - \alpha$	k = number of variances								
		2	3	4	5	6	7	8	9	10
4	.95	9.60	15.5	20.6	25.2	29.5	33.6	37.5	41.4	44.6
	.99	23.2	37.	49.	59.	69.	79.	89.	97.	106.
5	.95	7.15	10.8	13.7	16.3	18.7	20.8	22.9	24.7	26.5
	.99	14.9	22.	28.	33.	38.	42.	46.	50.	54.
6	.95	5.82	8.38	10.4	12.1	13.7	15.0	16.3	17.5	18.6
	.99	11.1	15.5	19.1	22.	25.	27.	30.	32.	34.
7	.95	4.99	6.94	8.44	9.70	10.8	11.8	12.7	13.5	14.3
	.99	8.89	12.1	14.5	16.5	18.4	20.	22.	23.	24.
8	.95	4.43	6.00	7.18	8.12	9.03	9.78	10.5	11.1	11.7
	.99	7.50	9.9	11.7	13.2	14.5	15.8	16.9	17.9	18.9
9	.95	4.03	5.34	6.31	7.11	7.80	8.41	8.95	9.45	9.91
	.99	6.54	8.5	9.9	11.1	12.1	13.1	13.9	14.7	15.3
10	.95	3.72	4.85	5.67	6.34	6.92	7.42	7.87	8.28	8.66
	.99	5.85	7.4	8.6	9.6	10.4	11.1	11.8	12.4	12.9
12	.95	3.28	4.16	4.79	5.30	5.72	6.09	6.42	6.72	7.00
	.99	4.91	6.1	6.9	7.6	8.2	8.7	9.1	9.5	9.9
15	.95	2.86	3.54	4.01	4.37	4.68	4.95	5.19	5.40	5.59
	.99	4.07	4.9	5.5	6.0	6.4	6.7	7.1	7.3	7.5
20	.95	2.46	2.95	3.29	3.54	3.76	3.94	4.10	4.24	4.37
	.99	3.32	3.8	4.3	4.6	4.9	5.1	5.3	5.5	5.6
30	.95	2.07	2.40	2.61	2.78	2.91	3.02	3.12	3.21	3.29
	.99	2.63	3.0	3.3	3.4	3.6	3.7	3.8	3.9	4.0
60	.95	1.67	1.85	1.96	2.04	2.11	2.17	2.22	2.26	2.30
	.99	1.96	2.2	2.3	2.4	2.4	2.5	2.5	2.6	2.6
∞	.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

* This table is abridged from Table 31 in E. S. Pearson and H. O. Hartley (eds.), "Biometrika Tables for Statisticians," 2d ed., vol. 1, Cambridge University Press, New York, 1958. Reproduced with the kind permission of E. S. Pearson and the trustees of *Biometrika*. Reprinted from B. J. Winer, "Statistical Principles in Experimental Design," McGraw-Hill Book Company, New York, 1962.

Table 6-9. Critical values for Cochran's test for homogeneity of variance*

$C = (\text{largest } s^2)/(\Sigma s_j^2)$

df for s_j^2	$1-\alpha$	k = number of variances										
		2	3	4	5	6	7	8	9	10	15	20
1	.95	.9985	.9669	.9065	.8412	.7808	.7271	.6798	.6385	.6020	.4709	.3894
	.99	.9999	.9933	.9676	.9279	.8828	.8376	.7945	.7544	.7175	.5747	.4799
2	.95	.9750	.8709	.7679	.6838	.6161	.5612	.5157	.4775	.4450	.3346	.2705
	.99	.9950	.9423	.8643	.7885	.7218	.6644	.6152	.5727	.5358	.4069	.3297
3	.95	.9392	.7977	.6841	.5981	.5321	.4800	.4377	.4027	.3733	.2758	.2205
	.99	.9794	.8831	.7814	.6957	.6258	.5685	.5209	.4810	.4469	.3317	.2654
4	.95	.9057	.7457	.6287	.5441	.4803	.4307	.3910	.3584	.3311	.2419	.1921
	.99	.9586	.8335	.7212	.6329	.5635	.5080	.4627	.4251	.3934	.2882	.2288
5	.95	.8772	.7071	.5895	.5065	.4447	.3974	.3595	.3286	.3029	.2195	.1735
	.99	.9373	.7933	.6761	.5875	.5195	.4659	.4226	.3870	.3572	.2593	.2048
6	.95	.8534	.6771	.5598	.4783	.4184	.3726	.3362	.3067	.2823	.2034	.1602
	.99	.9172	.7606	.6410	.5531	.4866	.4347	.3932	.3592	.3308	.2386	.1877
7	.95	.8332	.6530	.5365	.4564	.3980	.3535	.3185	.2901	.2666	.1911	.1501
	.99	.8988	.7335	.6129	.5259	.4608	.4105	.3704	.3378	.3106	.2228	.1748
8	.95	.8159	.6333	.5175	.4387	.3817	.3384	.3043	.2768	.2541	.1815	.1422
	.99	.8823	.7107	.5897	.5037	.4401	.3911	.3522	.3207	.2945	.2104	.1646
9	.95	.8010	.6167	.5017	.4241	.3682	.3259	.2926	.2659	.2439	.1736	.1357
	.99	.8674	.6912	.5702	.4854	.4229	.3751	.3373	.3067	.2813	.2002	.1567
16	.95	.7341	.5466	.4366	.3645	.3135	.2756	.2462	.2226	.2032	.1429	.1108
	.99	.7949	.6059	.4884	.4094	.3529	.3105	.2779	.2514	.2297	.1612	.1248
36	.95	.6602	.4748	.3720	.3066	.2612	.2278	.2022	.1820	.1655	.1144	.0879
	.99	.7067	.5153	.4057	.3351	.2858	.2494	.2214	.1992	.1811	.1251	.0960
144	.95	.5813	.4031	.3093	.2513	.2119	.1833	.1616	.1446	.1308	.0889	.0675
	.99	.6062	.4230	.3251	.2644	.2229	.1929	.1700	.1521	.1376	.0934	.0709

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* Reproduced with permission from C. Eisenhart, M. W. Hastay, and W. A. Wallis, "Techniques of Statistical Analysis," chap. 15, McGraw-Hill Book Company, New York, 1947. Reprinted from B. J. Winer "Statistical Principles in Experimental Design," McGraw-Hill Book Company, New York, 1962.

Table 6-10. Distribution of the studentized range statistic*

df for $s_{\bar{x}}$	$1-\alpha$	r = number of steps between ordered means													
		2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	.95	18.0	27.0	32.8	37.1	40.4	43.1	45.4	47.4	49.1	50.6	52.0	53.2	54.3	55.4
	.99	90.0	135	164	186	202	216	227	237	246	253	260	266	272	277
2	.95	6.09	8.3	9.8	10.9	11.7	12.4	13.0	13.5	14.0	14.4	14.7	15.1	15.4	15.7
	.99	14.0	19.0	22.3	24.7	26.6	28.2	29.5	30.7	31.7	32.6	33.4	34.1	34.8	35.4
3	.95	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.2	10.4	10.5
	.99	8.26	10.6	12.2	13.3	14.2	15.0	15.6	16.2	16.7	17.1	17.5	17.9	18.2	18.5
4	.95	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66
	.99	6.51	8.12	9.17	9.96	10.6	11.1	11.5	11.9	12.3	12.6	12.8	13.1	13.3	13.5
5	.95	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72
	.99	5.70	6.97	7.80	8.42	8.91	9.32	9.67	9.97	10.2	10.5	10.7	10.9	11.1	11.2
6	.95	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14
	.99	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30	9.49	9.65	9.81	9.95
7	.95	3.34	4.16	4.69	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76
	.99	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55	8.71	8.86	9.00	9.12
8	.95	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48
	.99	4.74	5.63	6.20	6.63	6.96	7.24	7.47	7.68	7.87	8.03	8.18	8.31	8.44	8.55
9	.95	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87	5.98	6.09	6.19	6.28
	.99	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.32	7.49	7.65	7.78	7.91	8.03	8.13
10	.95	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11
	.99	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36	7.48	7.60	7.71	7.81
11	.95	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.99
	.99	4.39	5.14	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13	7.26	7.36	7.46	7.56
12	.95	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51	5.62	5.71	5.80	5.88
	.99	4.32	5.04	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94	7.06	7.17	7.26	7.36

Table 6-10. Distribution of the studentized range statistic* (continued)

13	.95	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79
	.99	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79	6.90	7.01	7.10	7.19
14	.95	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	6.64	5.72
	.99	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66	6.77	6.87	6.96	7.05
16	.95	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59
	.99	4.13	4.78	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46	6.56	6.66	6.74	6.82
18	.95	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50
	.99	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31	6.41	6.50	6.58	6.65
20	.95	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43
	.99	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19	6.29	6.37	6.45	6.52
24	.95	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32
	.99	3.96	4.54	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02	6.11	6.19	6.26	6.33
30	.95	2.89	3.49	3.84	4.10	4.30	4.46	4.60	4.72	4.83	4.92	5.00	5.08	5.15	5.21
	.99	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.56	5.76	5.85	5.93	6.01	6.08	6.14
40	.95	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74	4.82	4.91	4.98	5.05	5.11
	.99	3.82	4.37	4.70	4.93	5.11	5.27	5.39	5.50	5.60	5.69	5.77	5.84	5.90	5.96
60	.95	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00
	.99	3.76	4.28	4.60	4.82	4.99	5.13	5.25	5.36	5.45	5.53	5.60	5.67	5.73	5.79
120	.95	2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.48	4.56	4.64	4.72	4.78	4.84	4.90
	.99	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.38	5.44	5.51	5.56	5.61
∞	.95	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80
	.99	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23	5.29	5.35	5.40	5.45

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* This table is abridged from Table II.2 in "The Probability Integrals of the Range and of the Studentized Range," prepared by H. Leon Harter, Donald S. Clemm, and Eugene H. Guthrie. These tables are published in WADC tech. Rep. 58–484, vol. 2, 1959, Wright Air Development Center, and are reproduced with the kind permission of the authors. Reprinted from B. J. Winer, "Statistical Principles in Experimental Design," McGraw-Hill Book Company, New York, 1962.

Table 6-11. Percentiles of the χ^2 distribution*

df	Percent									
	.5	1	2.5	5	10	90	95	97.5	99	99.5
1	.000039	.00016	.00098	.0039	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.1026	.2107	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.064	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
120	83.85	86.92	91.58	95.70	100.62	140.23	146.57	152.21	158.95	163.64

For large values of degrees of freedom the approximate formula

$$\chi_\alpha{}^2 = n\left(1 - \frac{2}{9n} + z_\alpha\sqrt{\frac{2}{9n}}\right)^3$$

where z_α is the normal deviate and n is the number of degrees of freedom, may be used. For example $\chi_{.99}{}^2 = 60[1 - .00370 + 2.326(.06086)]^3 = 60(1.1379)^3 = 88.4$ for the 99th percentile for 60 degrees of freedom.

* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1962.

Table 6-12. Percentiles of the χ^2/df distributions*

df	.05	.1	.5	1.0	2.5	5.0	10	20	30	40	50	60
1	$.0^{6}39$	$.0^{5}157$	$.0^{4}39$	$.0^{3}16$	$.0^{3}98$	$.0^{2}39$	.016	.064	.148	.275	.455	.708
2	.001	.001	.005	.010	.025	.052	.106	.223	.356	.511	.693	.916
3	.005	.008	.024	.038	.072	.117	.195	.335	.475	.623	.789	.982
4	.016	.023	.052	.074	.121	.178	.266	.412	.549	.688	.839	1.011
5	.032	.042	.082	.111	.166	.229	.322	.469	.600	.731	.870	1.03
6	.050	.064	.113	.145	.206	.272	.367	.512	.638	.762	.891	1.04
7	.069	.085	.141	.177	.241	.310	.405	.546	.667	.785	.907	1.04
8	.089	.107	.168	.206	.272	.342	.436	.574	.691	.803	.918	1.04
9	.108	.128	.193	.232	.300	.369	.463	.598	.710	.817	.927	1.05
10	.126	.148	.216	.256	.325	.394	.487	.618	.727	.830	.934	1.05
11	.144	.167	.237	.278	.347	.416	.507	.635	.741	.840	.940	1.05
12	.161	.184	.256	.298	.367	.436	.525	.651	.753	.848	.945	1.05
13	.177	.201	.274	.316	.385	.453	.542	.664	.764	.856	.949	1.05
14	.193	.217	.291	.333	.402	.469	.556	.676	.773	.863	.953	1.05
15	.207	.232	.307	.349	.418	.484	.570	.687	.781	.869	.956	1.05
16	.221	.246	.321	.363	.432	.498	.582	.697	.789	.874	.959	1.05
17	.234	.260	.335	.377	.445	.510	.593	.706	.796	.879	.961	1.05
18	.247	.272	.348	.390	.457	.522	.604	.714	.802	.883	.963	1.05
19	.258	.285	.360	.402	.469	.532	.613	.722	.808	.887	.965	1.05
20	.270	.296	.372	.413	.480	.543	.622	.729	.813	.890	.967	1.05
22	.291	.317	.393	.434	.499	.561	.638	.742	.823	.897	.970	1.05
24	.310	.337	.412	.452	.517	.577	.652	.753	.831	.902	.972	1.05
26	.328	.355	.429	.469	.532	.592	.665	.762	.838	.907	.974	1.05
28	.345	.371	.445	.484	.547	.605	.676	.771	.845	.911	.976	1.04
30	.360	.386	.460	.498	.560	.616	.687	.779	.850	.915	.978	1.04
35	.394	.420	.491	.529	.588	.642	.708	.795	.862	.922	.981	1.04
40	.423	.448	.518	.554	.611	.663	.726	.809	.872	.928	.983	1.04
45	.448	.472	.540	.576	.630	.680	.741	.820	.880	.933	.985	1.04
50	.469	.494	.560	.594	.647	.695	.754	.829	.886	.937	.987	1.04
55	.488	.512	.577	.610	.662	.708	.765	.837	.892	.941	.988	1.04
60	.506	.529	.592	.625	.675	.720	.774	.844	.897	.944	.989	1.04
70	.535	.558	.618	.649	.697	.739	.790	.856	.905	.949	.990	1.03
80	.560	.582	.640	.669	.714	.755	.803	.865	.911	.952	.992	1.03
90	.581	.602	.658	.686	.729	.768	.814	.873	.917	.955	.993	1.03
100	.599	.619	.673	.701	.742	.779	.824	.879	.921	.958	.993	1.03
120	.629	.648	.699	.724	.763	.798	.839	.890	.929	.962	.994	1.03
140	.653	.671	.719	.743	.780	.812	.850	.898	.934	.965	.995	1.03
160	.673	.690	.736	.758	.793	.824	.860	.905	.939	.968	.996	1.02
180	.689	.706	.749	.771	.804	.833	.868	.910	.942	.970	.996	1.02
200	.703	.719	.761	.782	.814	.841	.874	.915	.945	.972	.997	1.02
250	.732	.746	.785	.804	.832	.858	.887	.924	.951	.975	.997	1.02
300	.753	.767	.802	.820	.846	.870	.897	.931	.956	.977	.998	1.02
350	.770	.783	.816	.833	.857	.879	.904	.936	.959	.979	.998	1.02
400	.784	.796	.827	.843	.866	.887	.911	.940	.962	.981	.998	1.02
450	.795	.807	.837	.852	.874	.893	.916	.944	.964	.982	.999	1.02
500	.805	.816	.845	.859	.880	.898	.920	.946	.966	.983	.999	1.01
750	.839	.848	.872	.884	.901	.917	.934	.956	.972	.986	.999	1.01
1000	.859	.868	.889	.899	.914	.928	.943	.962	.976	.988	.999	1.01
5000	.936	.939	.949	.954	.961	.967	.974	.983	.989	.995	1.00	1.00
∞	1	1	1	1	1	1	1	1	1	1	1	1

Read $.0^{3}16$ as .00016, etc.

* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," McGraw-Hill Book Company, New York, 1962.

Table 6-12. Percentiles of the χ^2/df distributions (continued)

70	80	90	95	97.5	99	99.5	99.9	99.95	*df*
1.07	1.64	2.71	3.84	5.02	6.64	7.88	10.83	12.12	1
1.20	1.61	2.30	3.00	3.69	4.61	5.30	6.91	7.60	2
1.22	1.55	2.08	2.60	3.12	3.78	4.28	5.42	5.91	3
1.22	1.50	1.94	2.37	2.79	3.32	3.72	4.62	5.00	4
1.21	1.46	1.85	2.21	2.57	3.02	3.35	4.10	4.42	5
1.21	1.43	1.77	2.10	2.41	2.80	3.09	3.74	4.02	6
1.20	1.40	1.72	2.01	2.29	2.64	2.90	3.47	3.72	7
1.19	1.38	1.67	1.94	2.19	2.51	2.74	3.27	3.48	8
1.18	1.36	1.63	1.88	2.11	2.41	2.62	3.10	3.30	9
1.18	1.34	1.60	1.83	2.05	2.32	2.52	2.96	3.14	10
1.17	1.33	1.57	1.79	1.99	2.25	2.43	2.84	3.01	11
1.17	1.32	1.55	1.75	1.94	2.18	2.36	2.74	2.90	12
1.16	1.31	1.52	1.72	1.90	2.13	2.29	2.66	2.81	13
1.16	1.30	1.50	1.69	1.87	2.08	2.24	2.58	2.72	14
1.15	1.29	1.49	1.67	1.83	2.04	2.19	2.51	2.65	15
1.15	1.28	1.47	1.64	1.80	2.00	2.14	2.45	2.58	16
1.15	1.27	1.46	1.62	1.78	1.97	2.10	2.40	2.52	17
1.14	1.26	1.44	1.60	1.75	1.93	2.06	2.35	2.47	18
1.14	1.26	1.43	1.59	1.73	1.90	2.03	2.31	2.42	19
1.14	1.25	1.42	1.57	1.71	1.88	2.00	2.27	2.37	20
1.13	1.24	1.40	1.54	1.67	1.83	1.95	2.19	2.30	22
1.13	1.23	1.38	1.52	1.64	1.79	1.90	2.13	2.23	24
1.12	1.22	1.37	1.50	1.61	1.76	1.86	2.08	2.17	26
1.12	1.22	1.35	1.48	1.59	1.72	1.82	2.03	2.12	28
1.12	1.21	1.34	1.46	1.57	1.70	1.79	1.99	2.07	30
1.11	1.19	1.32	1.42	1.52	1.64	1.72	1.90	1.98	35
1.10	1.18	1.30	1.39	1.48	1.59	1.67	1.84	1.90	40
1.10	1.17	1.28	1.37	1.45	1.55	1.63	1.78	1.84	45
1.09	1.16	1.26	1.35	1.43	1.52	1.59	1.73	1.79	50
1.09	1.16	1.25	1.33	1.41	1.50	1.56	1.69	1.75	55
1.09	1.15	1.24	1.32	1.39	1.47	1.53	1.66	1.71	60
1.08	1.14	1.22	1.29	1.36	1.43	1.49	1.60	1.65	70
1.08	1.13	1.21	1.27	1.33	1.40	1.45	1.56	1.60	80
1.07	1.12	1.20	1.26	1.31	1.38	1.43	1.52	1.56	90
1.07	1.12	1.18	1.24	1.30	1.36	1.40	1.49	1.53	100
1.06	1.11	1.17	1.22	1.27	1.32	1.36	1.45	1.48	120
1.06	1.10	1.16	1.20	1.25	1.30	1.33	1.41	1.44	140
1.06	1.09	1.15	1.19	1.23	1.28	1.31	1.38	1.41	160
1.05	1.09	1.14	1.18	1.22	1.26	1.29	1.36	1.38	180
1.05	1.08	1.13	1.17	1.21	1.25	1.28	1.34	1.36	200
1.04	1.07	1.12	1.15	1.18	1.22	1.25	1.30	1.32	250
1.04	1.07	1.11	1.14	1.17	1.20	1.22	1.27	1.29	300
1.04	1.06	1.10	1.13	1.15	1.18	1.21	1.25	1.27	350
1.04	1.06	1.09	1.12	1.14	1.17	1.19	1.24	1.25	400
1.03	1.06	1.09	1.11	1.13	1.16	1.18	1.22	1.23	450
1.03	1.05	1.08	1.11	1.13	1.15	1.17	1.21	1.22	500
1.03	1.04	1.07	1.09	1.10	1.12	1.14	1.17	1.18	750
1.02	1.04	1.06	1.07	1.09	1.11	1.12	1.14	1.15	1000
1.01	1.02	1.02	1.03	1.04	1.05	1.05	1.06	1.07	5000
1	1	1	1	1	1	1	1	1	∞

section

7

NONPARAMETRIC AND DISTRIBUTION-FREE TESTS

Recent years have seen the development of a number of statistical test procedures which do not require strong assumptions as to the nature of population distributions. This is in contrast with most of the test procedures utilizing the tables in the preceding section, which require rather strong assumptions about distributions and/or parameter values. This section contains tables useful with a number of nonparametric tests. The test procedures described are all discussed more fully in refs. 2, 6, and 10.

Table 7-1 is in three parts. The first part is for the *Kolmogorov-Smirnov one-sample test,* and the second and third parts are for use with the *two-sample test.* All three tables use a statistic D. For the *one-sample case,* D is the maximum absolute difference in relative frequency between the sample cumulative-proportion step function and a completely specified cumulative probability distribution. The hypothesis that the sample was drawn from a population distributed according to the specified distribution is rejected at the α level of significance if the observed D exceeds the tabulated value for the desired level of significance and sample size. The test is against the composite alternative that the specified distribution is not correct; hence the test is sensitive to differences in location, in dispersion, in skewness, etc. The α level critical value of D for a given sample size may also be used to construct a confidence band with coefficient $(1 - \alpha)$ about the observed cumulative-proportion step function.

In the *two-sample test,* D is defined as the maximum difference in the predicted direction between the two-sample cumulative-proportion step functions for the one-tailed test and as the maximum absolute difference between the two for the two-tailed test. The one-tailed test examines the hypothesis that the two samples were drawn from the same population versus the alternative that one population is "stochastically larger" than the other, while the two-tailed test is sensitive to any kind of difference between the two populations. The value K actually tabulated in the second part of Table 7-1 is N times D, where N is the common sample size for both samples and hence is simple frequency difference. Note that n instead of N is used to indicate sample sizes in the third part of Table 7-1. A chi-square approximation can be used for the one-tailed test with large samples. The chi-square approximation makes use of the fact that the quantity $4D^2N_1N_2/(N_1 + N_2)$ is approximately distributed like the chi-square with 2 degrees of freedom when N_1 and N_2 are reasonably large.

Table 7-2 is in two parts for use with the *one-sample and two-sample runs tests.* The one-sample runs test is used to test the hypothesis of random sampling with a dichotomous (or artificially dichotomized) random variable, which takes either of two possible values. A *run* is defined as a sequence of one or more like results bracketed by unlike results, and the number of runs u is simply the number of such sequences. The two-sample runs test is used to test whether two samples have been drawn from populations having the same probability distribution versus the alternative that the two populations differ in any respect whatsoever. The total of $N_1 + N_2$ observations are put in rank order, and the number of runs in this ordered series are then determined where a run is defined as any sequence of observations from the same sample bracketed by observations from the other. The first part of Table 7-2 presents the cumulative probability distribution of u for various combinations of sample sizes N_1 and N_2, and the second part contains some fractiles or percentage points of the distribution. Both tables may be used with either the one-sample or two-sample test. For the one-sample test, the hypothesis of random sampling is rejected if there are either too few or too many runs, while in the two-sample test the null hypothesis is rejected only if there are too few runs.

The *Wilcoxon signed-ranks test* examines hypotheses about population medians. The hypothetical median is subtracted from each observation in a sample, and the resulting scores are put in rank order, disregarding the sign. The sign of the difference is then assigned to the corresponding rank, and the positive ranks are summed to obtain the statistic T. Table 7-3 contains selected percentage points of the distribution of T for various values of sample size N. Either one-tailed or two-tailed tests may be carried out, depending on the nature of the alternative hypothesis. For larger values of N, we may make use of the fact that T tends to be approximately normal with mean $N(N + 1)/4$ and variance $N(N + 1)(2N + 1)/24$. The observed value of T is converted into a standardized normal deviate using these values and a correction for continuity (see also Sec. 5).

The *signed-ranks test* is particularly valuable for testing the hypothesis that the median-difference score for matched or repeated observations is zero (or some other hypothetical value). This test is carried out by regarding the difference score as the variate of interest.

The *Walsh test* is used to test the hypothesis that a sample has been drawn from a population in which the true median is zero versus either a one- or two-tailed alternative. *A necessary assumption is that the probability distribution of the variate in question is symmetrical.*

Table 7-4 contains critical values of the Walsh test for selected significance levels and sample sizes. The symbol μ designates the median, and d_i refers to the ith value in the sample after the observations have been put in rank order. This test is particularly useful in testing hypotheses about median-difference scores for data obtained in matched pairs or by repeated observations. A slight modification of the procedure permits testing hypotheses other than those when the median is zero (ref. 2).

The *rank-sum or Mann-Whitney U test* is one of the most extensively used and powerful of the nonparametric tests. The rank-sum test is used to test the hypothesis that two independent samples of observations have been drawn from the same population against either a two-tailed alternative that the samples come from a different population, or a one-tailed alternative that a specified one of the samples came from a "stochastically larger" population than the other. Let N_1 denote the size of the smaller sample, and N_2 the size of the larger. The total of $N_1 + N_2$ observations are combined and put in rank order, the smallest observation being assigned a rank of 1, etc. The sum of the N_1 ranks, T', is computed for the smaller sample.

Table 7-5*a* contains *percentage points of the sampling distribution of* T' for various combinations of N_1 and N_2. Table 7-5*b* consists of critical values of a related statistic U, defined as the number of times that an observation from one sample precedes an observation from the other in the pooled ranking. U and T' are related by the fact that for the two-tailed test U equals the smaller of the two values $N_1N_2 + N_1(N_1 + 1)/2 - T'$ or $T' - N_1(N_1 + 1)/2$. The rejection region consists of values of U equal to or less than the appropriate tabulated value. Care must be taken when using either T' or U as a one-tailed test in order to keep the direction of the rejection region consistent with the direction of the alternative. To test against the alternative that sample A came from a stochastically larger population than sample B, U would be defined as the number of times an observation from B preceded an observation from A regardless of the sample sizes. The interpretation of T' is somewhat simpler. If the alternative specifies that the numerically smaller sample came from the stochastically larger population, we would accept the alternative for large values of T' and vice versa. For a two-tailed test, the rejection region consists of both "too large" and "too small" values of T'.

Table 7-5*b* complements Table 7-5*a* in that the critical values of U are listed for larger sample sizes than those available in Table 7-5*a* for T'. For even larger samples, T' is approximately normally distributed with mean $N_1(N_1 + N_2 + 1)/2$ and variance $N_1N_2(N_1 + N_2 + 1)/2$. A correction for continuity should be employed in converting T' to a standardized normal deviate (Sec. 5).

Table 7-6 is used in connection with the *two-way analysis of variance by ranks*. The data are cast in a two-way table with N rows and k columns. Each row represents a subject or matched subjects, and the columns represent various treatments or experimental conditions. The scores in each row are ranked, and the data consist of the rank values. The test procedure examines the hypothesis that the different *columns* of ranks come from the same population against the alternative that they come from different populations. The statistic utilized is defined as

$$\chi_r^2 = \frac{12}{Nk(k+1)} [\Sigma R_j^2 - 3N(k+1)]$$

where R_j denotes the sum of the ranks in the jth column.

Table 7-7 contains values of p (significance level) defined as the area to the right of the tabulated value in the sampling distribution of χ_r^2 for k equal 3 and 4 and selected values of N. For larger values of k and N, χ_r^2 is distributed approximately as a chi-square variate with $k - 1$ degrees of freedom. The rejection region consists of values of χ_r^2 as large as or larger than the value such that the probability of χ_r^2 exceeding the chosen value equals the desired significance level.

A *one-way analysis of variance by ranks* may be carried out when the data are in the form of k independent samples of observations, each perhaps corresponding to a different experimental condition or treatment, and it is desired to test the hypothesis that the k samples have been drawn from a common population of measurements versus the alternative that the samples come from different populations. Let n_j denote the size of the jth sample and N the sum of the n_j. The total of N observations is pooled and replaced by ranks with the smallest observation replaced by rank 1, etc. The sum of the ranks is computed separately in each sample, denoted by R_j and the statistic

$$H = \frac{12}{N(N+1)} [\Sigma (R_j^2/M_j) - 3(N+1)]$$

is computed. H is approximately distributed as chi-square with $df = k - 1$ and the null hypothesis is rejected for large values of H. When $k = 3$ and the number of cases in each sample is five or fewer, the chi-square approximation to the sampling distribution of H is not accurate. Table 7-7 contains *critical values of* H for various values of significance level p and combinations of sample sizes when $k = 3$. The significance levels tabulated give the probability of obtaining an observed value of H as large or larger than the associated value when the null hypothesis is true.

Table 7-8 can be used to construct $1 - \alpha$ *nonparametric confidence intervals for a sample median* based on a continuously distributed random variable. The observations are put in rank order, and we can be 100 $(1 - \alpha)$ percent confident that the population median lies between X_k and X_{N-k+1}, where k is given in the table as a function of N and α.

The *sign test* essentially tests the hypothesis that the median difference between matched pairs of observations is zero. To carry out the test, the numbers of positive and negative differences between pairs are recorded, the direction of difference always being the same, and the probability of occurrence of the observed numbers is evaluated by reference to a binomial distribution with p equal $\frac{1}{2}$ and N equal to the number of pairs. A more comprehensive discussion may be found in refs. 2 and 10.

Table 7-9*a* lists two-tailed percentage points of the binomial distribution for p equal ½ and various values of N. The numbers in the body of the table are numbers of "successes" or "failures," whichever is the less frequent, such that the probability of occurrence of this few or fewer of the less frequent occurrence is equal to or less than the corresponding column heading. Each row of the table corresponds to a different value of N, the total number of occurrences. Table 7-9*b* shows cumulative probabilities for various values of N.

TABLE 7-1. TABLES FOR KOLMOGOROV-SMIRNOV TESTS*

Table 7-1*a*. Table of critical values of D in the Kolmogorov-Smirnov one-sample test†

Sample size (N)	Level of significance for D = maximum $\lvert F_0(X) - S_N(X) \rvert$				
	.20	.15	.10	.05	.01
1	.900	.925	.950	.975	.995
2	.684	.726	.776	.842	.929
3	.565	.597	.642	.708	.828
4	.494	.525	.564	.624	.733
5	.446	.474	.510	.565	.669
6	.410	.436	.470	.521	.618
7	.381	.405	.438	.486	.577
8	.358	.381	.411	.457	.543
9	.339	.360	.388	.432	.514
10	.322	.342	.368	.410	.490
11	.307	.326	.352	.391	.468
12	.295	.313	.338	.375	.450
13	.284	.302	.325	.361	.433
14	.274	.292	.314	.349	.418
15	.266	.283	.304	.338	.404
16	.258	.274	.295	.328	.392
17	.250	.266	.286	.318	.381
18	.244	.259	.278	.309	.371
19	.237	.252	.272	.301	.363
20	.231	.246	.264	.294	.356
25	.21	.22	.24	.27	.32
30	.19	.20	.22	.24	.29
35	.18	.19	.21	.23	.27
Over 35	$\frac{1.07}{\sqrt{N}}$	$\frac{1.14}{\sqrt{N}}$	$\frac{1.22}{\sqrt{N}}$	$\frac{1.36}{\sqrt{N}}$	$\frac{1.63}{\sqrt{N}}$

* From S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

† Adapted from F. J. Massey, Jr., The Kolmogorov-Smirnov Test for Goodness of Fit, *J. Amer. Statist. Ass.*, vol. 46, p. 70, 1951, with the kind permission of the author and publisher.

Table 7-1*b*. Table of critical values of K_D in the Kolmogorov-Smirnov two-sample test

(Small samples)

N	One-tailed test*		Two-tailed test†	
	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$
3	3	—	—	—
4	4	—	4	—
5	4	5	5	5
6	5	6	5	6
7	5	6	6	6
8	5	6	6	7
9	6	7	6	7
10	6	7	7	8
11	6	8	7	8
12	6	8	7	8
13	7	8	7	9
14	7	8	8	9
15	7	9	8	9
16	7	9	8	10
17	8	9	8	10
18	8	10	9	10
19	8	10	9	10
20	8	10	9	11
21	8	10	9	11
22	9	11	9	11
23	9	11	10	11
24	9	11	10	12
25	9	11	10	12
26	9	11	10	12
27	9	12	10	12
28	10	12	11	13
29	10	12	11	13
30	10	12	11	13
35	11	13	12	
40	11	14	13	

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* Abridged from L. A. Goodman, Kolmogorov-Smirnov Tests for Psychological Research, *Psychol. Bull.*, vol. 51, p. 167, 1954, with the kind permission of the author and the American Psychological Association.

† Derived from Table 1 of F. J. Massey, Jr., The Distribution of the Maximum Deviation between Two Sample Cumulative Step Functions, *Ann. Math. Statist.*, vol. 22, pp. 126–127, 1951, with the kind permission of the author and the publisher.

Table 7-1c. Table of critical values of D in the Kolmogorov-Smirnov two-sample test

(Large samples: two-tailed test)*

Level of significance	Value of D so large as to call for rejection of H_0 at the indicated level of significance, where $D = \text{maximum } \lvert S_{n_1}(X) - S_{n_2}(X) \rvert$
.10	$1.22\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.05	$1.36\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.025	$1.48\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.01	$1.63\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.005	$1.73\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$
.001	$1.95\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$

* Adapted from N. Smirnov, Tables for Estimating the Goodness of Fit of Empirical Distributions, *Ann. Math. Statist.*, vol. 19, pp. 280–281, 1948, with the kind permission of the publisher.

TABLE 7-2. TABLES FOR THE RUNS TEST

Table 7-2*a*. Critical values of r in the Runs test*

Given in the bodies of Table 7-2*a* and Table 7-2*b* are various critical values of r for various values of n_1 and n_2. For the one-sample runs test, any value of r which is equal to or smaller than that shown in Table 7-2*a* or equal to or larger than that shown in Table 7-2*b* is significant at the .05 level. For the Wald-Wolfowitz two-sample runs test, any value of r which is equal to or smaller than that shown in Table 7-2*a* is significant at the .05 level.

n_1 \ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2	2	2	2	2	2	2	2	2
3					2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
4				2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
5			2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
6		2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
7		2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
8		2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
9		2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
10		2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
11		2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14

* Adapted from Frieda S. Swed and C. Eisenhart, Tables for Testing Randomness of Grouping in a Sequence of Alternatives, *Ann. Math. Statist.*, vol. 14, pp. 83–86, 1943, with the kind permission of the authors and publisher. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 7-2*b*. Critical values of r in the Runs test

n_1 \ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2																			
3																			
4				9	9														
5			9	10	10	11	11												
6			9	10	11	12	12	13	13	13	13								
7				11	12	13	13	14	14	14	14	15	15	15					
8				11	12	13	14	14	15	15	16	16	16	16	17	17	17	17	17
9					13	14	14	15	16	16	16	17	17	18	18	18	18	18	18
10					13	14	15	16	16	17	17	18	18	18	19	19	19	20	20
11					13	14	15	16	17	17	18	19	19	19	20	20	20	21	21
12					13	14	16	16	17	18	19	19	20	20	21	21	21	22	22
13						15	16	17	18	19	19	20	20	21	21	22	22	23	23
14						15	16	17	18	19	20	20	21	22	22	23	23	23	24
15						15	16	18	18	19	20	21	22	22	23	23	24	24	25
16							17	18	19	20	21	21	22	23	23	24	25	25	25
17							17	18	19	20	21	22	23	23	24	25	25	26	26
18							17	18	19	20	21	22	23	24	25	25	26	26	27
19							17	18	20	21	22	23	23	24	25	26	26	27	27
20							17	18	20	21	22	23	24	25	25	26	27	27	28

Table 7-2c. Distribution of the total number of runs u in samples of size (N_1,N_2)*

$(N_1,N_2) \backslash u$	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
(2,3)	.200	.500	.900	1.000															
(2,4)	.133	.400	.800	1.000															
(2,5)	.095	.333	.714	1.000															
(2,6)	.071	.286	.643	1.000															
(2,7)	.056	.250	.583	1.000															
(2,8)	.044	.222	.533	1.000															
(2,9)	.036	.200	.491	1.000															
(2,10)	.030	.182	.455	1.000															
(3,3)	.100	.300	.700	.900	1.000														
(3,4)	.057	.200	.543	.800	.971	1.000													
(3,5)	.036	.143	.429	.714	.929	1.000													
(3,6)	.024	.107	.345	.643	.881	1.000													
(3,7)	.017	.083	.283	.583	.833	1.000													
(3,8)	.012	.067	.236	.533	.788	1.000													
(3,9)	.009	.055	.200	.491	.745	1.000													
(3,10)	.007	.045	.171	.455	.706	1.000													
(4,4)	.029	.114	.371	.629	.886	.971	1.000												
(4,5)	.016	.071	.262	.500	.786	.929	.992	1.000											
(4,6)	.010	.048	.190	.405	.690	.881	.976	1.000											
(4,7)	.006	.033	.142	.333	.606	.833	.954	1.000											
(4,8)	.004	.024	.109	.279	.533	.788	.929	1.000											
(4,9)	.003	.018	.085	.236	.471	.745	.902	1.000											
(4,10)	.002	.014	.068	.203	.419	.706	.874	1.000											
(5,5)	.008	.040	.167	.357	.643	.833	.960	.992	1.000										
(5,6)	.004	.024	.110	.262	.522	.738	.911	.976	.998	1.000									
(5,7)	.003	.015	.076	.197	.424	.652	.854	.955	.992	1.000									
(5,8)	.002	.010	.054	.152	.347	.576	.793	.929	.984	1.000									
(5,9)	.001	.007	.039	.119	.287	.510	.734	.902	.972	1.000									
(5,10)	.001	.005	.029	.095	.239	.455	.678	.874	.958	1.000									
(6,6)	.002	.013	.067	.175	.392	.608	.825	.933	.987	.998	1.000								
(6,7)	.001	.008	.043	.121	.296	.500	.733	.879	.966	.992	.999	1.000							
(6,8)	.001	.005	.028	.086	.226	.413	.646	.821	.937	.984	.998	1.000							
(6,9)	.000	.003	.019	.063	.175	.343	.566	.762	.902	.972	.994	1.000							
(6,10)	.000	.002	.013	.047	.137	.288	.497	.706	.864	.958	.990	1.000							
(7,7)	.001	.004	.025	.078	.209	.383	.617	.791	.922	.975	.996	.999	1.000						
(7,8)	.000	.002	.015	.051	.149	.296	.514	.704	.867	.949	.988	.998	1.000	1.000					
(7,9)	.000	.001	.010	.035	.108	.231	.427	.622	.806	.916	.975	.994	.999	1.000					
(7,10)	.000	.001	.006	.024	.080	.182	.355	.549	.743	.879	.957	.990	.998	1.000					
(8,8)	.000	.001	.009	.032	.100	.214	.405	.595	.786	.900	.968	.991	.999	1.000	1.000				
(8,9)	.000	.001	.005	.020	.069	.157	.319	.500	.702	.843	.939	.980	.996	.999	1.000	1.000			
(8,10)	.000	.000	.003	.013	.048	.117	.251	.419	.621	.782	.903	.964	.990	.998	1.000	1.000			
(9,9)	.000	.000	.003	.012	.044	.109	.238	.399	.601	.762	.891	.956	.988	.997	1.000	1.000	1.000		
(9,10)	.000	.000	.002	.008	.029	.077	.179	.319	.510	.681	.834	.923	.974	.992	.999	1.000	1.000	1.000	
(10,10)	.000	.000	.001	.004	.019	.051	.128	.242	.414	.586	.758	.872	.949	.981	.996	.999	1.000	1.000	1.000

* Reproduced with permission from C. Eisenhart and F. Swed, Tables for Testing Randomness of Grouping in a Sequence of Alternatives, *Annals of Mathematical Statistics*, vol. 14, p. 66, 1943. Reprinted from W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 7-2d. Distribution of the total number of runs*

The values listed on the previous page give the chance that u or fewer runs will occur. For example, for two samples of size 4, the chance of three or fewer runs is .114. For sample sizes $N_1 = N_2$ larger than 10 the following table can be used. The columns headed 0.5, 1, 2.5, 5 give values of u such that u or fewer runs occur with chance less than the indicated percentage. For example, for $N_1 = N_2 = 12$ the chance of 8 or fewer runs is about .05. The columns headed 95, 97.5, 99, 99.5 give values of u for which the chance of u or more runs is less than 5, 2.5, 1, 0.5 percent.

$N_1 = N_2$	0.5	1	2.5	5	95	97.5	99	99.5	Mean	Var.	s.d.
11	5	6	7	7	16	16	17	18	12	5.24	2.29
12	6	7	7	8	17	18	18	19	13	5.74	2.40
13	7	7	8	9	18	19	20	20	14	6.24	2.50
14	7	8	9	10	19	20	21	22	15	6.74	2.60
15	8	9	10	11	20	21	22	23	16	7.24	2.69
16	9	10	11	11	22	22	23	24	17	7.74	2.78
17	10	10	11	12	23	24	25	25	18	8.24	2.87
18	10	11	12	13	24	25	26	27	19	8.74	2.96
19	11	12	13	14	25	26	27	28	20	9.24	3.04
20	12	13	14	15	26	27	28	29	21	9.74	3.12
25	16	17	18	19	32	33	34	35	26	12.24	3.50
30	20	21	22	24	37	39	40	41	31	14.75	3.84
35	24	25	27	28	43	44	46	47	36	17.25	4.15
40	29	30	31	33	48	50	51	52	41	19.75	4.44
45	33	34	36	37	54	55	57	58	46	22.25	4.72
50	37	38	40	42	59	61	63	64	51	24.75	4.97
55	42	43	45	46	65	66	68	69	56	27.25	5.22
60	46	47	49	51	70	72	74	75	61	29.75	5.45
65	50	52	54	56	75	77	79	81	66	32.25	5.68
70	55	56	58	60	81	83	85	86	71	34.75	5.89
75	59	61	63	65	86	88	90	92	76	37.25	6.10
80	64	65	68	70	91	93	96	97	81	39.75	6.30
85	68	70	72	74	97	99	101	103	86	42.25	6.50
90	73	74	77	79	102	104	107	108	91	44.75	6.69
95	77	79	82	84	107	109	112	114	96	47.25	6.87
100	82	84	86	88	113	115	117	119	101	49.75	7.05

For large values of N_1 and N_2, particularly for $N_1 = N_2$ greater than 10, a normal approximation may be used. The mean and variance are

$$\frac{2N_1N_2}{N_1 + N_2} + 1 \qquad \text{and} \qquad \frac{2N_1N_2(2N_1N_2 - N_1 - N_2)}{(N_1 + N_2)^2(N_1 + N_2 - 1)}$$

respectively. For example, for $N_1 = N_2 = 20$ the mean is 21, and the variance is 9.74. The 97.5 and 2.5 percentiles are $21 + 1.96\sqrt{9.74}$ and $21 - 1.96\sqrt{9.74}$ or 27.1 and 14.9. The approximation is improved if ½ is subtracted from the computed values. The resulting percentiles would then be 26.6 and 14.4. For two samples each of size N the mean is $N + 1$, and the variance is $N(N - 1)/(2N - 1)$.

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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 7-3. Critical values for the Wilcoxon signed-rank test[*]

The percentiles listed cover the range $\alpha = .005$ to $.125$ for every sample size up to $N = 20$. Values T_α are such that the probability is α that the signed rank statistic is less than or equal to T_α. The values $T_{1-\alpha}$ are such that the probability is α that T is greater than or equal to $T_{1-\alpha}$.

T_α	$T_{1-\alpha}$	α
	$N = 1$	
0	1	.500
	$N = 2$	
0	3	.250
	$N = 3$	
0	6	.125
	$N = 4$	
0	10	.062
1	9	.125
	$N = 5$	
0	15	.031
1	14	.062
2	13	.094
3	12	.156
	$N = 6$	
0	21	.016
1	20	.031
2	19	.047
3	18	.078
4	17	.109
5	16	.156
	$N = 7$	
0	28	.008
1	27	.016
2	26	.023
3	25	.039
4	24	.055
5	23	.078
6	22	.109
7	21	.148
	$N = 8$	
0	36	.004
1	35	.008
2	34	.012
3	33	.020
4	32	.027
5	31	.039
6	30	.055
7	29	.074
8	28	.098
9	27	.125
	$N = 9$	
1	44	.004
2	43	.006
3	42	.010
	$N = 9$ (cont.)	
4	41	.014
5	40	.020
6	39	.027
7	38	.037
8	37	.049
9	36	.064
10	35	.082
11	34	.102
12	33	.125
	$N = 10$	
3	52	.005
4	51	.007
5	50	.010
6	49	.014
7	48	.019
8	47	.024
9	46	.032
10	45	.042
11	44	.053
12	43	.065
13	42	.080
14	41	.097
15	40	.116
16	39	.138
	$N = 11$	
5	61	.005
6	60	.007
7	59	.009
8	58	.012
9	57	.016
10	56	.021
11	55	.027
12	54	.034
13	53	.042
14	52	.051
15	51	.062
16	50	.074
17	49	.087
18	48	.103
19	47	.120
20	46	.139
	$N = 12$	
7	71	.005
8	70	.006
	$N = 12$ (cont.)	
9	69	.008
10	68	.010
11	67	.013
12	66	.017
13	65	.021
14	64	.026
15	63	.032
16	62	.039
17	61	.046
18	60	.055
19	59	.065
20	58	.076
21	57	.088
22	56	.102
23	55	.117
24	54	.133
	$N = 13$	
9	82	.004
10	81	.005
11	80	.007
12	79	.009
13	78	.011
14	77	.013
15	76	.016
16	75	.020
17	74	.024
18	73	.029
19	72	.034
20	71	.040
21	70	.047
22	69	.055
23	68	.064
24	67	.073
25	66	.084
26	65	.095
27	64	.108
28	63	.122
29	62	.137
	$N = 14$	
12	93	.004
13	92	.005
14	91	.007
15	90	.008
16	89	.010
	$N = 14$ (cont.)	
17	88	.012
18	87	.015
19	86	.018
20	85	.021
21	84	.025
22	83	.029
23	82	.034
24	81	.039
25	80	.045
26	79	.052
27	78	.059
28	77	.068
29	76	.077
30	75	.086
31	74	.097
32	73	.108
33	72	.121
34	71	.134
	$N = 15$	
15	105	.004
16	104	.005
17	103	.006
18	102	.008
19	101	.009
20	100	.011
21	99	.013
22	98	.015
23	97	.018
24	96	.021
25	95	.024
26	94	.028
27	93	.032
28	92	.036
29	91	.042
30	90	.047
31	89	.053
32	88	.060
33	87	.068
34	86	.076
35	85	.084
36	84	.094
37	83	.104
38	82	.115
39	81	.126

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[*] From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 7-3. Critical values for the Wilcoxon signed-rank test (continued)

T_α	$T_{1-\alpha}$	α	T_α	$T_{1-\alpha}$	α	T_α	$T_{1-\alpha}$	α	T_α	$T_{1-\alpha}$	α
	$N = 16$			$N = 17$ (cont.)			$N = 18$ (cont.)			$N = 19$ (cont.)	
19	117	.005	36	117	.028	51	120	.071	64	126	.112
20	116	.005	37	116	.032	52	119	.077	65	125	.121
21	115	.007	38	115	.036	53	118	.084	66	124	.129
22	114	.008	39	114	.040	54	117	.091		$N = 20$	
23	113	.009	40	113	.044	55	116	.098	37	173	.005
24	112	.011	41	112	.049	56	115	.106	38	172	.005
25	111	.012	42	111	.054	57	114	.114	39	171	.006
26	110	.014	43	110	.060	58	113	.123	40	170	.007
27	109	.017	44	109	.066	59	112	.132	41	169	.008
28	108	.019	45	108	.072		$N = 19$		42	168	.009
29	107	.022	46	107	.080	32	158	.005	43	167	.010
30	106	.025	47	106	.087	33	157	.005	44	166	.011
31	105	.029	48	105	.095	34	156	.006	45	165	.012
32	104	.033	49	104	.103	35	155	.007	46	164	.013
33	103	.037	50	103	.112	36	154	.008	47	163	.015
34	102	.042	51	102	.122	37	153	.009	48	162	.016
35	101	.047	52	101	.132	38	152	.010	49	161	.018
36	100	.052		$N = 18$		39	151	.011	50	160	.020
37	99	.058	27	144	.004	40	150	.013	51	159	.022
38	98	.065	28	143	.005	41	149	.014	52	158	.024
39	97	.072	29	142	.006	42	148	.016	53	157	.027
40	96	.080	30	141	.007	43	147	.018	54	156	.029
41	95	.088	31	140	.008	44	146	.020	55	155	.032
42	94	.096	32	139	.009	45	145	.022	56	154	.035
43	93	.106	33	138	.010	46	144	.025	57	153	.038
44	92	.116	34	137	.012	47	143	.027	58	152	.041
45	91	.125	35	136	.013	48	142	.030	59	151	.045
46	90	.137	36	135	.015	49	141	.033	60	150	.049
	$N = 17$		37	134	.017	50	140	.036	61	149	.053
23	130	.005	38	133	.019	51	139	.040	62	148	.057
24	129	.005	39	132	.022	52	138	.044	63	147	.061
25	128	.006	40	131	.024	53	137	.048	64	146	.066
26	127	.008	41	130	.027	54	136	.052	65	145	.071
27	126	.009	42	129	.030	55	135	.057	66	144	.077
28	125	.010	43	128	.033	56	134	.062	67	143	.082
29	124	.012	44	127	.037	57	133	.067	68	142	.088
30	123	.013	45	126	.040	58	132	.072	69	141	.095
31	122	.015	46	125	.045	59	131	.078	70	140	.101
32	121	.017	47	124	.049	60	130	.084	71	139	.108
33	120	.020	48	123	.054	61	129	.091	72	138	.115
34	119	.022	49	122	.059	62	128	.098	73	137	.123
35	118	.025	50	121	.065	63	127	.105	74	136	.131

Table 7-4. Critical values for the Walsh test*

N	Significance level of tests		Tests	
			Two-tailed: accept $\mu_1 \neq 0$ if either	
	One-tailed	Two-tailed	One-tailed: accept $\mu_1 < 0$ if	One-tailed: accept $\mu_1 > 0$ if
4	.062	.125	$d_4 < 0$	$d_1 > 0$
5	.062	.125	$\frac{1}{2}(d_4 + d_5) < 0$	$\frac{1}{2}(d_1 + d_2) > 0$
	.031	.062	$d_5 < 0$	$d_1 > 0$
6	.047	.094	$\max [d_5, \frac{1}{2}(d_4 + d_6)] < 0$	$\min [d_2, \frac{1}{2}(d_1 + d_3)] > 0$
	.031	.062	$\frac{1}{2}(d_5 + d_6) < 0$	$\frac{1}{2}(d_1 + d_2) > 0$
	.016	.031	$d_6 < 0$	$d_1 > 0$
7	.055	.109	$\max [d_5, \frac{1}{2}(d_4 + d_7)] < 0$	$\min [d_3, \frac{1}{2}(d_1 + d_4)] > 0$
	.023	.047	$\max [d_6, \frac{1}{2}(d_5 + d_7)] < 0$	$\min [d_2, \frac{1}{2}(d_1 + d_3)] > 0$
	.016	.031	$\frac{1}{2}(d_6 + d_7) < 0$	$\frac{1}{2}(d_1 + d_2) > 0$
	.008	.016	$d_7 < 0$	$d_1 > 0$
8	.043	.086	$\max [d_6, \frac{1}{2}(d_4 + d_8)] > 0$	$\min [d_3, \frac{1}{2}(d_1 + d_5)] > 0$
	.027	.055	$\max [d_6, \frac{1}{2}(d_5 + d_8)] < 0$	$\min [d_3, \frac{1}{2}(d_1 + d_4)] > 0$
	.012	.023	$\max [d_7, \frac{1}{2}(d_6 + d_8)] < 0$	$\min [d_2, \frac{1}{2}(d_1 + d_3)] > 0$
	.008	.016	$\frac{1}{2}(d_7 + d_8) < 0$	$\frac{1}{2}(d_1 + d_2) > 0$
	.004	.008	$d_8 < 0$	$d_1 > 0$
9	.051	.102	$\max [d_6, \frac{1}{2}(d_4 + d_9)] < 0$	$\min [d_4, \frac{1}{2}(d_1 + d_6)] > 0$
	.022	.043	$\max [d_7, \frac{1}{2}(d_5 + d_9)] < 0$	$\min [d_3, \frac{1}{2}(d_1 + d_5)] > 0$
	.010	.020	$\max [d_8, \frac{1}{2}(d_5 + d_9)] < 0$	$\min [d_2, \frac{1}{2}(d_1 + d_5)] > 0$
	.006	.012	$\max [d_8, \frac{1}{2}(d_7 + d_9)] < 0$	$\min [d_2, \frac{1}{2}(d_1 + d_3)] > 0$
	.004	.008	$\frac{1}{2}(d_8 + d_9) < 0$	$\frac{1}{2}(d_1 + d_2) > 0$
10	.056	.111	$\max [d_6, \frac{1}{2}(d_4 + d_{10})] < 0$	$\min [d_5, \frac{1}{2}(d_1 + d_7)] > 0$
	.025	.051	$\max [d_7, \frac{1}{2}(d_5 + d_{10})] < 0$	$\min [d_4, \frac{1}{2}(d_1 + d_6)] > 0$
	.011	.021	$\max [d_8, \frac{1}{2}(d_6 + d_{10})] < 0$	$\min [d_3, \frac{1}{2}(d_1 + d_5)] > 0$
	.005	.010	$\max [d_9, \frac{1}{2}(d_6 + d_{10})] < 0$	$\min [d_2, \frac{1}{2}(d_1 + d_5)] > 0$
11	.048	.097	$\max [d_7, \frac{1}{2}(d_4 + d_{11})] < 0$	$\min [d_5, \frac{1}{2}(d_1 + d_8)] > 0$
	.028	.056	$\max [d_7, \frac{1}{2}(d_5 + d_{11})] < 0$	$\min [d_5, \frac{1}{2}(d_1 + d_7)] > 0$
	.011	.021	$\max [\frac{1}{2}(d_6 + d_{11}), \frac{1}{2}(d_8 + d_9)] < 0$	$\min [\frac{1}{2}(d_1 + d_6), \frac{1}{2}(d_3 + d_4)] > 0$
	.005	.011	$\max [d_9, \frac{1}{2}(d_7 + d_{11})] < 0$	$\min [d_3, \frac{1}{2}(d_1 + d_5)] > 0$
12	.047	.094	$\max [\frac{1}{2}(d_4 + d_{12}), \frac{1}{2}(d_5 + d_{11})] < 0$	$\min [\frac{1}{2}(d_1 + d_9), \frac{1}{2}(d_2 + d_8)] > 0$
	.024	.048	$\max [d_8, \frac{1}{2}(d_5 + d_{12})] < 0$	$\min [d_5, \frac{1}{2}(d_1 + d_8)] > 0$
	.010	.020	$\max [d_9, \frac{1}{2}(d_6 + d_{12})] < 0$	$\min [d_4, \frac{1}{2}(d_1 + d_7)] > 0$
	.005	.011	$\max [\frac{1}{2}(d_7 + d_{12}), \frac{1}{2}(d_9 + d_{10})] < 0$	$\min [\frac{1}{2}(d_1 + d_6), \frac{1}{2}(d_3 + d_4)] > 0$
13	.047	.094	$\max [\frac{1}{2}(d_4 + d_{13}), \frac{1}{2}(d_5 + d_{12})] < 0$	$\min [\frac{1}{2}(d_1 + d_{10}), \frac{1}{2}(d_2 + d_9)] > 0$
	.023	.047	$\max [\frac{1}{2}(d_5 + d_{13}), \frac{1}{2}(d_6 + d_{12})] < 0$	$\min [\frac{1}{2}(d_1 + d_9), \frac{1}{2}(d_2 + d_8)] > 0$
	.010	.020	$\max [\frac{1}{2}(d_6 + d_{13}), \frac{1}{2}(d_9 + d_{10})] < 0$	$\min [\frac{1}{2}(d_1 + d_8), \frac{1}{2}(d_4 + d_5)] > 0$
	.005	.010	$\max [d_{10}, \frac{1}{2}(d_7 + d_{13})] < 0$	$\min [d_4, \frac{1}{2}(d_1 + d_7)] > 0$
14	.047	.094	$\max [\frac{1}{2}(d_4 + d_{14}), \frac{1}{2}(d_5 + d_{13})] < 0$	$\min [\frac{1}{2}(d_1 + d_{11}), \frac{1}{2}(d_2 + d_{10})] > 0$
	.023	.047	$\max [\frac{1}{2}(d_5 + d_{14}), \frac{1}{2}(d_6 + d_{13})] < 0$	$\min [\frac{1}{2}(d_1 + d_{10}), \frac{1}{2}(d_2 + d_9)] > 0$
	.010	.020	$\max [d_{10}, \frac{1}{2}(d_6 + d_{14})] < 0$	$\min [d_5, \frac{1}{2}(d_1 + d_9)] > 0$
	.005	.010	$\max [\frac{1}{2}(d_7 + d_{14}), \frac{1}{2}(d_{10} + d_{11})] < 0$	$\min [\frac{1}{2}(d_1 + d_8), \frac{1}{2}(d_4 + d_5)] > 0$
15	.047	.094	$\max [\frac{1}{2}(d_4 + d_{15}), \frac{1}{2}(d_5 + d_{14})] < 0$	$\min [\frac{1}{2}(d_1 + d_{12}), \frac{1}{2}(d_2 + d_{11})] > 0$
	.023	.047	$\max [\frac{1}{2}(d_5 + d_{15}), \frac{1}{2}(d_6 + d_{14})] < 0$	$\min [\frac{1}{2}(d_1 + d_{11}), \frac{1}{2}(d_2 + d_{10})] > 0$
	.010	.020	$\max [\frac{1}{2}(d_6 + d_{15}), \frac{1}{2}(d_{10} + d_{11})] < 0$	$\min [\frac{1}{2}(d_1 + d_{10}), \frac{1}{2}(d_5 + d_6)] > 0$
	.005	.010	$\max [d_{11}, \frac{1}{2}(d_7 + d_{15})] < 0$	$\min [d_5, \frac{1}{2}(d_1 + d_9)] > 0$

* Adapted from J. E. Walsh, Applications of Some Significance Tests for the Median Which Are Valid under Very General Conditions, *J. Amer. Statist. Ass.*, vol. 44, p. 343, 1949, with the kind permission of the author and the publisher. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

TABLE 7-5. TABLES FOR THE MANN-WHITNEY TEST

Table 7-5*a*. Distribution of the rank sum T'*

The values of T'_α, $T'_{1-\alpha}$, and α are such that if the N_1 and N_2 observations are chosen at random from the same population the chance that the rank sum T' of the N_1 observations in the smaller sample is equal to or less than T'_α is α and the chance that T' is equal to or greater than $T'_{1-\alpha}$ is α. The sample sizes are shown in parentheses (N_1,N_2).

T'_α	$T'_{1-\alpha}$	α
	(1,1)	
1	2	.500
	(1,2)	
1	3	.333
2	2	.667
	(1,3)	
1	4	.250
2	3	.500
	(1,4)	
1	5	.200
2	4	.400
3	3	.600
	(1,5)	
1	6	.167
2	5	.333
3	4	.500
	(1,6)	
1	7	.143
2	6	.286
3	5	.428
4	4	.571
	(1,7)	
1	8	.125
2	7	.250
3	6	.375
4	5	.500
	(1,8)	
1	9	.111
2	8	.222
3	7	.333
4	6	.444
5	5	.556
	(1,9)	
1	10	.100
2	9	.200
3	8	.300
4	7	.400
5	6	.500
	(1,10)	
1	11	.091
2	10	.182
3	9	.273
4	8	.364
5	7	.455
6	6	.545
	(2,2)	
3	7	.167
4	6	.333
5	5	.667
	(2,3)	
3	9	.100
4	8	.200
5	7	.400
6	6	.600
	(2,4)	
3	11	.067
4	10	.133
5	9	.267
6	8	.400
7	7	.600
	(2,5)	
3	13	.047
4	12	.095
5	11	.190
6	10	.286
7	9	.429
8	8	.571
	(2,6)	
3	15	.036
4	14	.071
5	13	.143
6	12	.214
7	11	.321
8	10	.429
9	9	.571
	(2,7)	
3	17	.028
4	16	.056
5	15	.111
6	14	.167
7	13	.250
8	12	.333
9	11	.444
10	10	.556
	(2,8)	
3	19	.022
4	18	.044
5	17	.089
6	16	.133
7	15	.200
8	14	.267
9	13	.356
10	12	.444
11	11	.556
	(2,9)	
3	21	.018
4	20	.036
5	19	.073
6	18	.109
7	17	.164
8	16	.218
9	15	.291
10	14	.364
11	13	.455
12	12	.545
	(2,10)	
3	23	.015
4	22	.030
5	21	.061
6	20	.091
7	19	.136
8	18	.182
9	17	.242
10	16	.303
11	15	.379
12	14	.455
13	13	.545
	(3,3)	
6	15	.050
7	14	.100
8	13	.200
9	12	.350
10	11	.500
	(3,4)	
6	18	.028
7	17	.057
8	16	.114
9	15	.200
10	14	.314
11	13	.429
12	12	.571
	(3,5)	
6	21	.018
7	20	.036
8	19	.071
9	18	.125
10	17	.196
11	16	.286
12	15	.393
13	14	.500
	(3,6)	
6	24	.012
7	23	.024
8	22	.048
9	21	.083
10	20	.131
11	19	.190
12	18	.274
13	17	.357
14	16	.452
15	15	.548
	(3,7)	
6	27	.008
7	26	.017
8	25	.033
9	24	.058
10	23	.092
11	22	.133
12	21	.192
13	20	.258
14	19	.333
15	18	.417
16	17	.500
	(3,8)	
6	30	.006
7	29	.012
8	28	.024
9	27	.042
10	26	.067
11	25	.097
12	24	.139
13	23	.188
14	22	.248
15	21	.315
16	20	.387
17	19	.461
18	18	.539

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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 7-5*a*. Distribution of the rank sum T' (continued)

T'_{α}	$T'_{1-\alpha}$	α
	(3,9)	
6	33	.005
7	32	.009
8	31	.018
9	30	.032
10	29	.050
11	28	.073
12	27	.105
13	26	.141
14	25	.186
15	24	.241
16	23	.300
17	22	.363
18	21	.432
19	20	.500
	(3,10)	
6	36	.003
7	35	.007
8	34	.014
9	33	.024
10	32	.038
11	31	.056
12	30	.080
13	29	.108
14	28	.143
15	27	.185
16	26	.234
17	25	.287
18	24	.346
19	23	.406
20	22	.469
21	21	.531
	(4,4)	
10	26	.014
11	25	.029
12	24	.057
13	23	.100
14	22	.171
15	21	.243
16	20	.343
17	19	.443
18	18	.557
	(4,5)	
10	30	.008
11	29	.016
12	28	.032
13	27	.056
14	26	.095
15	25	.143
16	24	.206
	(4,5) (cont.)	
17	23	.278
18	22	.365
19	21	.452
20	20	.548
	(4,6)	
10	34	.005
11	33	.010
12	32	.019
13	31	.033
14	30	.057
15	29	.086
16	28	.129
17	27	.176
18	26	.238
19	25	.305
20	24	.381
21	23	.457
22	22	.545
	(4,7)	
10	38	.003
11	37	.006
12	36	.012
13	35	.021
14	34	.036
15	33	.055
16	32	.082
17	31	.115
18	30	.158
19	29	.206
20	28	.264
21	27	.324
22	26	.394
23	25	.464
24	24	.538
	(4,8)	
10	42	.002
11	41	.004
12	40	.008
13	39	.014
14	38	.024
15	37	.036
16	36	.055
17	35	.077
18	34	.107
19	33	.141
20	32	.184
21	31	.230
22	30	.285
23	29	.341
	(4,8) (cont.)	
24	28	.404
25	27	.467
26	26	.533
	(4,9)	
10	46	.001
11	45	.003
12	44	.006
13	43	.010
14	42	.017
15	41	.025
16	40	.038
17	39	.053
18	38	.074
19	37	.099
20	36	.130
21	35	.165
22	34	.207
23	33	.252
24	32	.302
25	31	.355
26	30	.413
27	29	.470
28	28	.530
	(4,10)	
10	50	.001
11	49	.002
12	48	.004
13	47	.007
14	46	.012
15	45	.018
16	44	.026
17	43	.038
18	42	.053
19	41	.071
20	40	.094
21	39	.120
22	38	.152
23	37	.187
24	36	.227
25	35	.270
26	34	.318
27	33	.367
28	32	.420
29	31	.473
30	30	.527
	(5,5)	
15	40	.004
16	39	.008
17	38	.016
	(5,5) (cont.)	
18	37	.028
19	36	.048
20	35	.075
21	34	.111
22	33	.155
23	32	.210
24	31	.274
25	30	.345
26	29	.421
27	28	.500
	(5,6)	
15	45	.002
16	44	.004
17	43	.009
18	42	.015
19	41	.026
20	40	.041
21	39	.063
22	38	.089
23	37	.123
24	36	.165
25	35	.214
26	34	.268
27	33	.331
28	32	.396
29	31	.465
30	30	.535
	(5,7)	
15	50	.001
16	49	.003
17	48	.005
18	47	.009
19	46	.015
20	45	.024
21	44	.037
22	43	.053
23	42	.074
24	41	.101
25	40	.134
26	39	.172
27	38	.216
28	37	.265
29	36	.319
30	35	.378
31	34	.438
32	33	.500
	(5,8)	
15	55	.001
16	54	.002

Table 7-5a. Distribution of the rank sum T' (continued)

T'_α	$T'_{1-\alpha}$	α
(5,8) (cont.)		
17	53	.003
18	52	.005
19	51	.009
20	50	.015
21	49	.023
22	48	.033
23	47	.047
24	46	.064
25	45	.085
26	44	.111
27	43	.142
28	42	.177
29	41	.217
30	40	.262
31	39	.311
32	38	.362
33	37	.416
34	36	.472
35	35	.528
(5,9)		
15	60	.000
16	59	.001
17	58	.002
18	57	.003
19	56	.006
20	55	.009
21	54	.014
22	53	.021
23	52	.030
24	51	.041
25	50	.056
26	49	.073
27	48	.095
28	47	.120
29	46	.149
30	45	.182
31	44	.219
32	43	.259
33	42	.303
34	41	.350
35	40	.399
36	39	.449
37	38	.500
(5,10)		
15	65	.000
16	64	.001
17	63	.001
18	62	.002
19	61	.004
(5,10) (cont.)		
20	60	.006
21	59	.010
22	58	.014
23	57	.020
24	56	.028
25	55	.038
26	54	.050
27	53	.065
28	52	.082
29	51	.103
30	50	.127
31	49	.155
32	48	.185
33	47	.220
34	46	.257
35	45	.297
36	44	.339
37	43	.384
38	42	.430
39	41	.477
40	40	.523
(6,6)		
21	57	.001
22	56	.002
23	55	.004
24	54	.008
25	53	.013
26	52	.021
27	51	.032
28	50	.047
29	49	.066
30	48	.090
31	47	.120
32	46	.155
33	45	.197
34	44	.242
35	43	.294
36	42	.350
37	41	.409
38	40	.469
39	39	.531
(6,7)		
21	63	.001
22	62	.001
23	61	.002
24	60	.004
25	59	.007
26	58	.011
27	57	.017
(6,7) (cont.)		
28	56	.026
29	55	.037
30	54	.051
31	53	.069
32	52	.090
33	51	.117
34	50	.147
35	49	.183
36	48	.223
37	47	.267
38	46	.314
39	45	.365
40	44	.418
41	43	.473
42	42	.527
(6,8)		
21	69	.000
22	68	.001
23	67	.001
24	66	.002
25	65	.004
26	64	.006
27	63	.010
28	62	.015
29	61	.021
30	60	.030
31	59	.041
32	58	.054
33	57	.071
34	56	.091
35	55	.114
36	54	.141
37	53	.172
38	52	.207
39	51	.245
40	50	.286
41	49	.331
42	48	.377
43	47	.426
44	46	.475
45	45	.525
(6,9)		
21	75	.000
22	74	.000
23	73	.001
24	72	.001
25	71	.002
26	70	.004
27	69	.006
(6,9) (cont.)		
28	68	.009
29	67	.013
30	66	.018
31	65	.025
32	64	.033
33	63	.044
34	62	.057
35	61	.072
36	60	.091
37	59	.112
38	58	.136
39	57	.164
40	56	.194
41	55	.228
42	54	.264
43	53	.303
44	52	.344
45	51	.388
46	50	.432
47	49	.477
48	48	.523
(6,10)		
21	81	.000
22	80	.000
23	79	.000
24	78	.001
25	77	.001
26	76	.002
27	75	.004
28	74	.005
29	73	.008
30	72	.011
31	71	.016
32	70	.021
33	69	.028
34	68	.036
35	67	.047
36	66	.059
37	65	.074
38	64	.090
39	63	.110
40	62	.132
41	61	.157
42	60	.184
43	59	.214
44	58	.246
45	57	.281
46	56	.318
47	55	.356

Table 7-5*a*. Distribution of the rank sum T' (continued)

T'_α	$T'_{1-\alpha}$	α
(6,10) (cont.)		
48	54	.396
49	53	.437
50	52	.479
51	51	.521
	(7,7)	
28	77	.000
29	76	.001
30	75	.001
31	74	.002
32	73	.003
33	72	.006
34	71	.009
35	70	.013
36	69	.019
37	68	.027
38	67	.036
39	66	.049
40	65	.064
41	64	.082
42	63	.104
43	62	.130
44	61	.159
45	60	.191
46	59	.228
47	58	.267
48	57	.310
49	56	.355
50	55	.402
51	54	.451
52	53	.500
	(7,8)	
28	84	.000
29	83	.000
30	82	.001
31	81	.001
32	80	.002
33	79	.003
34	78	.005
35	77	.007
36	76	.010
37	75	.014
38	74	.020
39	73	.027
40	72	.036
41	71	.047
42	70	.060
43	69	.076
44	68	.095
45	67	.116

T'_α	$T'_{1-\alpha}$	α
(7,8) (cont.)		
46	66	.140
47	65	.168
48	64	.198
49	63	.232
50	62	.268
51	61	.306
52	60	.347
53	59	.389
54	58	.433
55	57	.478
56	56	.522
	(7,9)	
28	91	.000
29	90	.000
30	89	.000
31	88	.001
32	87	.001
33	86	.002
34	85	.003
35	84	.004
36	83	.006
37	82	.008
38	81	.011
39	80	.016
40	79	.021
41	78	.027
42	77	.036
43	76	.045
44	75	.057
45	74	.071
46	73	.087
47	72	.105
48	71	.126
49	70	.150
50	69	.175
51	68	.204
52	67	.235
53	66	.268
54	65	.303
55	64	.340
56	63	.379
57	62	.419
58	61	.459
59	60	.500
	(7,10)	
28	98	.000
29	97	.000
30	96	.000
31	95	.000

T'_α	$T'_{1-\alpha}$	α
(7,10) (cont.)		
32	94	.001
33	93	.001
34	92	.001
35	91	.002
36	90	.003
37	89	.005
38	88	.007
39	87	.009
40	86	.012
41	85	.017
42	84	.022
43	83	.028
44	82	.035
45	81	.044
46	80	.054
47	79	.067
48	78	.081
49	77	.097
50	76	.115
51	75	.135
52	74	.157
53	73	.182
54	72	.209
55	71	.237
56	70	.268
57	69	.300
58	68	.335
59	67	.370
60	66	.406
61	65	.443
62	64	.481
63	63	.519
	(8,8)	
36	100	.000
37	99	.000
38	98	.000
39	97	.001
40	96	.001
41	95	.001
42	94	.002
43	93	.003
44	92	.005
45	91	.007
46	90	.010
47	89	.014
48	88	.019
49	87	.025
50	86	.032
51	85	.041

T'_α	$T'_{1-\alpha}$	α
(8,8) (cont.)		
52	84	.052
53	83	.065
54	82	.080
55	81	.097
56	80	.117
57	79	.139
58	78	.164
59	77	.191
60	76	.221
61	75	.253
62	74	.287
63	73	.323
64	72	.360
65	71	.399
66	70	.439
67	69	.480
68	68	.520
	(8,9)	
36	108	.000
40	104	.000
41	103	.001
42	102	.001
43	101	.002
44	100	.003
45	99	.004
46	98	.006
47	97	.008
48	96	.010
49	95	.014
50	94	.018
51	93	.023
52	92	.030
53	91	.037
54	90	.046
55	89	.057
56	88	.069
57	87	.084
58	86	.100
59	85	.118
60	84	.138
61	83	.161
62	82	.185
63	81	.212
64	80	.240
65	79	.271
66	78	.303
67	77	.336
68	76	.371
69	75	.407

Table 7-5a. Distribution of the rank sum T' (continued)

T'_α	$T'_{1-\alpha}$	α
(8,9) (cont.)		
70	74	.444
71	73	.481
72	72	.519
(8,10)		
36	116	.000
41	111	.000
42	110	.001
43	109	.001
44	108	.002
45	107	.002
46	106	.003
47	105	.004
48	104	.006
49	103	.008
50	102	.010
51	101	.013
52	100	.017
53	99	.022
54	98	.027
55	97	.034
56	96	.042
57	95	.051
58	94	.061
59	93	.073
60	92	.086
61	91	.102
62	90	.118
63	89	.137
64	88	.158
65	87	.180
66	86	.204
67	85	.230
68	84	.257
69	83	.286
70	82	.317
71	81	.348
72	80	.381
73	79	.414
74	78	.448
75	77	.483
76	76	.517
(9,9)		
45	126	.000
50	121	.000
51	120	.001
52	119	.001
53	118	.001
54	117	.002
55	116	.003
56	115	.004
57	114	.005
58	113	.007
59	112	.009
60	111	.012
61	110	.016
62	109	.020
63	108	.025
64	107	.031
65	106	.039
66	105	.047
67	104	.057
68	103	.068
69	102	.081
70	101	.095
71	100	.111
72	99	.129
73	98	.149
74	97	.170
75	96	.193
76	95	.218
77	94	.245
78	93	.273
79	92	.302
80	91	.333
81	90	.365
82	89	.398
83	88	.432
84	87	.466
85	86	.500
(9,10)		
45	135	.000
52	128	.000
53	127	.001
(9,10) (cont.)		
54	126	.001
55	125	.001
56	124	.002
57	123	.003
58	122	.004
59	121	.005
60	120	.007
61	119	.009
62	118	.011
63	117	.014
64	116	.017
65	115	.022
66	114	.027
67	113	.033
68	112	.039
69	111	.047
70	110	.056
71	109	.067
72	108	.078
73	107	.091
74	106	.106
75	105	.121
76	104	.139
77	103	.158
78	102	.178
79	101	.200
80	100	.223
81	99	.248
82	98	.274
83	97	.302
84	96	.330
85	95	.360
86	94	.390
87	93	.421
88	92	.452
89	91	.484
90	90	.516
(10,10)		
55	155	.000
63	147	.000
64	146	.001
(10,10) (cont.)		
65	145	.001
66	144	.001
67	143	.001
68	142	.002
69	141	.003
70	140	.003
71	139	.004
72	138	.006
73	137	.007
74	136	.009
75	135	.012
76	134	.014
77	133	.018
78	132	.022
79	131	.026
80	130	.032
81	129	.038
82	128	.045
83	127	.053
84	126	.062
85	125	.072
86	124	.083
87	123	.095
88	122	.109
89	121	.124
90	120	.140
91	119	.157
92	118	.176
93	117	.197
94	116	.218
95	115	.241
96	114	.264
97	113	.289
98	112	.315
99	111	.342
100	110	.370
101	109	.398
102	108	.427
103	107	.456
104	106	.485
105	105	.515

For sample sizes greater than 10 the chance that the statistic T' will be less than or equal to an integer k is given approximately by the area under the standard normal curve to the left of

$$z = \frac{k + \frac{1}{2} - N_1(N_1 + N_2 + 1)/2}{\sqrt{N_1 N_2 (N_1 + N_2 + 1)/12}}$$

Table 7-5*b*. Table of critical values of U in the Mann-Whitney test*

Critical values of U for a one-tailed test at $\alpha = .001$ or for a two-tailed test at $\alpha = .002$

n_1 \ n_2	9	10	11	12	13	14	15	16	17	18	19	20
1												
2												
3									0	0	0	0
4		0	0	0	1	1	1	2	2	3	3	3
5	1	1	2	2	3	3	4	5	5	6	7	7
6	2	3	4	4	5	6	7	8	9	10	11	12
7	3	5	6	7	8	9	10	11	13	14	15	16
8	5	6	8	9	11	12	14	15	17	18	20	21
9	7	8	10	12	14	15	17	19	21	23	25	26
10	8	10	12	14	17	19	21	23	25	27	29	32
11	10	12	15	17	20	22	24	27	29	32	34	37
12	12	14	17	20	23	25	28	31	34	37	40	42
13	14	17	20	23	26	29	32	35	38	42	45	48
14	15	19	22	25	29	32	36	39	43	46	50	54
15	17	21	24	28	32	36	40	43	47	51	55	59
16	19	23	27	31	35	39	43	48	52	56	60	65
17	21	25	29	34	38	43	47	52	57	61	66	70
18	23	27	32	37	42	46	51	56	61	66	71	76
19	25	29	34	40	45	50	55	60	66	71	77	82
20	26	32	37	42	48	54	59	65	70	76	82	88

* Adapted and abridged from Tables 1, 3, 5, and 7 of D. Auble, Extended Tables for the Mann-Whitney Statistic, *Bulletin of the Institute of Educational Research at Indiana University*, vol. 1, No. 2, 1953, with the kind permission of the author and the publisher. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 7-5*b*. Table of critical values of U in the Mann-Whitney test (continued)

Critical values of U for a one-tailed test at $\alpha = .01$ or for a two-tailed test at $\alpha = .02$

n_1 \ n_2	9	10	11	12	13	14	15	16	17	18	19	20
1												
2					0	0	0	0	0	0	1	1
3	1	1	1	2	2	2	3	3	4	4	4	5
4	3	3	4	5	5	6	7	7	8	9	9	10
5	5	6	7	8	9	10	11	12	13	14	15	16
6	7	8	9	11	12	13	15	16	18	19	20	22
7	9	11	12	14	16	17	19	21	23	24	26	28
8	11	13	15	17	20	22	24	26	28	30	32	34
9	14	16	18	21	23	26	28	31	33	36	38	40
10	16	19	22	24	27	30	33	36	38	41	44	47
11	18	22	25	28	31	34	37	41	44	47	50	53
12	21	24	28	31	35	38	42	46	49	53	56	60
13	23	27	31	35	39	43	47	51	55	59	63	67
14	26	30	34	38	43	47	51	56	60	65	69	73
15	28	33	37	42	47	51	56	61	66	70	75	80
16	31	36	41	46	51	56	61	66	71	76	82	87
17	33	38	44	49	55	60	66	71	77	82	88	93
18	36	41	47	53	59	65	70	76	82	88	94	100
19	38	44	50	56	63	69	75	82	88	94	101	107
20	40	47	53	60	67	73	80	87	93	100	107	114

Table 7-5*b*. Table of critical values of U in the Mann-Whitney test (continued)

Critical values of U for a one-tailed test at $\alpha = .025$ or for a two-tailed test at $\alpha = .05$

n_2 n_1	9	10	11	12	13	14	15	16	17	18	19	20
1												
2	0	0	0	1	1	1	1	1	2	2	2	2
3	2	3	3	4	4	5	5	6	6	7	7	8
4	4	5	6	7	8	9	10	11	11	12	13	13
5	7	8	9	11	12	13	14	15	17	18	19	20
6	10	11	13	14	16	17	19	21	22	24	25	27
7	12	14	16	18	20	22	24	26	28	30	32	34
8	15	17	19	22	24	26	29	31	34	36	38	41
9	17	20	23	26	28	31	34	37	39	42	45	48
10	20	23	26	29	33	36	39	42	45	48	52	55
11	23	26	30	33	37	40	44	47	51	55	58	62
12	26	29	33	37	41	45	49	53	57	61	65	69
13	28	33	37	41	45	50	54	59	63	67	72	76
14	31	36	40	45	50	55	59	64	67	74	78	83
15	34	39	44	49	54	59	64	70	75	80	85	90
16	37	42	47	53	59	64	70	75	81	86	92	98
17	39	45	51	57	63	67	75	81	87	93	99	105
18	42	48	55	61	67	74	80	86	93	99	106	112
19	45	52	58	65	72	78	85	92	99	106	113	119
20	48	55	62	69	76	83	90	98	105	112	119	127

Table 7-5*b*. Table of critical values of U in the Mann-Whitney test (continued)

Critical values of U for a one-tailed test at $\alpha = .05$ or for a two-tailed test at $\alpha = .10$

n_1 / n_2	9	10	11	12	13	14	15	16	17	18	19	20
1											0	0
2	1	1	1	2	2	2	3	3	3	4	4	4
3	3	4	5	5	6	7	7	8	9	9	10	11
4	6	7	8	9	10	11	12	14	15	16	17	18
5	9	11	12	13	15	16	18	19	20	22	23	25
6	12	14	16	17	19	21	23	25	26	28	30	32
7	15	17	19	21	24	26	28	30	33	35	37	39
8	18	20	23	26	28	31	33	36	39	41	44	47
9	21	24	27	30	33	36	39	42	45	48	51	54
10	24	27	31	34	37	41	44	48	51	55	58	62
11	27	31	34	38	42	46	50	54	57	61	65	69
12	30	34	38	42	47	51	55	60	64	68	72	77
13	33	37	42	47	51	56	61	65	70	75	80	84
14	36	41	46	51	56	61	66	71	77	82	87	92
15	39	44	50	55	61	66	72	77	83	88	94	100
16	42	48	54	60	65	71	77	83	89	95	101	107
17	45	51	57	64	70	77	83	89	96	102	109	115
18	48	55	61	68	75	82	88	95	102	109	116	123
19	51	58	65	72	80	87	94	101	109	116	123	130
20	54	62	69	77	84	92	100	107	115	123	130	138

Table 7-6. **Table of probabilities associated with values as large as observed values of χ_r^2 in the Friedman two-way analysis of variance by ranks***

$k = 3$

$N = 2$		$N = 3$		$N = 4$		$N = 5$	
χ_r^2	p	χ_r^2	p	χ_r^2	p	χ_r^2	p
0	1.000	.000	1.000	.0	1.000	.0	1.000
1	.833	.667	.944	.5	.931	.4	.954
3	.500	2.000	.528	1.5	.653	1.2	.691
4	.167	2.667	.361	2.0	.431	1.6	.522
		4.667	.194	3.5	.273	2.8	.367
		6.000	.028	4.5	.125	3.6	.182
				6.0	.069	4.8	.124
				6.5	.042	5.2	.093
				8.0	.0046	6.4	.039
						7.6	.024
						8.4	.0085
						10.0	.00077

$N = 6$		$N = 7$		$N = 8$		$N = 9$	
χ_r^2	p	χ_r^2	p	χ_r^2	p	χ_r^2	p
.00	1.000	.000	1.000	.00	1.000	.000	1.000
.33	.956	.286	.964	.25	.967	.222	.971
1.00	.740	.857	.768	.75	.794	.667	.814
1.33	.570	1.143	.620	1.00	.654	.889	.865
2.33	.430	2.000	.486	1.75	.531	1.556	.569
3.00	.252	2.571	.305	2.25	.355	2.000	.398
4.00	.184	3.429	.237	3.00	.285	2.667	.328
4.33	.142	3.714	.192	3.25	.236	2.889	.278
5.33	.072	4.571	.112	4.00	.149	3.556	.187
6.33	.052	5.429	.085	4.75	.120	4.222	.154
7.00	.029	6.000	.052	5.25	.079	4.667	.107
8.33	.012	7.143	.027	6.25	.047	5.556	.069
9.00	.0081	7.714	.021	6.75	.038	6.000	.057
9.33	.0055	8.000	.016	7.00	.030	6.222	.048
10.33	.0017	8.857	.0084	7.75	.018	6.889	.031
12.00	.00013	10.286	.0036	9.00	.0099	8.000	.019
		10.571	.0027	9.25	.0080	8.222	.016
		11.143	.0012	9.75	.0048	8.667	.010
		12.286	.00032	10.75	.0024	9.556	.0060
		14.000	.000021	12.00	.0011	10.667	.0035
				12.25	.00086	10.889	.0029
				13.00	.00026	11.556	.0013
				14.25	.000061	12.667	.00066
				16.00	.0000036	13.556	.00035
						14.000	.00020
						14.222	.000097
						14.889	.000054
						16.222	.000011
						18.000	.0000006

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* Adapted from M. Friedman, The Use of Ranks to Avoid the Assumption of Normality Implicit in the Analysis of Variance, *J. Amer. Statist. Ass.*, vol. 32, pp. 688–689, 1937, with the kind permission of the author and the publisher. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 7-6. Table of probabilities associated with values as large as observed values of χ_r^2 in the Friedman two-way analysis of variance by ranks (continued)

$k = 4$

$N = 2$		$N = 3$		$N = 4$			
χ_r^2	p	χ_r^2	p	χ_r^2	p	χ_r^2	p
.0	1.000	.2	1.000	.0	1.000	5.7	.141
.6	.958	.6	.958	.3	.992	6.0	.105
1.2	.834	1.0	.910	.6	.928	6.3	.094
1.8	.792	1.8	.727	.9	.900	6.6	.077
2.4	.625	2.2	.608	1.2	.800	6.9	.068
3.0	.542	2.6	.524	1.5	.754	7.2	.054
3.6	.458	3.4	.446	1.8	.677	7.5	.052
4.2	.375	3.8	.342	2.1	.649	7.8	.036
4.8	.208	4.2	.300	2.4	.524	8.1	.033
5.4	.167	5.0	.207	2.7	.508	8.4	.019
6.0	.042	5.4	.175	3.0	.432	8.7	.014
		5.8	.148	3.3	.389	9.3	.012
		6.6	.075	3.6	.355	9.6	.0069
		7.0	.054	3.9	.324	9.9	.0062
		7.4	.033	4.5	.242	10.2	.0027
		8.2	.017	4.8	.200	10.8	.0016
		9.0	.0017	5.1	.190	11.1	.00094
				5.4	.158	12.0	.000072

Table 7-7. Table of probabilities associated with values as large as observed values of H in the Kruskal-Wallis one-way analysis of variance by ranks*

Sample sizes			H	p
n_1	n_2	n_3		
2	1	1	2.7000	.500
2	2	1	3.6000	.200
2	2	2	4.5714	.067
			3.7143	.200
3	1	1	3.2000	.300
3	2	1	4.2857	.100
			3.8571	.133
3	2	2	5.3572	.029
			4.7143	.048
			4.5000	.067
			4.4643	.105
3	3	1	5.1429	.043
			4.5714	.100
			4.0000	.129
3	3	2	6.2500	.011
			5.3611	.032
			5.1389	.061
			4.5556	.100
			4.2500	.121
3	3	3	7.2000	.004
			6.4889	.011
			5.6889	.029
			5.6000	.050
			5.0667	.086
			4.6222	.100
4	1	1	3.5714	.200
4	2	1	4.8214	.057
			4.5000	.076
			4.0179	.114
4	2	2	6.0000	.014
			5.3333	.033
			5.1250	.052
			4.4583	.100
			4.1667	.105
4	3	1	5.8333	.021
			5.2083	.050
			5.0000	.057
			4.0556	.093
			3.8889	.129
4	3	2	6.4444	.008
			6.3000	.011
			5.4444	.046
			5.4000	.051
			4.5111	.098
			4.4444	.102
4	3	3	6.7455	.010
			6.7091	.013
			5.7909	.046
			5.7273	.050
			4.7091	.092
			4.7000	.101
4	4	1	6.6667	.010
			6.1667	.022
			4.9667	.048
			4.8667	.054
			4.1667	.082
			4.0667	.102
4	4	2	7.0364	.006
			6.8727	.011
			5.4545	.046
			5.2364	.052
			4.5545	.098
			4.4455	.103
4	4	3	7.1439	.010
			7.1364	.011
			5.5985	.049
			5.5758	.051
			4.5455	.099
			4.4773	.102
4	4	4	7.6538	.008
			7.5385	.011
			5.6923	.049
			5.6538	.054
			4.6539	.097
			4.5001	.104
5	1	1	3.8571	.143
5	2	1	5.2500	.036
			5.0000	.048
			4.4500	.071
			4.2000	.095
			4.0500	.119

Table 7-7. **Table of probabilities associated with values as large as observed values of *H* in the Kruskal-Wallis one-way analysis of variance by ranks* (continued)**

Sample sizes			H	p
n_1	n_2	n_3		
5	2	2	6.5333	.008
			6.1333	.013
			5.1600	.034
			5.0400	.056
			4.3733	.090
			4.2933	.122
5	3	1	6.4000	.012
			4.9600	.048
			4.8711	.052
			4.0178	.095
			3.8400	.123
5	3	2	6.9091	.009
			6.8218	.010
			5.2509	.049
			5.1055	.052
			4.6509	.091
			4.4945	.101
5	3	3	7.0788	.009
			6.9818	.011
			5.6485	.049
			5.5152	.051
			4.5333	.097
			4.4121	.109
5	4	1	6.9545	.008
			6.8400	.011
			4.9855	.044
			4.8600	.056
			3.9873	.098
			3.9600	.102
5	4	2	7.2045	.009
			7.1182	.010
			5.2727	.049
			5.2682	.050
			4.5409	.098
			4.5182	.101
5	4	3	7.4449	.010
			7.3949	.011
			5.6564	.049
			5.6308	.050
			4.5487	.099
			4.5231	.103
5	4	4	7.7604	.009
			7.7440	.011
			5.6571	.049
			5.6176	.050
			4.6187	.100
			4.5527	.102
5	5	1	7.3091	.009
			6.8364	.011
			5.1273	.046
			4.9091	.053
			4.1091	.086
			4.0364	.105
5	5	2	7.3385	.010
			7.2692	.010
			5.3385	.047
			5.2462	.051
			4.6231	.097
			4.5077	.100
5	5	3	7.5780	.010
			7.5429	.010
			5.7055	.046
			5.6264	.051
			4.5451	.100
			4.5363	.102
5	5	4	7.8229	.010
			7.7914	.010
			5.6657	.049
			5.6429	.050
			4.5229	.099
			4.5200	.101
5	5	5	8.0000	.009
			7.9800	.010
			5.7800	.049
			5.6600	.051
			4.5600	.100
			4.5000	.102

Adapted and abridged from W. H. Kruskal and W. A. Wallis. Use of Ranks in One-criterion Variance Analysis, *J. Amer. Statist. Ass.*, vol. 47, pp. 614–617, 1952, with the kind permission of the authors and the publisher. (The corrections to this table given by the authors in Errata, *J. Amer. Statist. Ass.*, vol. 48, p. 910, have been incorporated.) Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 7-8. Nonparametric confidence intervals for the median*

N	Largest k	$\alpha \leq .05$	Largest k	$\alpha \leq .01$	N	Largest k	$\alpha \leq .05$	Largest k	$\alpha \leq .01$
6	1	.031			36	12	.029	10	.004
7	1	.016			37	13	.047	11	.008
8	1	.008	1	.008	38	13	.034	11	.005
9	2	.039	1	.004	39	13	.024	12	.009
10	2	.021	1	.002	40	14	.038	12	.006
11	2	.012	1	.001	41	14	.028	12	.004
12	3	.039	2	.006	42	15	.044	13	.008
13	3	.022	2	.003	43	15	.032	13	.005
14	3	.013	2	.002	44	16	.049	14	.010
15	4	.035	3	.007	45	16	.036	14	.007
16	4	.021	3	.004	46	16	.026	14	.005
17	5	.049	3	.002	47	17	.040	15	.008
18	5	.031	4	.008	48	17	.029	15	.006
19	5	.019	4	.004	49	18	.044	16	.009
20	6	.041	4	.003	50	18	.033	16	.007
21	6	.027	5	.007	51	19	.049	16	.005
22	6	.017	5	.004	52	19	.036	17	.008
23	7	.035	5	.003	53	19	.027	17	.005
24	7	.023	6	.007	54	20	.040	18	.009
25	8	.043	6	.004	55	20	.030	18	.006
26	8	.029	7	.009	56	21	.044	18	.005
27	8	.019	7	.006	57	21	.033	19	.008
28	9	.036	7	.004	58	22	.048	19	.005
29	9	.024	8	.008	59	22	.036	20	.009
30	10	.043	8	.005	60	22	.027	20	.006
31	10	.029	8	.003	61	23	.040	21	.010
32	10	.020	9	.007	62	23	.030	21	.007
33	11	.035	9	.005	63	24	.043	21	.005
34	11	.024	10	.009	64	24	.033	22	.008
35	12	.041	10	.006	65	25	.046	22	.006

If the observations are arranged in order of size $X_1 < X_2 < X_3 < \cdots < X_N$, then we are $100(1 - \alpha)\%$ confident of the population median being between X_k and X_{N-k+1}, where k and α are given above.

* Reproduced by permission of S. K. Banerjee and K. R. Nair, Table of Confidence Interval for the Median in Samples from Any Continuous Population, *Sankhya*, vol. 4, pp. 551–558, 1940. Reprinted from W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

TABLE 7-9. TABLES FOR THE SIGN TEST*

Table 7-9*a*. Critical values of *r* for the Sign test*

(Two-tail percentage points for the binomial for $p = .5$)

N	1%	5%	10%	25%	*N*	1%	5%	10%	25%
1					46	13	15	16	18
2					47	14	16	17	19
3				0	48	14	16	17	19
4				0	49	15	17	18	19
5			0	0	50	15	17	18	20
6		0	0	1	51	15	18	19	20
7		0	0	1	52	16	18	19	21
8	0	0	1	1	53	16	18	20	21
9	0	1	1	2	54	17	19	20	22
10	0	1	1	2	55	17	19	20	22
11	0	1	2	3	56	17	20	21	23
12	1	2	2	3	57	18	20	21	23
13	1	2	3	3	58	18	21	22	24
14	1	2	3	4	59	19	21	22	24
15	2	3	3	4	60	19	21	23	25
16	2	3	4	5	61	20	22	23	25
17	2	4	4	5	62	20	22	24	25
18	3	4	5	6	63	20	23	24	26
19	3	4	5	6	64	21	23	24	26
20	3	5	5	6	65	21	24	25	27
21	4	5	6	7	66	22	24	25	27
22	4	5	6	7	67	22	25	26	28
23	4	6	7	8	68	22	25	26	28
24	5	6	7	8	69	23	25	27	29
25	5	7	7	9	70	23	26	27	29
26	6	7	8	9	71	24	26	28	30
27	6	7	8	10	72	24	27	28	30
28	6	8	9	10	73	25	27	28	31
29	7	8	9	10	74	25	28	29	31
30	7	9	10	11	75	25	28	29	32
31	7	9	10	11	76	26	28	30	32
32	8	9	10	12	77	26	29	30	32
33	8	10	11	12	78	27	29	31	33
34	9	10	11	13	79	27	30	31	33
35	9	11	12	13	80	28	30	32	34
36	9	11	12	14	81	28	31	32	34
37	10	12	13	14	82	28	31	33	35
38	10	12	13	14	83	29	32	33	35
39	11	12	13	15	84	29	32	33	36
40	11	13	14	15	85	30	32	34	36
41	11	13	14	16	86	30	33	34	37
42	12	14	15	16	87	31	33	35	37
43	12	14	15	17	88	31	34	35	38
44	13	15	16	17	89	31	34	36	38
45	13	15	16	18	90	32	35	36	39

For values of N larger than 90, approximate values of r may be found by taking the nearest integer less than $(N - 1)/2 - k\sqrt{N + 1}$, where k is 1.2879, 0.9800, 0.8224, 0.5752 for the 1, 5, 10, 25% values, respectively.

* Reprinted from W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 7-9*b*. Distribution for the sign test

(Percentage points for the binomial for $p = .5$)

The percentiles listed cover the range from $\alpha = .005$ to $\alpha = .125$ for every value of N up to 100. Percentiles from $\alpha = .875$ to $\alpha = .995$ may be read by entering the table with $N - x$ and $1 - \alpha$. x is the number of plus signs.

x	α
$N = 3$	
0	.125
$N = 4$	
0	.062
1	.312
$N = 5$	
0	.031
1	.188
$N = 6$	
0	.016
1	.109
2	.344
$N = 7$	
0	.008
1	.062
2	.227
$N = 8$	
0	.004
1	.035
2	.145
$N = 9$	
0	.002
1	.020
2	.090
3	.254
$N = 10$	
0	.001
1	.011
2	.055
3	.172
$N = 11$	
0	.000
1	.006
2	.033
3	.113
4	.274

x	α
$N = 12$	
1	.003
2	.019
3	.073
4	.194
$N = 13$	
1	.002
2	.011
3	.046
4	.133
$N = 14$	
1	.001
2	.006
3	.029
4	.090
5	.212
$N = 15$	
1	.000
2	.004
3	.018
4	.059
5	.151
$N = 16$	
2	.002
3	.011
4	.038
5	.105
6	.227
$N = 17$	
2	.001
3	.006
4	.025
5	.072
6	.166
$N = 18$	
3	.004
4	.015
5	.048
6	.119
7	.240

x	α
$N = 19$	
3	.002
4	.010
5	.032
6	.084
7	.180
$N = 20$	
3	.001
4	.006
5	.021
6	.058
7	.132
$N = 21$	
4	.004
5	.013
6	.039
7	.095
8	.192
$N = 22$	
4	.002
5	.008
6	.026
7	.067
8	.143
$N = 23$	
4	.001
5	.005
6	.017
7	.047
8	.105
9	.202
$N = 24$	
5	.003
6	.011
7	.032
8	.076
9	.154

x	α
$N = 25$	
5	.002
6	.007
7	.022
8	.054
9	.115
10	.212
$N = 26$	
6	.005
7	.014
8	.038
9	.084
10	.163
$N = 27$	
6	.003
7	.010
8	.026
9	.061
10	.124
11	.221
$N = 28$	
6	.002
7	.006
8	.018
9	.044
10	.092
11	.172
$N = 29$	
7	.004
8	.012
9	.031
10	.068
11	.132
$N = 30$	
7	.003
8	.008
9	.021
10	.049
11	.100
12	.181

x	α
$N = 31$	
7	.002
8	.005
9	.015
10	.035
11	.075
12	.141
$N = 32$	
8	.004
9	.010
10	.025
11	.055
12	.108
13	.189
$N = 33$	
8	.002
9	.007
10	.018
11	.040
12	.081
13	.148
$N = 34$	
9	.005
10	.012
11	.029
12	.061
13	.115
14	.196
$N = 35$	
9	.003
10	.008
11	.020
12	.045
13	.088
14	.155
$N = 36$	
9	.002
10	.006
11	.014
12	.033
13	.066
14	.121
15	.203

x	α
$N = 37$	
10	.004
11	.010
12	.024
13	.049
14	.094
15	.162
$N = 38$	
10	.003
11	.007
12	.017
13	.036
14	.072
15	.128
$N = 39$	
11	.005
12	.012
13	.027
14	.054
15	.100
16	.168
$N = 40$	
11	.003
12	.008
13	.019
14	.040
15	.077
16	.134
$N = 41$	
11	.002
12	.006
13	.014
14	.030
15	.059
16	.106
17	.174
$N = 42$	
12	.004
13	.010
14	.022
15	.044
16	.082
17	.140

Table 7-9*b*. Distribution for the sign test (continued)

x	α
$N = 43$	
12	.003
13	.007
14	.016
15	.033
16	.063
17	.111
18	.180
$N = 44$	
13	.005
14	.011
15	.024
16	.048
17	.087
18	.146
$N = 45$	
13	.003
14	.008
15	.018
16	.036
17	.068
18	.116
19	.186
$N = 46$	
13	.002
14	.006
15	.013
16	.027
17	.052
18	.092
19	.151
$N = 47$	
14	.004
15	.009
16	.020
17	.039
18	.072
19	.121
20	.191
$N = 48$	
14	.003
15	.007
16	.015
17	.030
18	.056
19	.097
20	.156

x	α
$N = 49$	
15	.005
16	.011
17	.022
18	.043
19	.076
20	.126
$N = 50$	
15	.003
16	.008
17	.016
18	.032
19	.059
20	.101
21	.161
$N = 51$	
15	.002
16	.005
17	.012
18	.024
19	.046
20	.080
21	.131
$N = 52$	
16	.004
17	.009
18	.018
19	.035
20	.063
21	.106
22	.166
$N = 53$	
16	.003
17	.006
18	.014
19	.027
20	.049
21	.084
22	.136
$N = 54$	
17	.005
18	.010
19	.020
20	.038
21	.067
22	.110
23	.170

x	α
$N = 55$	
17	.003
18	.007
19	.015
20	.029
21	.052
22	.089
23	.140
$N = 56$	
17	.002
18	.005
19	.011
20	.022
21	.041
22	.070
23	.114
24	.175
$N = 57$	
18	.004
19	.008
20	.017
21	.031
22	.056
23	.092
24	.145
$N = 58$	
18	.003
19	.006
20	.012
21	.024
22	.043
23	.074
24	.119
25	.179
$N = 59$	
19	.004
20	.009
21	.018
22	.034
23	.059
24	.096
25	.149

x	α
$N = 60$	
19	.003
20	.007
21	.014
22	.026
23	.046
24	.078
25	.123
26	.183
$N = 61$	
20	.005
21	.010
22	.020
23	.036
24	.062
25	.100
26	.153
$N = 62$	
20	.004
21	.008
22	.015
23	.028
24	.049
25	.081
26	.126
$N = 63$	
20	.003
21	.006
22	.011
23	.021
24	.038
25	.065
26	.104
27	.157
$N = 64$	
21	.004
22	.008
23	.016
24	.030
25	.052
26	.084
27	.130

x	α
$N = 65$	
21	.003
22	.006
23	.012
24	.023
25	.041
26	.068
27	.107
28	.161
$N = 66$	
22	.005
23	.009
24	.018
25	.032
26	.054
27	.088
28	.134
$N = 67$	
22	.003
23	.007
24	.014
25	.025
26	.043
27	.071
28	.111
29	.164
$N = 68$	
22	.002
23	.005
24	.010
25	.019
26	.034
27	.057
28	.091
29	.137
$N = 69$	
23	.004
24	.008
25	.015
26	.027
27	.046
28	.074
29	.114
30	.168

x	α
$N = 70$	
23	.003
24	.006
25	.011
26	.021
27	.036
28	.060
29	.094
30	.141
$N = 71$	
24	.004
25	.008
26	.016
27	.028
28	.048
29	.077
30	.118
31	.171
$N = 72$	
24	.003
25	.006
26	.012
27	.022
28	.038
29	.062
30	.097
31	.144
$N = 73$	
25	.005
26	.009
27	.017
28	.030
29	.050
30	.080
31	.121
32	.175
$N = 74$	
25	.004
26	.007
27	.013
28	.024
29	.040
30	.065
31	.100
32	.148

Table 7-9*b*. Distribution for the sign test (continued)

x	α
$N = 75$	
25	.003
26	.005
27	.010
28	.018
29	.032
30	.053
31	.083
32	.124
33	.179
$N = 76$	
26	.004
27	.008
28	.014
29	.025
30	.042
31	.068
32	.103
33	.151
$N = 77$	
26	.003
27	.006
28	.011
29	.020
30	.034
31	.055
32	.086
33	.127
$N = 78$	
27	.004
28	.008
29	.015
30	.027
31	.044
32	.070
33	.106
34	.154
$N = 79$	
27	.003
28	.006
29	.012
30	.021
31	.036
32	.057
33	.088
34	.130
$N = 80$	
28	.005
29	.009
30	.016
31	.028
32	.046
33	.073
34	.109
35	.157
$N = 81$	
28	.004
29	.007
30	.013
31	.022
32	.037
33	.060
34	.091
35	.133
$N = 82$	
28	.003
29	.005
30	.010
31	.018
32	.030
33	.049
34	.075
35	.112
36	.160
$N = 83$	
29	.004
30	.008
31	.014
32	.024
33	.039
34	.062
35	.094
36	.136
$N = 84$	
29	.003
30	.006
31	.011
32	.019
33	.031
34	.051
35	.078
36	.115
37	.163
$N = 85$	
30	.004
31	.008
32	.015
33	.025
34	.041
35	.064
36	.096
37	.139
$N = 86$	
30	.003
31	.006
32	.011
33	.020
34	.033
35	.053
36	.080
37	.118
38	.166
$N = 87$	
31	.005
32	.009
33	.016
34	.027
35	.043
36	.066
37	.099
38	.142
$N = 88$	
31	.004
32	.007
33	.012
34	.012
35	.035
36	.055
37	.083
38	.120
39	.169
$N = 89$	
31	.003
32	.005
33	.010
34	.017
35	.028
36	.045
37	.069
38	.102
39	.145
$N = 90$	
32	.004
33	.007
34	.013
35	.022
36	.036
37	.057
38	.085
39	.123
40	.171
$N = 91$	
32	.003
33	.006
34	.010
35	.018
36	.029
37	.046
38	.071
39	.104
40	.147
$N = 92$	
33	.004
34	.008
35	.014
36	.024
37	.038
38	.059
39	.087
40	.126
$N = 93$	
33	.003
34	.006
35	.011
36	.019
37	.031
38	.048
39	.073
40	.107
41	.150
$N = 94$	
34	.005
35	.009
36	.015
37	.025
38	.039
39	.061
40	.090
41	.128
$N = 95$	
34	.004
35	.007
36	.012
37	.020
38	.032
39	.050
40	.075
41	.109
42	.152
$N = 96$	
34	.003
35	.005
36	.009
37	.016
38	.026
39	.041
40	.063
41	.092
42	.131
$N = 97$	
35	.004
36	.007
37	.012
38	.021
39	.034
40	.052
41	.077
42	.111
43	.155
$N = 98$	
35	.003
36	.006
37	.010
38	.017
39	.027
40	.043
41	.065
42	.094
43	.133
$N = 99$	
36	.004
37	.008
38	.013
39	.022
40	.035
41	.054
42	.080
43	.114
44	.157
$N = 100$	
36	.003
37	.006
38	.010
39	.018
40	.028
41	.044
42	.067
43	.097
44	.136

section

8

CORRELATION AND ASSOCIATION

The tables in this section concern statistics that either quantify the "degree of relationship" between two or more random variables and/or permit testing a hypothesis about the relationship between two or more random variables. The applications of Tables 8-1 through 8-4 require the assumption that the variables of interest are distributed according to the bivariate or multivariate normal probability density. Tables 8-6 through 8-10 involve the sampling distributions of certain nonparametric measures of association. When using these statistics to test the null hypothesis that variables are independent, one should keep in mind that the only reasonable alternative hypotheses are that the regression of one variable on the other is nondecreasing or nonincreasing (or either in the case of a two-tailed test), and vice versa. The tests are not sensitive to other kinds of alternatives.

The general sampling distribution of the correlation coefficient based on normally distributed material has proven quite intractable except for the case of $\rho = 0$, where ρ denotes the theoretical value of the correlation coefficient. Fisher, however, demonstrated that the sampling distribution of

$$z_r = \tfrac{1}{2}[\log_e (1 + r) - \log_e (1 - r)]$$

is approximately normal with mean $\mu = z\rho$ and variance $\sigma^2 = 1/(N - 3)$ (ref. 1). Consequently, the z_r transformation can be used to test hypotheses about ρ, the difference between correlation coefficients characterizing two populations of measurements provided the corresponding sample r's are based on independent samples, and to construct confidence intervals for ρ.

Table 8-1 contains values of z_r as a function of r. For negative values of r, attach a negative sign to the tabulated value of z_r which corresponds to the absolute value of the observed correlation. When $\rho = 0$ in the bivariate normal distribution, the sampling distribution of r is relatively manageable. Similarly, the sampling distribution of the multiple correlation coefficient based on normally distributed variates is easily handled if the theoretical value of the multiple correlation coefficient is zero.

Table 8-2 consists of critical values corresponding to various values of $N - 2$ (df) for testing the hypothesis that a bivariate correlation has been determined in a sample drawn from a population in which the true value of the correlation is zero versus either a one-tailed alternative $\rho > 0$ or a two-tailed alternative $|\rho| > 0$. In the two-tailed test the null hypothesis is rejected if the absolute value of the observed r exceeds the tabled value for the appropriate degrees of freedom, $N - 2$, and selected significance level. To test against the one-tailed alternative $\rho < 0$, negative signs are attached to the critical values in Table 8-2.

Table 8-3 shows critical values of the sampling distribution of a multiple correlation when the population value is zero. The significance levels correspond to a one-tailed test, since a multiple r is, by definition, greater than or equal to zero. An exception to this rule is the column for two variables, which contains two-tailed critical values for an ordinary bivariate correlation coefficient. The parameter or degrees of freedom in the sampling distribution of a multiple correlation is $N - m$, where m is the number of variables and N the sample size.

The *tetrachoric correlation coefficient* is frequently used with dichotomous data. Suppose the data on two dichotomous variables x and y are arranged in the following way:

$y\backslash x$	$+$	$-$
$+$	a	b
$-$	c	d

where a, b, c and d refer to proportions. Suppose we can assume that a bivariate normal probability distribution underlies the observable joint distribution of x and y. In other words, suppose that the frequencies represent counts in quadrants of the scatter plot of a sample drawn from a bivariate normal distribution. The tetrachoric correlation estimates, from the frequencies a, b, c and d, the value of ρ in the underlying bivariate normal distribution. Tetrachoric r's are difficult to compute, but for many purposes an approximation called the *cosine-pi approximation* to the tetrachoric is quite satisfactory. The estimate is defined as the quantity

$$r_{\text{cos-pi}} = \cos\left(\pi \frac{\sqrt{bc}}{\sqrt{ad} + \sqrt{bc}}\right)$$

Table 8-4 contains values of $r_{\text{cos-pi}}$ expressed as functions of the ratio ad/bc. If ad/bc is smaller than 1, the reciprocal bc/ad is determined, and a negative sign is attached to the value of $r_{\text{cos-pi}}$ corresponding to this value. The cosine-pi approximation to a tetrachoric should not be used if the proportion of $+$'s differs greatly for the two variables, if it is known that the tetrachoric correlation is very large, or if the marginal proportions are extremely different from ½.

The Pearson product-moment correlation computed for a pair of dichotomous variables is frequently called the *phi coefficient*. Unless the marginal proportions for the two dichotomies are exactly the same, the maximum value of the phi coefficient is less than 1. Table 8-5 presents a graph for determining the maximum value of a phi coefficient for a given marginal split. Suppose the data are arranged in the following way:

$y\backslash x$	0	1	
0	a	b	q_j
1	c	d	p_j
	q_i	p_i	

where a, b, c, and d denote relative frequencies or proportions, $p_i = b + d$, etc., and $p_i \geq p_j$. The maximum value of phi is given by the quantity $\sqrt{(p_j/q_j)(q_i/p_i)}$ when $p_i \geq p_j$. Table 8-5 can be used to evaluate the maximum value of phi when both p_i and p_j are greater than $\frac{1}{2}$. If both are less than $\frac{1}{2}$, Table 8-5 can be used if one reverses the direction of both x and y, so that q_i and q_j replace p_i and p_j. If $p_i > \frac{1}{2}$ and $p_j < \frac{1}{2}$, Table 8-5 cannot be used. For such cases, the values of $\sqrt{p/q}$ and $\sqrt{q/p}$ in Table 5-5 can be used to rapidly calculate the maximum value of phi. The minimum (negative) value of the phi coefficient is not -1 unless $q_i = p_j$. Neither is the minimum value of phi equal to minus the maximum value. One can determine the minimum value of phi by reversing the direction of either x or y, but not both, so that p_i is replaced by q_i, or p_j by q_j; one finds the maximum value of phi for this situation and attaches a negative sign to the result.

Tables 8-6 through 8-10 all contain critical values for *nonparametric* measures of correlation. One obtains *Spearman rank-difference correlation* by first assigning ranks to N observations on one variable and then independently assigning ranks to the N paired observations on the other variable and computing the statistic

$$r_s = 1 - \frac{6\Sigma d_i^2}{N(N^2 - 1)}$$

where d_i denotes the difference between the ranks assigned the ith pair of observations.

Table 8-6 contains *selected percentage points of the distribution of* Σd_i^2 *under the hypothesis that the two sets of rankings are statistically independent.* The probabilities are listed in terms of $\alpha = 1 -$ cumulative probability. For two-tailed tests, the rejection region consists of values of r_s greater than or equal in absolute value to the value of r_s obtained when the value of Σd_i^2 in Table 8-6 corresponding to $\frac{1}{2}\alpha$ is substituted in the equation for r_s. One-tailed tests against a negative alternative are carried out analogously.

Table 8-7 contains *critical values of the distribution of* r_s *under the hypothesis that the two sets of rankings are statistically independent.* The table values are for a one-tailed test against a positive alternative. One comes out two-tailed tests at the 2α level of significance by rejecting the null hypothesis if the absolute value of the observed r_s exceeds the table value for the α significance level. For larger values of N, the quantity $r_s\sqrt{(N-2)/(1-r_s^2)}$ is approximately distributed like t with $N - 2$ degrees of freedom.

Kendall's rank-correlation coefficient τ is used in the same kind of situation as the Spearman rank correlation. N paired observations are separately ranked on each variable, the largest value being assigned the rank of 1, etc. The ranks assigned to one variable are then arranged in a sequence according to the rank order on the other variable. This sequence of ranks is then examined. For each position in the sequence, we determine the number of ranks to the right which are larger than the rank in the position in question and subtract from this the number of ranks to the right which are smaller. These quantities are then summed over all positions and the result S is divided by $N(N - 1)/2$ to obtain τ. Table 8-8 lists *values* $\alpha = 1 -$ *cumulative probability corresponding to values of* S *for* N *between* 4 *and* 10, *given that the two sets of rankings are statistically independent.* Since the distribution of S is symmetrical under the null hypothesis, Table 8-7 can be used to carry out two-tailed tests or one-tailed tests against a negative alternative. For larger values of N, τ is approximately normally distributed with mean zero and variance $2(2N + 5)/9N(N - 1)$ when the hypothesis of independence is true. Consequently, τ may be converted to a standardized normal deviate using these values.

Kendall's coefficient of concordance can be used to evaluate the agreement between k sets of experimentally independent rankings of the same N objects or individuals. Let the data be arranged in an N by k table, where the N rows correspond to N objects or individuals, the k columns correspond to k judges or raters, and the entries in each column consist of each rater's rankings of the N objects. Let R_j denote the sum of the ranks in the jth row; then Kendall's coefficient of concordance is defined as

$$W = \frac{12\Sigma(R_j - \Sigma R_j/N)^2}{k^2(N^3 - N)}$$

The quantity $\Sigma(R_j - \Sigma R_j/N)^2$ is denoted by s. Since W is entirely determined by s once N and k are fixed, s can be used to evaluate the significance of W. Table 8-9 contains *critical values of s for the 0.05 and 0.01 levels of significance under the assumption that the* k *sets of rankings are statistically independent.* The critical values correspond to a one-tailed critical region as the only reasonable alternative hypothesis is that there is agreement among the rankings, i.e., $W > 0$. Critical values are listed for N ranging from 3 to 7 and several values of k. For larger values of N, the quantity $k(N - 1)W$ is approximately distributed as a chi-square variate with $N - 1$ degrees of freedom.

Suppose joint observations on two continuous random variables, say x and y, are plotted on a scatter diagram; the scatter diagram is divided into quadrants by determining the two medians, and the first and third quadrants are labeled with positive signs, while the second and fourth are labeled with negative signs. With this setup a test, called the *corner test*, for the hypothesis that x and y are independent can be performed. Let the abscissa be x and the ordinate y. Starting at the extreme right (largest x) and moving toward the center, we count the number of points (observations) until a point is found across the y median. This total is given the sign of the quadrant in which the points occur. If the value of y associated with the largest observed x is greater than the y median, the points counted will be in quadrant one; otherwise, they will be in quadrant four. Analogous counts are determined by starting from the top (largest y), left (smallest x), and bottom (smallest y); the resulting four numbers are summed to obtain a statistic S. Note that S may be either positive or negative, and that some observations may be counted twice in S. Ties may occur in the sense that the value of x associated with the point that "crosses the y median" may also occur in another observation, where the associated y is not across the y median, etc. Such a tie is given the value of $\frac{1}{2}$ in the corresponding count. If the sample size is odd, the two median points are replaced by a single point whose coordinates are the nonmedian values of the two, and the sample size is reduced by one. Table 8-10 lists fractiles of S for even-numbered samples under the hypothesis that x and y are independent. Since the distribution of S is symmetrical, about zero when the null hypothesis is true, Table 8-10 can be used to conduct either a two-tailed or one-tailed test.

Table 8-1. Fisher's z-transformation of r to z_r*

r	z_r	r	z_r	r	z_r	r	z_r	r	z_r
.000	.000	.200	.203	.400	.424	.600	.693	.800	1.099
.005	.005	.205	.208	.405	.430	.605	.701	.805	1.113
.010	.010	.210	.213	.410	.436	.610	.709	.810	1.127
.015	.015	.215	.218	.415	.442	.615	.717	.815	1.142
.020	.020	.220	.224	.420	.448	.620	.725	.820	1.157
.025	.025	.225	.229	.425	.454	.625	.733	.825	1.172
.030	.030	.230	.234	.430	.460	.630	.741	.830	1.188
.035	.035	.235	.239	.435	.466	.635	.750	.835	1.204
.040	.040	.240	.245	.440	.472	.640	.758	.840	1.221
.045	.045	.245	.250	.445	.478	.645	.767	.845	1.238
.050	.050	.250	.255	.450	.485	.650	.775	.850	1.256
.055	.055	.255	.261	.455	.491	.655	.784	.855	1.274
.060	.060	.260	.266	.460	.497	.660	.793	.860	1.293
.065	.065	.265	.271	.465	.504	.665	.802	.865	1.313
.070	.070	.270	.277	.470	.510	.670	.811	.870	1.333
.075	.075	.275	.282	.475	.517	.675	.820	.875	1.354
.080	.080	.280	.288	.480	.523	.680	.829	.880	1.376
.085	.085	.285	.293	.485	.530	.685	.838	.885	1.398
.090	.090	.290	.299	.490	.536	.690	.848	.890	1.422
.095	.095	.295	.304	.495	.543	.695	.858	.895	1.447
.100	.100	.300	.310	.500	.549	.700	.867	.900	1.472
.105	.105	.305	.315	.505	.556	.705	.877	.905	1.499
.110	.110	.310	.321	.510	.563	.710	.887	.910	1.528
.115	.116	.315	.326	.515	.570	.715	.897	.915	1.557
.120	.121	.320	.332	.520	.576	.720	.908	.920	1.589
.125	.126	.325	.337	.525	.583	.725	.918	.925	1.623
.130	.131	.330	.343	.530	.590	.730	.929	.930	1.658
.135	.136	.335	.348	.535	.597	.735	.940	.935	1.697
.140	.141	.340	.354	.540	.604	.740	.950	.940	1.738
.145	.146	.345	.360	.545	.611	.745	.962	.945	1.783
.150	.151	.350	.365	.550	.618	.750	.973	.950	1.832
.155	.156	.355	.371	.555	.626	.755	.984	.955	1.886
.160	.161	.360	.377	.560	.633	.760	.996	.960	1.946
.165	.167	.365	.383	.565	.640	.765	1.008	.965	2.014
.170	.172	.370	.388	.570	.648	.770	1.020	.970	2.092
.175	.177	.375	.394	.575	.655	.775	1.033	.975	2.185
.180	.182	.380	.400	.580	.662	.780	1.045	.980	2.298
.185	.187	.385	.406	.585	.670	.785	1.058	.985	2.443
.190	.192	.390	.412	.590	.678	.790	1.071	.990	2.647
.195	.198	.395	.418	.595	.685	.795	1.085	.995	2.994

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* Reprinted, by permission, from Allen L. Edwards, "Statistical Methods for the Behavioral Sciences," Rinehart & Company, Inc., New York. Reprinted from G. Ferguson, "Statistical Analysis in Psychology and Education," McGraw-Hill Book Company, New York, 1959.

Table 8-2. Critical values for correlation and *t* ratios significant at the .05 level (lightface type) and at the .01 level (boldface type) for varying degrees of freedom*

Degrees of freedom	*Number of variables*									*t*
	2	*3*	*4*	*5*	*6*	*7*	*9*	*13*	*25*	
1	.997	.999	.999	.999	1.000	1.000	1.000	1.000	1.000	12.706
	1.000	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**1.000**	**63.657**
2	.950	.975	.983	.987	.990	.992	.994	.996	.998	4.303
	.990	**.995**	**.997**	**.998**	**.998**	**.998**	**.999**	**.999**	**1.000**	**9.925**
3	.878	.930	.950	.961	.968	.973	.979	.986	.993	3.182
	.959	**.976**	**.983**	**.987**	**.990**	**.991**	**.993**	**.995**	**.998**	**5.841**
4	.811	.881	.912	.930	.942	.950	.961	.973	.986	2.776
	.917	**.949**	**.962**	**.970**	**.975**	**.979**	**.984**	**.989**	**.994**	**4.604**
5	.754	.836	.874	.898	.914	.925	.941	.958	.978	2.571
	.874	**.917**	**.937**	**.949**	**.957**	**.963**	**.971**	**.980**	**.989**	**4.032**
6	.707	.795	.839	.867	.886	.900	.920	.943	.969	2.447
	.834	**.886**	**.911**	**.927**	**.938**	**.946**	**.957**	**.969**	**.983**	**3.707**
7	.666	.758	.807	.838	.860	.876	.900	.927	.960	2.365
	.798	**.855**	**.885**	**.904**	**.918**	**.928**	**.942**	**.958**	**.977**	**3.499**
8	.632	.726	.777	.811	.835	.854	.880	.912	.950	2.306
	.765	**.827**	**.860**	**.882**	**.898**	**.909**	**.926**	**.946**	**.970**	**3.355**
9	.602	.697	.750	.786	.812	.832	.861	.897	.941	2.262
	.735	**.800**	**.836**	**.861**	**.878**	**.891**	**.911**	**.934**	**.963**	**3.250**
10	.576	.671	.726	.763	.790	.812	.843	.882	.932	2.228
	.708	**.776**	**.814**	**.840**	**.859**	**.874**	**.895**	**.922**	**.955**	**3.169**
11	.553	.648	.703	.741	.770	.792	.826	.868	.922	2.201
	.684	**.753**	**.793**	**.821**	**.841**	**.857**	**.880**	**.910**	**.948**	**3.106**
12	.532	.627	.683	.722	.751	.774	.809	.854	.913	2.179
	.661	**.732**	**.773**	**.802**	**.824**	**.841**	**.866**	**.898**	**.940**	**3.055**
13	.514	.608	.664	.703	.733	.757	.794	.840	.904	2.160
	.641	**.712**	**.755**	**.785**	**.807**	**.825**	**.852**	**.886**	**.932**	**3.012**
14	.497	.590	.646	.686	.717	.741	.779	.828	.895	2.145
	.623	**.694**	**.737**	**.768**	**.792**	**.810**	**.838**	**.875**	**.924**	**2.977**
15	.482	.574	.630	.670	.701	.726	.765	.815	.886	2.131
	.606	**.677**	**.721**	**.752**	**.776**	**.796**	**.825**	**.864**	**.917**	**2.947**
16	.468	.559	.615	.655	.686	.712	.751	.803	.878	2.120
	.590	**.662**	**.706**	**.738**	**.762**	**.782**	**.813**	**.853**	**.909**	**2.921**
17	.456	.545	.601	.641	.673	.698	.738	.792	.869	2.110
	.575	**.647**	**.691**	**.724**	**.749**	**.769**	**.800**	**.842**	**.902**	**2.898**
18	.444	.532	.587	.628	.660	.686	.726	.781	.861	2.101
	.561	**.633**	**.678**	**.710**	**.736**	**.756**	**.789**	**.832**	**.894**	**2.878**
19	.433	.520	.575	.615	.647	.674	.714	.770	.853	2.093
	.549	**.620**	**.665**	**.698**	**.723**	**.744**	**.778**	**.822**	**.887**	**2.861**
20	.423	.509	.563	.604	.636	.662	.703	.760	.845	2.086
	.537	**.608**	**.652**	**.685**	**.712**	**.733**	**.767**	**.812**	**.880**	**2.845**
21	.413	.498	.552	.592	.624	.651	.693	.750	.837	2.080
	.526	**.596**	**.641**	**.674**	**.700**	**.722**	**.756**	**.803**	**.873**	**2.831**
22	.404	.488	.542	.582	.614	.640	.682	.740	.830	2.074
	.515	**.585**	**.630**	**.663**	**.690**	**.712**	**.746**	**.794**	**.866**	**2.819**
23	.396	.479	.532	.572	.604	.630	.673	.731	.823	2.069
	.505	**.574**	**.619**	**.652**	**.679**	**.701**	**.736**	**.785**	**.859**	**2.807**

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*Adapted from H. A. Wallace and G. W. Snedecor, "Correlation and Machine Calculation," Iowa State College, Ames, Iowa, 1931, by courtesy of the authors. Reprinted from J. P. Guilford, "Fundamental Statistics in Psychology and Education," 4th ed., McGraw-Hill Book Company, New York, 1965.

Table 8-2. Critical values for correlation and *t* ratios significant at the .05 level (lightface type) and at the .01 level (boldface type) for varying degrees of freedom (continued)

Degrees of freedom	*Number of variables*									*t*
	2	*3*	*4*	*5*	*6*	*7*	*9*	*13*	*25*	
24	.388	.470	.523	.562	.594	.621	.663	.722	.815	2.064
	.496	**.565**	**.609**	**.642**	**.669**	**.692**	**.727**	**.776**	**.852**	**2.797**
25	.381	.462	.514	.553	.585	.612	.654	.714	.808	2.060
	.487	**.555**	**.600**	**.633**	**.660**	**.682**	**.718**	**.768**	**.846**	**2.787**
26	.374	.454	.506	.545	.576	.603	.645	.706	.802	2.056
	.478	**.546**	**.590**	**.624**	**.651**	**.673**	**.709**	**.760**	**.839**	**2.779**
27	.367	.446	.498	.536	.568	.594	.637	.698	.795	2.052
	.470	**.538**	**.582**	**.615**	**.642**	**.664**	**.701**	**.752**	**.833**	**2.771**
28	.361	.439	.490	.529	.560	.586	.629	.690	.788	2.048
	.463	**.530**	**.573**	**.606**	**.634**	**.656**	**.692**	**.744**	**.827**	**2.763**
29	.355	.432	.482	.521	.552	.579	.621	.682	.782	2.045
	.456	**.522**	**.565**	**.598**	**.625**	**.648**	**.685**	**.737**	**.821**	**2.756**
30	.349	.426	.476	.514	.545	.571	.614	.675	.776	2.042
	.449	**.514**	**.558**	**.591**	**.618**	**.640**	**.677**	**.729**	**.815**	**2.750**
35	.325	.397	.445	.482	.512	.538	.580	.642	.746	2.030
	.418	**.481**	**.523**	**.556**	**.582**	**.605**	**.642**	**.696**	**.786**	**2.724**
40	.304	.373	.419	.455	.484	.509	.551	.613	.720	2.021
	.393	**.454**	**.494**	**.526**	**.552**	**.575**	**.612**	**.667**	**.761**	**2.704**
45	.288	.353	.397	.432	.460	.485	.526	.587	.696	2.014
	.372	**.430**	**.470**	**.501**	**.527**	**.549**	**.586**	**.640**	**.737**	**2.690**
50	.273	.336	.379	.412	.440	.464	.504	.565	.674	2.008
	.354	**.410**	**.449**	**.479**	**.504**	**.526**	**.562**	**.617**	**.715**	**2.678**
60	.250	.308	.348	.380	.406	.429	.467	.526	.636	2.000
	.325	**.377**	**.414**	**.442**	**.466**	**.488**	**.523**	**.577**	**.677**	**2.660**
70	.233	.286	.324	.354	.379	.401	.438	.495	.604	1.994
	.302	**.351**	**.386**	**.413**	**.436**	**.456**	**.491**	**.544**	**.644**	**2.648**
80	.217	.269	.304	.332	.356	.377	.413	.469	.576	1.990
	.283	**.330**	**.362**	**.389**	**.411**	**.431**	**.464**	**.516**	**.615**	**2.638**
90	.205	.254	.288	.315	.338	.358	.392	.446	.552	1.987
	.267	**.312**	**.343**	**.368**	**.390**	**.409**	**.441**	**.492**	**.590**	**2.632**
100	.195	.241	.274	.300	.322	.341	.374	.426	.530	1.984
	.254	**.297**	**.327**	**.351**	**.372**	**.390**	**.421**	**.470**	**.568**	**2.626**
125	.174	.216	.246	.269	.290	.307	.338	.387	.485	1.979
	.228	**.266**	**.294**	**.316**	**.335**	**.352**	**.381**	**.428**	**.521**	**2.616**
150	.159	.198	.225	.247	.266	.282	.310	.356	.450	1.976
	.208	**.244**	**.270**	**.290**	**.308**	**.324**	**.351**	**.395**	**.484**	**2.609**
200	.138	.172	.196	.215	.231	.246	.271	.312	.398	1.972
	.181	**.212**	**.234**	**.253**	**.269**	**.283**	**.307**	**.347**	**.430**	**2.601**
300	.113	.141	.160	.176	.190	.202	.223	.258	.332	1.968
	.148	**.174**	**.192**	**.208**	**.221**	**.233**	**.253**	**.287**	**.359**	**2.592**
400	.098	.122	.139	.153	.165	.176	.194	.225	.291	1.966
	.128	**.151**	**.167**	**.180**	**.192**	**.202**	**.220**	**.250**	**.315**	**2.588**
500	.088	.109	.124	.137	.148	.157	.174	.202	.262	1.965
	.115	**.135**	**.150**	**.162**	**.172**	**.182**	**.198**	**.225**	**.284**	**2.586**
1,000	.062	.077	.088	.097	.105	.112	.124	.144	.188	1.962
	.081	**.096**	**.106**	**.115**	**.122**	**.129**	**.141**	**.160**	**.204**	**2.581**
∞										1.960
										2.576

Table 8-3. Critical values of the correlation coefficient*

df	Level of significance for one-tailed test			
	.05	.025	.01	.005
	Level of significance for two-tailed test			
	.10	.05	.02	.01
1	.988	.997	.9995	.9999
2	.900	.950	.980	.990
3	.805	.878	.934	.959
4	.729	.811	.882	.917
5	.669	.754	.833	.874
6	.622	.707	.789	.834
7	.582	.666	.750	.798
8	.549	.632	.716	.765
9	.521	.602	.685	.735
10	.497	.576	.658	.708
11	.476	.553	.634	.684
12	.458	.532	.612	.661
13	.441	.514	.592	.641
14	.426	.497	.574	.623
15	.412	.482	.558	.606
16	.400	.468	.542	.590
17	.389	.456	.528	.575
18	.378	.444	.516	.561
19	.369	.433	.503	.549
20	.360	.423	.492	.537
21	.352	.413	.482	.526
22	.344	.404	.472	.515
23	.337	.396	.462	.505
24	.330	.388	.453	.496
25	.323	.381	.445	.487
26	.317	.374	.437	.479
27	.311	.367	.430	.471
28	.306	.361	.423	.463
29	.301	.355	.416	.456
30	.296	.349	.409	.449
35	.275	.325	.381	.418
40	.257	.304	.358	.393
45	.243	.288	.338	.372
50	.231	.273	.322	.354
60	.211	.250	.295	.325
70	.195	.232	.274	.303
80	.183	.217	.256	.283
90	.173	.205	.242	.267
100	.164	.195	.230	.254

* Abridged from R. A. Fisher and F. Yates, "Statistical Tables for Biological, Agricultural, and Medical Research," Oliver & Boyd, Ltd., Edinburgh, by permission of the authors and publishers. Reprinted from G. A. Ferguson, "Statistical Analysis in Psychology and Education," McGraw-Hill Book Company, New York, 1959.

Table 8-4. Values to facilitate the estimation of the cosine-pi coefficient of correlation, with two-place accuracy*

$\frac{ad}{bc}$	$r_{\text{cos-pi}}$	$\frac{ad}{bc}$	$r_{\text{cos-pi}}$	$\frac{ad}{bc}$	$r_{\text{cos-pi}}$	$\frac{ad}{bc}$	$r_{\text{cos-pi}}$
1.013	.005†	1.940	.255	4.067	.505	11.512	.755
1.039	.015	1.993	.265	4.205	.515	12.177	.765
1.066	.025	2.048	.275	4.351	.525	12.906	.775
1.093	.035	2.105	.285	4.503	.535	13.702	.785
1.122	.045	2.164	.295	4.662	.545	14.592	.795
1.150	.055	2.225	.305	4.830	.555	15.573	.805
1.180	.065	2.288	.315	5.007	.565	16.670	.815
1.211	075	2.353	.325	5.192	.575	17.900	.825
1.242	.085	2.421	.335	5.388	.585	19.288	.835
1.275	.095	2.490	.345	5.595	.595	20.866	.845
1.308	.105	2.563	.355	5.813	.605	22.675	.855
1.342	.115	2.638	.365	6.043	.615	24.768	.865
1.377	.125	2.716	.375	6.288	.625	27.212	.875
1.413	.135	2.797	.385	6.547	.635	30.106	.885
1.450	.145	2.881	.395	6.822	.645	33.578	.895
1.488	.155	2.957	.405	7.115	.655	37.818	.905
1.528	.165	3.095	.415	7.428	.665	43.100	.915
1.568	.175	3.153	.425	7.761	.675	49.851	.925
1.610	.185	3.251	.435	8.117	.685	58.765	.935
1.653	.195	3.353	.445	8.499	.695	71.046	.945
1.697	.205	3.460	.455	8.910	.705	88.984	.955
1.743	.215	3.571	.465	9.351	.715	117.52	.965
1.790	.225	3.690	.475	9.828	.725	169.60	.975
1.838	.235	3.808	.485	10.344	.735	293.28	.985
1.888	.245	3.935	.495	10.903	.745	934.06	.995

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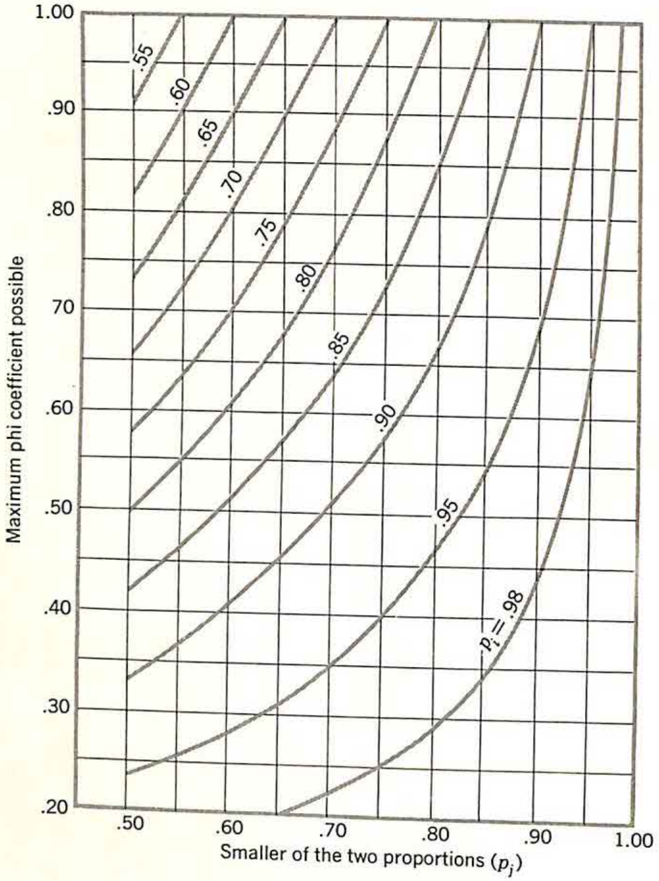
* Based upon a more detailed tabulation of the same values by N. C. Perry, N. W. Kettner, A. F. Hertzka, and E. A. Bouvier, Estimating the Tetrachoric Correlation Coefficient Via a Cosine-pi Table, Technical Memorandum No. 2, University of Southern California, Los Angeles, 1953. Reprinted from J. P. Guilford, "Fundamental Statistics in Psychology and Education," 4th ed., McGraw-Hill Book Company, New York, 1965.

† Example: If an obtained ratio *ad/bc* equals 3.472, we find that this value lies between tabled values of 3.460 and 3.571. The cosine-pi coefficient is therefore between .455 and .465; that is to say, it is .46. If *bc* is greater than *ad*, find the ratio *bc/ad* and attach a negative sign to $r_{\text{cos-pi}}$.

Table 8-5. **Maximal phi coefficients for different combinations of proportions of cases on the categories in X and Y having the largest marginal frequency p_i and its corresponding marginal frequency p_j***



* From F. S. Guilford, "Fundamental Statistics in Psychology and Education," 4th ed., McGraw-Hill Book Company, New York, 1965.

Table 8-6. Distribution of Σd_i^2 used in the rank-correlation coefficient*

The chance is α that Σd_i^2 equals or exceeds the value listed.

Σd_i^2	α	Σd_i^2	α	Σd_i^2	α
$N = 4$		$N = 7$		$N = 9$	
18	.167	82	.151	172	.125
20	.042	86	.118	180	.089
$N = 5$		90	.083	188	.060
32	.175	94	.055	196	.038
34	.117	98	.033	204	.022
36	.067	102	.017	212	.011
38	.042	106	.006	220	.004
40	.008	110	.001	228	.001
$N = 6$		$N = 8$		$N = 10$	
56	.121	120	.150	228	.139
58	.088	126	.108	238	.102
60	.068	132	.076	248	.072
62	.051	138	.048	258	.048
64	.029	144	.029	268	.030
66	.017	150	.014	278	.017
68	.008	156	.005	288	.009
70	.001	162	.001	298	.004

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* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 8-7. Critical values of r_S, the Spearman rank-correlation coefficient*

N	Significance level (one-tailed test)	
	.05	.01
4	1.000	
5	.900	1.000
6	.829	.943
7	.714	.893
8	.643	.833
9	.600	.783
10	.564	.746
12	.506	.712
14	.456	.645
16	.425	.601
18	.399	.564
20	.377	.534
22	359	.508
24	.343	.485
26	.329	.465
28	.317	.448
30	.306	.432

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* Adapted from E. G. Olds, Distributions of Sums of Squares of Rank Differences for Small Numbers of Individuals, *Ann. Math. Statist.*, vol. 9, pp. 133–148, 1938, and from E. G. Olds, The 5% Significance Levels for Sums of Squares of Rank Differences and a Correction, *Ann. Math. Statist.*, vol. 20, pp. 117–118, 1949, with the kind permission of the author and the publisher. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 8-8. **Probabilities associated with values as large as observed values of S in the Kendall rank-correlation coefficient***

S	Values of N				S	Values of N		
	4	5	8	9		6	7	10
0	.625	.592	.548	.540	1	.500	.500	.500
2	.375	.408	.452	.460	3	.360	.386	.431
4	.167	.242	.360	.381	5	.235	.281	.364
6	.042	.117	.274	.306	7	.136	191	.300
8		.042	.199	.238	9	.068	.119	.242
10		.0083	.138	.179	11	.028	.068	.190
12			.089	.130	13	.0083	.035	.146
14			.054	.090	15	.0014	.015	.108
16			.031	.060	17		.0054	.078
18			.016	.038	19		.0014	.054
20			.0071	.022	21		.00020	.036
22			.0028	.012	23			.023
24			.00087	.0063	25			.014
26			.00019	.0029	27			.0083
28			.000025	.0012	29			.0046
30				.00043	31			.0023
32				.00012	33			.0011
34				.000025	35			.00047
36				.0000028	37			.00018
					39			.000058
					41			.000015
					43			.0000028
					45			.00000028

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* Reproduced from M. G. Kendall, "Rank Correlation Methods," 2d ed., Charles Griffin & Company, Ltd., London, 1955, by permission of author and publishers. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

Table 8-9. Critical values of s in the Kendall coefficient of concordance*

k	N					Additional values for $N = 3$	
	3†	4	5	6	7	k	s
			Values at the .05 level of significance				
3			64.4	103.9	157.3	9	54.0
4		49.5	88.4	143.3	217.0	12	71.9
5		62.6	112.3	182.4	276.2	14	83.8
6		75.7	136.1	221.4	335.2	16	95.8
8	48.1	101.7	183.7	299.0	453.1	18	107.7
10	60.0	127.8	231.2	376.7	571.0		
15	89.8	192.9	349.8	570.5	864.9		
20	119.7	258.0	468.5	764.4	1,158.7		
			Values at the .01 level of significance				
3			75.6	122.8	185.6	9	75.9
4		61.4	109.3	176.2	265.0	12	103.5
5		80.5	142.8	229.4	343.8	14	121.9
6		99.5	176.1	282.4	422.6	16	140.2
8	66.8	137.4	242.7	388.3	579.9	18	158.6
10	85.1	175.3	309.1	494.0	737.0		
15	131.0	269.8	475.2	758.2	1,129.5		
20	177.0	364.2	641.2	1,022.2	1,521.9		

* Adapted from M. Friedman, A Comparison of Alternative Tests of Significance for the Problem of m Rankings, *Ann. Math. Statist.*, vol. 11, pp. 86–92, 1940, with the kind permission of the author and the publisher. Reprinted from S. Siegel, "Nonparametric Statistics," McGraw-Hill Book Company, New York, 1956.

† Notice that additional critical values of s for $N = 3$ are given in the right-hand column of this table.

Table 8-10. Distribution of quadrant sum for corner test for association for samples of size $2N$*

The table gives α, the chance that the quadrant sum equals or exceeds S.

$2N$ S	6	8	10	14	∞
8	.111	.126	.133	.132	.122
9	.100	.084	.093	.092	.081
10	.100	.055	.064	.063	.053
11	.100	.038	.044	.043	.034
12	.100	.030	.029	.030	.022
13	.000	.029	.019	.020	.013
14		.029	.013	.014	.008
15		.029	.010	.010	.005
16		.029	.008	.007	.003
17		.000	.008	.005	.002
18			.008	.003	.001
19			.008	.002	.001
20			.008	.001	.000
21			.000	.001	.000
22				.001	.000
23				.001	.000

* From W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

section

9

MISCELLANEOUS STATISTICAL TABLES

The simplified statistics of Table 9-1 can be used to facilitate several varieties of statistical analysis, provided that the data are at least approximately normal (ref. 2).

Tables 9-2 and 9-3 are tables of random numbers. The two-digit numbers in Table 9-2 appear with *equal frequency*. By combining numbers, random numbers with larger numbers of digits may be obtained. Table 9-3 contains random normal numbers calculated to three decimal places, with $\mu = 0$ and $\sigma = 1$.

An analysis-of-variance experiment is sometimes designed so that the shape of the regression of the dependent variable on an independent variable can be ascertained. If the levels of the independent variable are in equal steps, the technique of *orthogonal polynomials* can be used. In this method, linear, quadratic, cubic, etc. components of the regression may be separately evaluated for statistical significance with quasi-independence between the tests. In order to compute the mean squares for the linear, quadratic, etc., regression effects, the coefficients of orthogonal polynomials appropriate to the number of levels of the independent variable are needed. Table 9-4 contains such coefficients for evaluating up to fifth degree components and for up to 10 levels of the independent variable. Table 9-4 also contains the sums of squares of the coefficients used in computing the mean squares for the corresponding component and quantities λ, which are the multipliers for determining regression coefficients (ref. 2).

Quite frequently, the basic data in an experiment will consist of proportions or percentages, and it is desired to subject this data to an analysis of variance or similar analysis. In this case it usually is desirable to transform the basic observations so as to eliminate the relationship between mean and variance. The customary transformation in this situation is the *arcsin transformation* $\phi = 2 \arcsin \sqrt{x}$. Table 9-5 lists values of ϕ for values of X from 0.001 to 0.999.

Information theory, as applied in communication systems, makes use of the concept of the amount of information associated with the transmission of a message x. The amount of information is $-\log P(x)$ where $P(x)$ denotes the probability of the message x. The *average amount of information per message* for a discrete source is then $-\sum_x P(x) \log_2 P(x)$. Information theory has had numerous applications in the behavioral sciences, and the connections between information theory and statistics have been extensively explored. Of particular interest is the use of the information concept in testing hypotheses about the goodness of fit of a sample of observations from a discrete random variable to some hypothetical distribution and in testing hypotheses about contingency tables. Excellent discussions of these procedures can be found in Kullback, "Information Theory and Statistics," or in Garner, "Uncertainty and Structure as Psychological Concepts." The many applications of information theory require use of the functions $-\log_2 P$, $-P \log P$, and $H(P) = -(P \log_2 P + Q \log_2 Q)$ where P is a probability or proportion. Statistical applications frequently make use of the quantity $n \log_2 n$. If $P = n/N$, then $P \log_2 P$ and $n \log_2 n$ are related by the equation

$$n \log_2 n = NP(\log_2 P + \log_2 N)$$

Table 9-6 contains values of $-\log_2 P$, $-P \log_2 P$, H, $-\log_2 Q$, and $-Q \log_2 Q$ for values of P from 0.0001 to 0.5000. For larger values of P, P and Q are interchanged. All logarithms are to the base 2 in Table 9-6.

Table 9-1. Simplified statistics and statistics based on sample ranges*

(1) Percentile estimates in large samples

(*a*) Mean

	Percentile estimate	Eff.
1	P_{50}	.64
2	$.5(P_{25} + P_{75})$	.81
3	$.3333(P_{17} + P_{50} + P_{83})$	.88
4	$.25(P_{12.5} + P_{37.5} + P_{62.5} + P_{87.5})$	.91
5	$.20(P_{10} + P_{30} + P_{50} + P_{70} + P_{90})$	.93
. . .	. . . .	. . .
10	$.10(P_{05} + P_{15} + P_{25} + P_{35} + P_{45} + P_{55} + P_{65} + P_{75} + P_{85} + P_{95})$	.97

(*b*) Standard deviation

	Percentile estimate	Eff.
2	$.3388(P_{93} - P_{07})$	.65
4	$.1714(P_{97} + P_{85} - P_{15} - P_{03})$	.80
6	$.1180(P_{98} + P_{91} + P_{80} - P_{20} - P_{09} - P_{02})$	.87
8	$.0935(P_{98} + P_{93} + P_{86} + P_{77} - P_{23} - P_{14} - P_{07} - P_{02})$	.90
10	$.0739(P_{98.5} + P_{95} + P_{90} + P_{84} + P_{75} - P_{25} - P_{16} - P_{10} - P_{05} - P_{01.5})$	.92

(*c*) Mean and standard deviation

	Percentile	Efficiency		K
		Mean	Standard deviation	
2	15, 85	.73	.56	.4824
4	05, 30, 70, 95	.80	.74	.2305
6	05, 15, 40, 60, 85, 95	.89	.80	.1704
8	03, 10, 25, 45, 55, 75, 90, 97	.90	.86	.1262
10	03, 10, 20, 30, 50, 50, 70, 80, 90, 97	.94	.87	.1104

(2) Estimates of mean and dispersion in small samples

(*a*) Unbiased estimate of σ using the sample range w (variance to be multiplied by σ^2)

Sample size	Estimate	Variance	Eff.	Sample size	Estimate	Variance	Eff.
2	$.886w$	.571	1.000	11	$.315w$	.0616	.831
3	$.591w$	.275	.992	12	$.307w$	.0571	.814
4	$.486w$	.183	.975	13	$.300w$	.0533	.797
5	$.430w$	.138	.955	14	$.294w$	.0502	.781
6	$.395w$	.112	.933	15	$.288w$	.0474	.766
7	$.370w$	.0949	.911	16	$.283w$	.0451	.751
8	$.351w$	.0829	.890	17	$.279w$	.0430	.738
9	$.337w$	.0740	.869	18	$.275w$	.0412	.725
10	$.325w$	.0671	.850	19	$.271w$	.0395	.712
				20	$.268w$	.0381	.700

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* Reprinted from W. J. Dixon and F. J. Massey, "Introduction to Statistical Analysis," 2d ed., McGraw-Hill Book Company, New York, 1957.

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(*b*) Percentiles of the distribution of w/σ

Sample size	$P_{0.1}$	$P_{0.5}$	P_{01}	$P_{02.5}$	P_{05}	P_{10}	P_{90}	P_{95}	$P_{97.5}$	P_{99}	$P_{99.5}$	$P_{99.9}$
2	.00	.01	.02	.04	.09	.18	2.33	2.77	3.17	3.64	3.97	4.65
3	.06	.13	.19	.30	.43	.62	2.90	3.31	3.68	4.12	4.42	5.06
4	.20	.34	.43	.59	.76	.98	3.24	3.63	3.98	4.40	4.69	5.31
5	.37	.55	.66	.85	1.03	1.26	3.48	3.86	4.20	4.60	4.89	5.48
6	.54	.75	.87	1.06	1.25	1.49	3.66	4.03	4.36	4.76	5.03	5.62
7	.69	.92	1.05	1.25	1.44	1.68	3.81	4.17	4.49	4.88	5.15	5.73
8	.83	1.08	1.20	1.41	1.60	1.83	3.93	4.29	4.61	4.99	5.26	5.82
9	.96	1.21	1.34	1.55	1.74	1.97	4.04	4.39	4.70	5.08	5.34	5.90
10	1.08	1.33	1.47	1.67	1.86	2.09	4.13	4.47	4.79	5.16	5.42	5.97
11	1.20	1.45	1.58	1.78	1.97	2.20	4.21	4.55	4.86	5.23	5.49	6.04
12	1.30	1.55	1.68	1.88	2.07	2.30	4.29	4.62	4.92	5.29	5.54	6.09
13	1.38	1.64	1.77	1.97	2.16	2.39	4.35	4.69	4.98	5.35	5.60	6.15
14	1.47	1.72	1.85	2.06	2.24	2.47	4.41	4.74	5.04	5.40	5.65	6.20
15	1.55	1.80	1.93	2.14	2.32	2.54	4.47	4.80	5.09	5.45	5.70	6.23
16	1.62	1.87	2.00	2.21	2.39	2.61	4.52	4.85	5.14	5.49	5.74	6.27
17	1.69	1.94	2.07	2.27	2.45	2.67	4.57	4.89	5.18	5.54	5.79	6.30
18	1.75	2.01	2.14	2.33	2.52	2.73	4.61	4.93	5.22	5.58	5.82	6.35
19	1.82	2.07	2.20	2.39	2.57	2.79	4.65	4.97	5.26	5.61	5.86	6.38
20	1.88	2.13	2.25	2.45	2.63	2.84	4.69	5.01	5.30	5.65	5.89	6.40

(*c*) Unbiased estimate of σ based on s

Sample size	Estimate	Variance	Sample size	Estimate	Variance
2	$1.253s$	$.571\sigma^2$	7	$1.042s$	$.0865\sigma^2$
3	$1.128s$	$.273\sigma^2$	8	$1.036s$	$.0738\sigma^2$
4	$1.085s$	$.178\sigma^2$	9	$1.032s$	$.0643\sigma^2$
5	$1.064s$	$.132\sigma^2$	10	$1.028s$	$.0570\sigma^2$
6	$1.051s$	$.104\sigma^2$	∞	$\left[1 + \frac{1}{4(N-1)}\right] s$	$\sigma^2/2N$

(*d*) Mean deviation estimate of σ

Sample size	Estimate	Eff.
2	$.8862(X_2 - X_1)$	1.00
3	$.5908(X_3 - X_1)$	.99
4	$.3770(X_4 + X_3 - X_2 - X_1)$	.91
5	$.3016(X_5 + X_4 - X_2 - X_1)$	.94
6	$.2369(X_6 + X_5 + X_4 - X_3 - X_2 - X_1)$	.90
7	$.2031(X_7 + X_6 + X_5 - X_3 - X_2 - X_1)$	.92
8	$.1723(X_8 + X_7 + X_6 + X_5 - X_4 - X_3 - X_2 - X_1)$	.90
9	$.1532(X_9 + X_8 + X_7 + X_6 - X_4 - X_3 - X_2 - X_1)$	.91
10	$.1353(X_{10} + X_9 + X_8 + X_7 + X_6 - X_5 - X_4 - X_3 - X_2 - X_1)$	.89

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(*e*) Modified linear estimate of σ (variance to be multiplied by σ^2)

Sample size	Estimate	Variance	Eff.
2	$.8862(X_2 - X_1)$	.571	1.000
3	$.5908(X_3 - X_1)$	.275	.992
4	$.4857(X_4 - X_1)$	.183	.975
5	$.4299(X_5 - X_1)$	.138	.955
6	$.2619(X_6 + X_5 - X_2 - X_1)$	.109	.957
7	$.2370(X_7 + X_6 - X_2 - X_1)$	.0895	.967
8	$.2197(X_8 + X_7 - X_2 - X_1)$	.0761	.970
9	$.2068(X_9 + X_8 - X_2 - X_1)$	.0664	.968
10	$.1968(X_{10} + X_9 - X_2 - X_1)$	.0591	.964
11	$.1608(X_{11} + X_{10} + X_8 - X_4 - X_2 - X_1)$	.0529	.967
12	$.1524(X_{12} + X_{11} + X_9 - X_4 - X_2 - X_1)$	.0478	.972
13	$.1456(X_{13} + X_{12} + X_{10} - X_4 - X_2 - X_1)$	.0436	.975
14	$.1399(X_{14} + X_{13} + X_{11} - X_4 - X_2 - X_1)$	.0401	.977
15	$.1352(X_{15} + X_{14} + X_{12} - X_4 - X_2 - X_1)$	.0372	.977
16	$.1311(X_{16} + X_{15} + X_{13} - X_4 - X_2 - X_1)$	.0347	.975
17	$.1050(X_{17} + X_{16} + X_{15} + X_{13} - X_5 - X_3 - X_2 - X_1)$	.0325	.978
18	$.1020(X_{18} + X_{17} + X_{16} + X_{14} - X_5 - X_3 - X_2 - X_1)$	.0305	.978
19	$.09939(X_{19} + X_{18} + X_{17} + X_{15} - X_5 - X_3 - X_2 - X_1)$	.0288	.979
20	$.09706(X_{20} + X_{19} + X_{18} + X_{16} - X_5 - X_3 - X_2 - X_1)$	.0272	.978

(*f*) Several estimates of the mean (variance to be multiplied by σ^2)

N	Median		Midrange		Av. of best two			$(X_2 + X_3 + \cdots + X_{N-1})/(N - 2)$	
	Var.	Eff.	Var.	Eff.	Statistic	Var.	Eff.	Var.	Eff.
2	.500	1.000	.500	1.000	$\frac{1}{2}(X_1 + X_2)$	.500	1.000		
3	.449	.743	.362	.920	$\frac{1}{2}(X_1 + X_3)$	.362	.920	.449	.743
4	.298	.838	.298	.838	$\frac{1}{2}(X_2 + X_3)$	.298	.838	.298	.838
5	.287	.697	.261	.767	$\frac{1}{2}(X_2 + X_4)$	.231	.867	.227	.881
6	.215	.776	.236	.706	$\frac{1}{2}(X_2 + X_5)$	.193	.865	.184	.906
7	.210	.679	.218	.654	$\frac{1}{2}(X_2 + X_6)$	.168	.849	.155	.922
8	.168	.743	.205	.610	$\frac{1}{2}(X_3 + X_6)$	.149	.837	.134	.934
9	.166	.669	.194	.572	$\frac{1}{2}(X_3 + X_7)$	.132	.843	.118	.942
10	.138	.723	.186	.539	$\frac{1}{2}(X_3 + X_8)$	.119	.840	.105	.949
11	.137	.663	.178	.510	$\frac{1}{2}(X_3 + X_9)$	.109	.832	.0952	.955
12	.118	.709	.172	.484	$\frac{1}{2}(X_4 + X_9)$	.100	.831	.0869	.959
13	.117	.659	.167	.461	$\frac{1}{2}(X_4 + X_{10})$	.0924	.833	.0799	.963
14	.102	.699	.162	.440	$\frac{1}{2}(X_4 + X_{11})$	.0860	.830	.0739	.966
15	.102	.656	.158	.422	$\frac{1}{2}(X_4 + X_{12})$	.0808	.825	.0688	.969
16	.0904	.692	.154	.392	$\frac{1}{2}(X_5 + X_{12})$	.0756	.827	.0644	.971
17	.0901	.653	.151	.389	$\frac{1}{2}(X_5 + X_{13})$	.0711	.827	.0605	.973
18	.0810	.686	.148	.375	$\frac{1}{2}(X_5 + X_{14})$	.0673	.825	.0570	.975
19	.0808	.651	.145	.362	$\frac{1}{2}(X_6 + X_{14})$	.0640	.823	.0539	.976
20	.0734	.681	.143	.350	$\frac{1}{2}(X_6 + X_{15})$	.0607	.824	.0511	.978
∞	$1.57/N$	.637		.000	$\frac{1}{2}(P_{25} + P_{75})$	$1.24/N$	.808		1.000

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(*g*) Mean values of the order statistics. Values given are in deviations from the mean in standard-deviation units.

Sample size	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
2	− .564	.564								
3	− .846	.000	.846							
4	−1.029	− .297	.297	1.029						
5	−1.163	− .495	.000	.495	1.163					
6	−1.267	− .642	− .202	.202	.642	1.267				
7	−1.352	− .757	− .353	.000	.353	.757	1.352			
8	−1.424	− .852	− .473	−.153	.153	.473	.852	1.424		
9	−1.485	− .932	− .572	−.275	.000	.275	.572	.932	1.485	
10	−1.539	−1.001	− .656	−.376	−.123	.123	.376	.656	1.001	1.539
11	−1.586	−1.062	− .729	−.462	−.225	.000	.225	.462	.729	1.062
12	−1.629	−1.116	− .793	−.537	−.312	−.103	.103	.312	.537	.793
13	−1.668	−1.164	− .850	−.603	−.388	−.191	.000	.191	.388	.603
14	−1.703	−1.208	− .901	−.662	−.456	−.267	−.088	.088	.267	.456
15	−1.736	−1.248	− .948	−.715	−.516	−.335	−.165	.000	.165	.335
16	−1.766	−1.285	− .990	−.763	−.570	−.396	−.234	−.077	.077	.234
17	−1.794	−1.319	−1.029	−.807	−.619	−.451	−.295	−.146	.000	.146
18	−1.820	−1.350	−1.066	−.848	−.665	−.502	−.351	−.208	−.069	.069
19	−1.844	−1.380	−1.099	−.886	−.707	−.548	−.402	−.264	−.131	.000
20	−1.867	−1.408	−1.131	−.921	−.745	−.590	−.448	−.315	−.187	−.062

For sample sizes greater than 10 the mean values of X_{11}, X_{12}, etc., can be obtained by symmetry. For example, for $N = 17$ the expected value of X_{15} is the same except for sign as the expected value of X_3.

(*h*) Best linear estimate of σ

Sample size	Estimate	Eff.
2	$.8862(X_2 - X_1)$	1.000
3	$.5908(X_3 - X_1)$	.992
4	$.4539(X_4 - X_1) + .1102(X_3 - X_2)$	.989
5	$.3724(X_5 - X_1) + .1352(X_4 - X_2)$	.988
6	$.3175(X_6 - X_1) + .1386(X_5 - X_2) + .0432(X_4 - X_3)$	.988
7	$.2778(X_7 - X_1) + .1351(X_6 - X_2) + .0625(X_5 - X_3)$	.989
8	$.2476(X_8 - X_1) + .1294(X_7 - X_2) + .0713(X_6 - X_3) + .0230(X_5 - X_4)$	.989
9	$.2237(X_9 - X_1) + .1233(X_8 - X_2) + .0751(X_7 - X_3) + .0360(X_6 - X_4)$	.989
10	$.2044(X_{10} - X_1) + .1172(X_9 - X_2) + .0763(X_8 - X_3) + .0436(X_7 - X_4) + .0142(X_6 - X_5)$	.990

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(3) Substitute t ratios

(*a*) Percentiles* for $\tau_1 = \dfrac{\bar{X} - \mu}{w}$, where w is the sample range

Sample size	P_{95}	$P_{97.5}$	P_{99}	$P_{99.5}$	$P_{99.9}$	$P_{99.95}$
2	3.175	6.353	15.910	31.828	159.16	318.31
3	.885	1.304	2.111	3.008	6.77	9.58
4	.529	.717	1.023	1.316	2.29	2.85
5	.388	.507	.685	.843	1.32	1.58
6	.312	.399	.523	.628	.92	1.07
7	.263	.333	.429	.507	.71	.82
8	.230	.288	.366	.429	.59	.67
9	.205	.255	.322	.374	.50	.57
10	.186	.230	.288	.333	.44	.50
11	.170	.210	.262	.302	.40	.44
12	.158	.194	.241	.277	.36	.40
13	.147	.181	.224	.256	.33	.37
14	.138	.170	.209	.239	.31	.34
15	.131	.160	.197	.224	.29	.32
16	.124	.151	.186	.212	.27	.30
17	.118	.144	.177	.201	.26	.28
18	.113	.137	.168	.191	.24	.26
19	.108	.131	.161	.182	.23	.25
20	.104	.126	.154	.175	.22	.24
	$-P_{05}$	$-P_{02.5}$	$-P_{01}$	$-P_{0.5}$	$-P_{0.1}$	$-P_{0.05}$

* When the table is read from the foot, the tabled values are to be prefixed with a negative sign.

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(*b*) Percentiles* for $\tau_d = \dfrac{\bar{X}_1 - \bar{X}_2}{\frac{1}{2}(w_1 + w_2)}$ obtained from two samples

Sample sizes $N_1 = N_2$	P_{95}	$P_{97.5}$	P_{99}	$P_{99.5}$	$P_{99.9}$	$P_{99.95}$
2	2.322	3.427	5.553	7.916	17.81	25.23
3	.974	1.272	1.715	2.093	3.27	4.18
4	.644	.813	1.047	1.237	1.74	1.99
5	.493	.613	.772	.896	1.21	1.35
6	.405	.499	.621	.714	.94	1.03
7	.347	.426	.525	.600	.77	.85
8	.306	.373	.459	.521	.67	.73
9	.275	.334	.409	.464	.59	.64
10	.250	.304	.371	.419	.53	.58
11	.233	.280	.340	.384	.48	.52
12	.214	.260	.315	.355	.44	.48
13	.201	.243	.294	.331	.41	.45
14	.189	.228	.276	.311	.39	.42
15	.179	.216	.261	.293	.36	.39
16	.170	.205	.247	.278	.34	.37
17	.162	.195	.236	.264	.33	.35
18	.155	.187	.225	.252	.31	.34
19	.149	.179	.216	.242	.30	.32
20	.143	.172	.207	.232	.29	.31
	$-P_{05}$	$-P_{02.5}$	$-P_{01}$	$-P_{0.5}$	$-P_{0.1}$	$-P_{0.05}$

* When the table is read from the foot, the tabled values are to be prefixed with a negative sign.

(*c*) Percentiles for $\tau_2 = \dfrac{\frac{1}{2}(X_1 + X_N) - \mu}{w}$

Sample size	P_{95}	$P_{97.5}$	P_{99}	$P_{99.5}$
2	3.16	6.35	15.91	31.83
3	.90	1.30	2.11	3.02
4	.55	.74	1.04	1.37
5	.42	.52	.71	.85
6	.35	.43	.56	.66
7	.30	.37	.47	.55
8	.26	.33	.42	.47
9	.24	.30	.38	.42
10	.22	.27	.35	.39
	$-P_{05}$	$-P_{02.5}$	$-P_{01}$	$-P_{0.5}$

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(4) Substitute F ratio, ratio of ranges

Sample size for denominator	Cum. prop.	Sample size for numerator								
		2	3	4	5	6	7	8	9	10
2	.005	.0078	.096	.21	.30	.38	.44	.49	.54	.57
	.01	.0157	.136	.26	.38	.46	.53	.59	.64	.68
	.025	.039	.217	.37	.50	.60	.68	.74	.79	.83
	.05	.079	.31	.50	.62	.74	.80	.86	.91	.95
	.95	12.7	19.1	23	26	29	30	32	34	35
	.975	25.5	38.2	52	57	60	62	64	67	68
	.99	63.7	95	116	132	142	153	160	168	174
	.995	127	191	230	250	260	270	280	290	290
3	.005	.0052	.071	.16	.24	.32	.38	.43	.47	.50
	.01	.0105	.100	.20	.30	.37	.43	.49	.53	.57
	.025	.026	.160	.28	.39	.47	.54	.59	.64	.68
	.05	.052	.23	.37	.49	.57	.64	.70	.75	.80
	.95	3.19	4.4	5.0	5.7	6.2	6.6	6.9	7.2	7.4
	.975	4.61	6.3	7.3	8.0	8.7	9.3	9.8	10.2	10.5
	.99	7.37	10	12	13	14	15	15	16	17
	.995	10.4	14	17	18	20	21	22	23	25
4	.005	.0043	.059	.14	.22	.28	.34	.39	.43	.46
	.01	.0086	.084	.18	.26	.33	.39	.44	.48	.52
	.025	.019	.137	.25	.34	.42	.48	.53	.57	.61
	.05	.043	.20	.32	.42	.50	.57	.62	.67	.70
	.95	2.02	2.7	3.1	3.4	3.6	3.8	4.0	4.2	4.4
	.975	2.72	3.5	4.0	4.4	4.7	5.0	5.2	5.4	5.6
	.99	3.83	5.0	5.5	6.0	6.4	6.7	7.0	7.2	7.5
	.995	4.85	6.1	7.0	7.6	8.1	8.5	8.8	9.3	9.6
5	.005	.0039	.054	.13	.20	.26	.32	.36	.40	.44
	.01	.0076	.079	.17	.24	.31	.36	.41	.45	.49
	.025	.018	.124	.23	.32	.38	.44	.49	.53	.57
	.05	.038	.18	.29	.40	.46	.52	.57	.61	.65
	.95	1.61	2.1	2.4	2.6	2.8	2.9	3.0	3.1	3.2
	.975	2.01	2.6	2.9	3.2	3.4	3.6	3.7	3.8	3.9
	.99	2.64	3.4	3.8	4.1	4.3	4.6	4.7	4.9	5.0
	.995	3.36	4.1	4.6	4.9	5.2	5.5	5.7	5.9	6.1

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

Sample size for denominator	cum. prop.	Sample size for numerator								
		2	3	4	5	6	7	8	9	10
6	.005	.0038	.051	.12	.19	.25	.30	.35	.38	.42
	.01	.0070	.073	.16	.23	.29	.34	.39	.43	.46
	.025	.017	.115	.21	.30	.36	.42	.46	.50	.54
	.05	.035	.16	.27	.36	.43	.49	.54	.58	.61
	.95	1.36	1.8	2.0	2.2	2.3	2.4	2.5	2.6	2.7
	.975	1.67	2.1	2.4	2.6	2.8	2.9	3.0	3.1	3.2
	.99	2.16	2.7	3.0	3.2	3.4	3.6	3.7	3.8	3.9
	.995	2.67	3.1	3.5	3.8	4.0	4.1	4.3	4.5	4.6
7	.005	.0037	.048	.12	.18	.24	.29	.33	.37	.40
	.01	.0066	.069	.15	.22	.28	.33	.37	.41	.45
	.025	.016	.107	.20	.28	.34	.40	.44	.48	.52
	.05	.032	.15	.26	.35	.41	.47	.51	.55	.59
	.95	1.26	1.6	1.8	1.9	2.0	2.1	2.2	2.3	2.4
	.975	1.48	1.9	2.1	2.3	2.4	2.5	2.6	2.7	2.8
	.99	1.87	2.3	2.6	2.8	2.9	3.0	3.1	3.2	3.3
	.995	2.28	2.7	2.9	3.1	3.3	3.5	3.6	3.7	3.8
8	.005	.0036	.045	.11	.18	.23	.28	.32	.36	.39
	.01	.0063	.065	.14	.21	.27	.32	.36	.40	.43
	.025	.016	.102	.19	.27	.33	.38	.43	.47	.50
	.05	.031	.14	.25	.33	.40	.45	.50	.53	.57
	.95	1.17	1.4	1.6	1.8	1.9	1.9	2.0	2.1	2.1
	.975	1.36	1.7	1.9	2.0	2.2	2.3	2.3	2.4	2.5
	.99	1.69	2.1	2.3	2.4	2.6	2.7	2.8	2.8	2.9
	.995	2.03	2.3	2.6	2.7	2.9	3.0	3.1	3.2	3.3
9	.005	.0035	.042	.11	.17	.22	.27	.31	.35	.38
	.01	.0060	.062	.14	.21	.26	.31	.35	.39	.42
	.025	.015	.098	.18	.26	.32	.37	.42	.46	.49
	.05	.030	.14	.24	.32	.38	.44	.48	.52	.55
	.95	1.10	1.3	1.5	1.6	1.7	1.8	1.9	1.9	2.0
	.975	1.27	1.6	1.8	1.9	2.0	2.1	2.1	2.2	2.3
	.99	1.56	1.9	2.1	2.2	2.3	2.4	2.5	2.6	2.6
	.995	1.87	2.1	2.3	2.5	2.6	2.7	2.8	2.9	3.0
10	.005	.0034	.041	.10	.16	.22	.26	.30	.34	.37
	.01	.0058	.060	.13	.20	.26	.30	.34	.38	.41
	.025	.015	.095	.18	.25	.31	.36	.41	.44	.48
	.05	.029	.13	.23	.31	.37	.43	.47	.51	.54
	.95	1.05	1.3	1.4	1.5	1.6	1.7	1.8	1.8	1.9
	.975	1.21	1.5	1.6	1.8	1.9	1.9	2.0	2.0	2.1
	.99	1.47	1.8	1.9	2.1	2.2	2.2	2.3	2.4	2.4
	.995	1.75	2.0	2.2	2.3	2.4	2.5	2.6	2.6	2.7

Table 9-1. Simplified statistics and statistics based on sample ranges (continued)

(5) Criteria for testing for extreme mean (testing for outliers)

Statistic	Number of obs., k	Critical values						
		$\alpha =$ .30	$\alpha =$ .20	$\alpha =$ .10	$\alpha =$ .05	$\alpha =$ .02	$\alpha =$ .01	$\alpha =$ .005
$r_{10} = \dfrac{X_2 - X_1}{X_k - X_1}$	3	.684	.781	.886	.941	.976	.988	.994
	4	.471	.560	.679	.765	.846	.889	.926
	5	.373	.451	.557	.642	.729	.780	.821
	6	.318	.386	.482	.560	.644	.698	.740
	7	.281	.344	.434	.507	.586	.637	.680
$r_{11} = \dfrac{X_2 - X_1}{X_{k-1} - X_1}$	8	.318	.385	.479	.554	.631	.683	.725
	9	.288	.352	.441	.512	.587	.635	.677
	10	.265	.325	.409	.477	.551	.597	.639
$r_{21} = \dfrac{X_3 - X_1}{X_{k-1} - X_1}$	11	.391	.442	.517	.576	.638	.679	.713
	12	.370	.419	.490	.546	.605	.642	.675
	13	.351	.399	.467	.521	.578	.615	.649
$r_{22} = \dfrac{X_3 - X_1}{X_{k-2} - X_1}$	14	.370	.421	.492	.546	.602	.641	.674
	15	.353	.402	.472	.525	.579	.616	.647
	16	.338	.386	.454	.507	.559	.595	.624
	17	.325	.373	.438	.490	.542	.577	.605
	18	.314	.361	.424	.475	.527	.561	.589
	19	.304	.350	.412	.462	.514	.547	.575
	20	.295	.340	.401	.450	.502	.535	.562
	21	.287	.331	.391	.440	.491	.524	.551
	22	.280	.323	.382	.430	.481	.514	.541
	23	.274	.316	.374	.421	.472	.505	.532
	24	.268	.310	.367	.413	.464	.497	.524
	25	.262	.304	.360	.406	.457	.489	.516

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Table 9-2. Random numbers*

10 09 73 25 33	76 52 01 35 86	34 67 35 48 76	80 95 90 91 17	39 29 27 49 45
37 54 20 48 05	64 89 47 42 96	24 80 52 40 37	20 63 61 04 02	00 82 29 16 65
08 42 26 89 53	19 64 50 93 03	23 20 90 25 60	15 95 33 47 64	35 08 03 36 06
99 01 90 25 29	09 37 67 07 15	38 31 13 11 65	88 67 67 43 97	04 43 62 76 59
12 80 79 99 70	80 15 73 61 47	64 03 23 66 53	98 95 11 68 77	12 17 17 68 33
66 06 57 47 17	34 07 27 68 50	36 69 73 61 70	65 81 33 98 85	11 19 92 91 70
31 06 01 08 05	45 57 18 24 06	35 30 34 26 14	86 79 90 74 39	23 40 30 97 32
85 26 97 76 02	02 05 16 56 92	68 66 57 48 18	73 05 38 52 47	18 62 38 85 79
63 57 33 21 35	05 32 54 70 48	90 55 35 75 48	28 46 82 87 09	83 49 12 56 24
73 79 64 57 53	03 52 96 47 78	35 80 83 42 82	60 93 52 03 44	35 27 38 84 35
98 52 01 77 67	14 90 56 86 07	22 10 94 05 58	60 97 09 34 33	50 50 07 39 98
11 80 50 54 31	39 80 82 77 32	50 72 56 82 48	29 40 52 42 01	52 77 56 78 51
83 45 29 96 34	06 28 89 80 83	13 74 67 00 78	18 47 54 06 10	68 71 17 78 17
88 68 54 02 00	86 50 75 84 01	36 76 66 79 51	90 36 47 64 93	29 60 91 10 62
99 59 46 73 48	87 51 76 49 69	91 82 60 89 28	93 78 56 13 68	23 47 83 41 13
65 48 11 76 74	17 46 85 09 50	58 04 77 69 74	73 03 95 71 86	40 21 81 65 44
80 12 43 56 35	17 72 70 80 15	45 31 82 23 74	21 11 57 82 53	14 38 55 37 63
74 35 09 98 17	77 40 27 72 14	43 23 60 02 10	45 52 16 42 37	96 28 60 26 55
69 91 62 68 03	66 25 22 91 48	36 93 68 72 03	76 62 11 39 90	94 40 05 64 18
09 89 32 05 05	14 22 56 85 14	46 42 75 67 88	96 29 77 88 22	54 38 21 45 98
91 49 91 45 23	68 47 92 76 86	46 16 28 35 54	94 75 08 99 23	37 08 92 00 48
80 33 69 45 98	26 94 03 68 58	70 29 73 41 35	53 14 03 33 40	42 05 08 23 41
44 10 48 19 49	85 15 74 79 54	32 97 92 65 75	57 60 04 08 81	22 22 20 64 13
12 55 07 37 42	11 10 00 20 40	12 86 07 46 97	96 64 48 94 39	28 70 72 58 15
63 60 64 93 29	16 50 53 44 84	40 21 95 25 63	43 65 17 70 82	07 20 73 17 90
61 19 69 04 46	26 45 74 77 74	51 92 43 37 29	65 39 45 95 93	42 58 26 05 27
15 47 44 52 66	95 27 07 99 53	59 36 78 38 48	82 39 61 01 18	33 21 15 94 66
94 55 72 85 73	67 89 75 43 87	54 62 24 44 31	91 19 04 25 92	92 92 74 59 73
42 48 11 62 13	97 34 40 87 21	16 86 84 87 67	03 07 11 20 59	25 70 14 66 70
23 52 37 83 17	73 20 88 98 37	68 93 59 14 16	26 25 22 96 63	05 52 28 25 62
04 49 35 24 94	75 24 63 38 24	45 86 25 10 25	61 96 27 93 35	65 33 71 24 72
00 54 99 76 54	64 05 18 81 59	96 11 96 38 96	54 69 28 23 91	23 28 72 95 29
35 96 31 53 07	26 89 80 93 54	33 35 13 54 62	77 97 45 00 24	90 10 33 93 33
59 80 80 83 91	45 42 72 68 42	83 60 94 97 00	13 02 12 48 92	78 56 52 01 06
46 05 88 52 36	01 39 09 22 86	77 28 14 40 77	93 91 08 36 47	70 61 74 29 41
32 17 90 05 97	87 37 92 52 41	05 56 70 70 07	86 74 31 71 57	85 39 41 18 38
69 23 46 14 06	20 11 74 52 04	15 95 66 00 00	18 74 39 24 23	97 11 89 63 38
19 56 54 14 30	01 75 87 53 79	40 41 92 15 85	66 67 43 68 06	84 96 28 52 07
45 15 51 49 38	19 47 60 72 46	43 66 79 45 43	59 04 79 00 33	20 82 66 95 41
94 86 43 19 94	36 16 81 08 51	34 88 88 15 53	01 54 03 54 56	05 01 45 11 76
98 08 62 48 26	45 24 02 84 04	44 99 90 88 96	39 09 47 34 07	35 44 13 18 80
33 18 51 62 32	41 94 15 09 49	89 43 54 85 81	88 69 54 19 94	37 54 87 30 43
80 95 10 04 06	96 38 27 07 74	20 15 12 33 87	25 01 62 52 98	94 62 46 11 71
79 75 24 91 40	71 96 12 82 96	69 86 10 25 91	74 85 22 05 39	00 38 75 95 79
18 63 33 25 37	98 14 50 65 71	31 01 02 46 74	05 45 56 14 27	77 93 89 19 36
74 02 94 39 02	77 55 73 22 70	97 79 01 71 19	52 52 75 80 21	80 81 45 17 48
54 17 84 56 11	80 99 33 71 43	05 33 51 29 69	56 12 71 92 55	36 04 09 03 24
11 66 44 98 83	52 07 98 48 27	59 38 17 15 39	09 97 33 34 40	88 46 12 33 56
48 32 47 79 28	31 24 96 47 10	02 29 53 68 70	32 30 75 75 46	15 02 00 99 94
69 07 49 41 38	87 63 79 19 76	35 58 40 44 01	10 51 82 16 15	01 84 87 69 38

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Table 9-2. Random numbers (continued)

09 18 82 00 97	32 82 53 95 27	04 22 08 63 04	83 38 98 73 74	64 27 85 80 44
90 04 58 54 97	51 98 15 06 54	94 93 88 19 97	91 87 07 61 50	68 47 66 46 59
73 18 95 02 07	47 67 72 62 69	62 29 06 44 64	27 12 46 70 18	41 36 18 27 60
75 76 87 64 90	20 97 18 17 49	90 42 91 22 72	95 37 50 58 71	93 82 34 31 78
54 01 64 40 56	66 28 13 10 03	00 68 22 73 98	20 71 45 32 95	07 70 61 78 13
08 35 86 99 10	78 54 24 27 85	13 66 15 88 73	04 61 89 75 53	31 22 30 84 20
28 30 60 32 64	81 33 31 05 91	40 51 00 78 93	32 60 46 04 75	94 11 90 18 40
53 84 08 62 33	81 59 41 36 28	51 21 59 02 90	28 46 66 87 95	77 76 22 07 91
91 75 75 37 41	61 61 36 22 69	50 26 39 02 12	55 78 17 65 14	83 48 34 70 55
89 41 59 26 94	00 39 75 83 91	12 60 71 76 46	48 94 97 23 06	94 54 13 74 08
77 51 30 38 20	86 83 42 99 01	68 41 48 27 74	51 90 81 39 80	72 89 35 55 07
19 50 23 71 74	69 97 92 02 88	55 21 02 97 73	74 28 77 52 51	65 34 46 74 15
21 81 85 93 13	93 27 88 17 57	05 68 67 31 56	07 08 28 50 46	31 85 33 84 52
51 47 46 64 99	68 10 72 36 21	94 04 99 13 45	42 83 60 91 91	08 00 74 54 49
99 55 96 83 31	62 53 52 41 70	69 77 71 28 30	74 81 97 81 42	43 86 07 28 34
33 71 34 80 07	93 58 47 28 69	51 92 66 47 21	58 30 32 98 22	93 17 49 39 72
85 27 48 68 93	11 30 32 92 70	28 83 43 41 37	73 51 59 04 00	71 14 84 36 43
84 13 38 96 40	44 03 55 21 66	73 85 27 00 91	61 22 26 05 61	62 32 71 84 23
56 73 21 62 34	17 39 59 61 31	10 12 39 16 22	85 49 65 75 60	81 60 41 88 80
65 13 85 68 06	87 64 88 52 61	34 31 36 58 61	45 87 52 10 69	85 64 44 72 77
38 00 10 21 76	81 71 91 17 11	71 60 29 29 37	74 21 96 40 49	65 58 44 96 98
37 40 29 63 97	01 30 47 75 86	56 27 11 00 86	47 32 46 26 05	40 03 03 74 38
97 12 54 03 48	87 08 33 14 17	21 81 53 92 50	75 23 76 20 47	15 50 12 95 78
21 82 64 11 34	47 14 33 40 72	64 63 88 59 02	49 13 90 64 41	03 85 65 45 52
73 13 54 27 42	95 71 90 90 35	85 79 47 42 96	08 78 98 81 56	64 69 11 92 02
07 63 87 79 29	03 06 11 80 72	96 20 74 41 56	23 82 19 95 38	04 71 36 69 94
60 52 88 34 41	07 95 41 98 14	59 17 52 06 95	05 53 35 21 39	61 21 20 64 55
83 59 63 56 55	06 95 89 29 83	05 12 80 97 19	77 43 35 37 83	92 30 15 04 98
10 85 06 27 46	99 59 91 05 07	13 49 90 63 19	53 07 57 18 39	06 41 01 93 62
39 82 09 89 52	43 62 26 31 47	64 42 18 08 14	43 80 00 93 51	31 02 47 31 67
59 58 00 64 78	75 56 97 88 00	88 83 55 44 86	23 76 80 61 56	04 11 10 84 08
38 50 80 73 41	23 79 34 87 63	90 82 29 70 22	17 71 90 42 07	95 95 44 99 53
30 69 27 06 68	94 68 81 61 27	56 19 68 00 91	82 06 76 34 00	05 46 26 92 00
65 44 39 56 59	18 28 82 74 37	49 63 22 40 41	08 33 76 56 76	96 29 99 08 36
27 26 75 02 64	13 19 27 22 94	07 47 74 46 06	17 98 54 89 11	97 34 13 03 58
91 30 70 69 91	19 07 22 42 10	36 69 95 37 28	28 82 53 57 93	28 97 66 62 52
68 43 49 46 88	84 47 31 36 22	62 12 69 84 08	12 84 38 25 90	09 81 59 31 46
48 90 81 58 77	54 74 52 45 91	35 70 00 47 54	83 82 45 26 92	54 13 05 51 60
06 91 34 51 97	42 67 27 86 01	11 88 30 95 28	63 01 19 89 01	14 97 44 03 44
10 45 51 60 19	14 21 03 37 12	91 34 23 78 21	88 32 58 08 51	43 66 77 08 83
12 88 39 73 43	65 02 76 11 84	04 28 50 13 92	17 97 41 50 77	90 71 22 67 69
21 77 83 09 76	38 80 73 69 61	31 64 94 20 96	63 28 10 20 23	08 81 64 74 49
19 52 35 95 15	65 12 25 96 59	86 28 36 82 58	69 57 21 37 98	16 43 59 15 29
67 24 55 26 70	35 58 31 65 63	79 24 68 66 86	76 46 33 42 22	26 65 59 08 02
60 58 44 73 77	07 50 03 79 92	45 13 42 65 29	26 76 08 36 37	41 32 64 43 44
53 85 34 13 77	36 06 69 48 50	58 83 87 38 59	49 36 47 33 31	96 24 04 36 42
24 63 73 87 36	74 38 48 93 42	52 62 30 79 92	12 36 91 86 01	03 74 28 38 73
83 08 01 24 51	38 99 22 28 15	07 75 95 17 77	97 37 72 75 85	51 97 23 78 67
16 44 42 43 34	36 15 19 90 73	27 49 37 09 39	85 13 03 25 52	54 84 65 47 59
60 79 01 81 57	57 17 86 57 62	11 16 17 85 76	45 81 95 29 79	65 13 00 48 60

Table 9-2. Random numbers (continued)

03 99 11 04 61	93 71 61 68 94	66 08 32 46 53	84 60 95 82 32	88 61 81 91 61
38 55 59 55 54	32 88 65 97 80	08 35 56 08 60	29 73 54 77 62	71 29 92 38 53
17 54 67 37 04	92 05 24 62 15	55 12 12 92 81	59 07 60 79 36	27 95 45 89 09
32 64 35 28 61	95 81 90 68 31	00 91 19 89 36	76 35 59 37 79	80 86 30 05 14
69 57 26 87 77	39 51 03 59 05	14 06 04 06 19	29 54 96 96 16	33 56 46 07 80
24 12 26 65 91	27 69 90 64 94	14 84 54 66 72	61 95 87 71 00	90 89 97 57 54
61 19 63 02 31	92 96 26 17 73	41 83 95 53 82	17 26 77 09 43	78 03 87 02 67
30 53 22 17 04	10 27 41 22 02	39 68 52 33 09	10 06 16 88 29	55 98 66 64 85
03 78 89 75 99	75 86 72 07 17	74 41 65 31 66	35 20 83 33 74	87 53 90 88 23
48 22 86 33 79	85 78 34 76 19	53 15 26 74 33	35 66 35 29 72	16 81 86 03 11
60 36 59 46 53	35 07 53 39 49	42 61 42 92 97	01 91 82 83 16	98 95 37 32 31
83 79 94 24 02	56 62 33 44 42	34 99 44 13 74	70 07 11 47 36	09 95 81 80 65
32 96 00 74 05	36 40 98 32 32	99 38 54 16 00	11 13 30 75 86	15 91 70 62 53
19 32 25 38 45	57 62 05 26 06	66 49 76 86 46	78 13 86 65 59	19 64 09 94 13
11 22 09 47 47	07 39 93 74 08	48 50 92 39 29	27 48 24 54 76	85 24 43 51 59
31 75 15 72 60	68 98 00 53 39	15 47 04 83 55	88 65 12 25 96	03 15 21 91 21
88 49 29 93 82	14 45 40 45 04	20 09 49 89 77	74 84 39 34 13	22 10 97 85 08
30 93 44 77 44	07 48 18 38 28	73 78 80 65 33	28 59 72 04 05	94 20 52 03 80
22 88 84 88 93	27 49 99 87 48	60 53 04 51 28	74 02 28 46 17	82 03 71 02 68
78 21 21 69 93	35 90 29 13 86	44 37 21 54 86	65 74 11 40 14	87 48 13 72 20
41 84 98 45 47	46 85 05 23 26	34 67 75 83 00	74 91 06 43 45	19 32 58 15 49
46 35 23 30 49	69 24 89 34 60	45 30 50 75 21	61 31 83 18 55	14 41 37 09 51
11 08 79 62 94	14 01 33 17 92	59 74 76 72 77	76 50 33 45 13	39 66 37 75 44
52 70 10 83 37	56 30 38 73 15	16 52 06 96 76	11 65 49 98 93	02 18 16 81 61
57 27 53 68 98	81 30 44 85 85	68 65 22 73 76	92 85 25 58 66	88 44 80 35 84
20 85 77 31 56	70 28 42 43 26	79 37 59 52 20	01 15 96 32 67	10 62 24 83 91
15 63 38 49 24	90 41 59 36 14	33 52 12 66 65	55 82 34 76 41	86 22 53 17 04
92 69 44 82 97	39 90 40 21 15	59 58 94 90 67	66 82 14 15 75	49 76 70 40 37
77 61 31 90 19	88 15 20 00 80	20 55 49 14 09	96 27 74 82 57	50 81 69 76 16
38 68 83 24 86	45 13 46 35 45	59 40 47 20 59	43 94 75 16 80	43 85 25 96 93
25 16 30 18 89	70 01 41 50 21	41 29 06 73 12	71 85 71 59 57	68 97 11 14 03
65 25 10 76 29	37 23 93 32 95	05 87 00 11 19	92 78 42 63 40	18 47 76 56 22
36 81 54 36 25	18 63 73 75 09	82 44 49 90 05	04 92 17 37 01	14 70 79 39 97
64 39 71 16 92	05 32 78 21 62	20 24 78 17 59	45 19 72 53 32	83 74 52 25 67
04 51 52 56 24	95 09 66 79 46	48 46 08 55 58	15 19 11 87 82	16 93 03 33 61
83 76 16 08 73	43 25 38 41 45	60 83 32 59 83	01 29 14 13 49	20 36 80 71 26
14 38 70 63 45	80 85 40 92 79	43 52 90 63 18	38 38 47 47 61	41 19 63 74 80
51 32 19 22 46	80 08 87 70 74	88 72 25 67 36	66 16 44 94 31	66 91 93 16 78
72 47 20 00 08	80 89 01 80 02	94 81 33 19 00	54 15 58 34 36	35 35 25 41 31
05 46 65 53 06	93 12 81 84 64	74 45 79 05 61	72 84 81 18 34	79 98 26 84 16
39 52 87 24 84	82 47 42 55 93	48 54 53 52 47	18 61 91 36 74	18 61 11 92 41
81 61 61 87 11	53 34 24 42 76	75 12 21 17 24	74 62 77 37 07	58 31 91 59 97
07 58 61 61 20	82 64 12 28 20	92 90 41 31 41	32 39 21 97 63	61 19 96 79 40
90 76 70 42 35	13 57 41 72 00	69 90 26 37 42	78 46 42 25 01	18 62 79 08 72
40 18 82 81 93	29 59 38 86 27	94 97 21 15 98	62 09 53 67 87	00 44 15 89 97
34 41 48 21 57	86 88 75 50 87	19 15 20 00 23	12 30 28 07 83	32 62 46 86 91
63 43 97 53 63	44 98 91 68 22	36 02 40 08 67	76 37 84 16 05	65 96 17 34 88
67 04 90 90 70	93 39 94 55 47	94 45 87 42 84	05 04 14 98 07	20 28 83 40 60
79 49 50 41 46	52 16 29 02 86	54 15 83 42 43	46 97 83 54 82	59 36 29 59 38
91 70 43 05 52	04 73 72 10 31	75 05 19 30 29	47 66 56 43 82	99 78 29 34 78

Table 9-2. Random numbers (continued)

94 01 54 68 74	32 44 44 82 77	59 82 09 61 63	64 65 42 58 43	41 14 54 28 20
74 10 88 82 22	88 57 07 40 15	25 70 49 10 35	01 75 51 47 50	48 96 83 86 03
62 88 08 78 73	95 16 05 92 21	22 30 49 03 14	72 87 71 73 34	39 28 30 41 49
11 74 81 21 02	80 58 04 18 67	17 71 05 96 21	06 55 40 78 50	73 95 07 95 52
17 94 40 56 00	60 47 80 33 43	25 85 25 89 05	57 21 63 96 18	49 85 69 93 26
66 06 74 27 92	95 04 35 26 80	46 78 05 64 87	09 97 15 94 81	37 00 62 21 86
54 24 49 10 30	45 54 77 08 18	59 84 99 61 69	61 45 92 16 47	87 41 71 71 98
30 94 55 75 89	31 73 25 72 60	47 67 00 76 54	46 37 62 53 66	94 74 64 95 80
69 17 03 74 03	86 99 59 03 07	94 30 47 18 03	26 82 50 55 11	12 45 99 13 14
08 34 58 89 75	35 84 18 57 71	08 10 55 99 87	87 11 22 14 76	14 71 37 11 81
27 76 74 35 84	85 30 18 89 77	29 49 06 97 14	73 03 54 12 07	74 69 90 93 10
13 02 51 43 38	54 06 61 52 43	47 72 46 67 33	47 43 14 39 05	31 04 85 66 99
80 21 73 62 92	98 52 52 43 35	24 43 22 48 96	43 27 75 88 74	11 46 61 60 82
10 87 56 20 04	90 39 16 11 05	57 41 10 63 68	53 85 63 07 43	08 67 08 47 41
54 12 75 73 26	26 62 91 90 87	24 47 28 87 79	30 54 02 78 86	61 73 27 54 54
60 31 14 28 24	37 30 14 26 78	45 99 04 32 42	17 37 45 20 03	70 70 77 02 14
49 73 97 14 84	92 00 39 80 86	76 66 87 32 09	59 20 21 19 73	02 90 23 32 50
78 62 65 15 94	16 45 39 46 14	39 01 49 70 66	83 01 20 98 32	25 57 17 76 28
66 69 21 39 86	99 83 70 05 82	81 23 24 49 87	09 50 49 64 12	90 19 37 95 68
44 07 12 80 91	07 36 29 77 03	76 44 74 25 37	98 52 49 78 31	65 70 40 95 14
41 46 88 51 49	49 55 41 79 94	14 92 43 96 50	95 29 40 05 56	70 48 10 69 05
94 55 93 75 59	49 67 85 31 19	70 31 20 56 82	66 98 63 40 99	74 47 42 07 40
41 61 57 03 60	64 11 45 86 60	90 85 06 46 18	80 62 05 17 90	11 43 63 80 72
50 27 39 31 13	41 79 48 68 61	24 78 18 96 83	55 41 18 56 67	77 53 59 98 92
41 39 68 05 04	90 67 00 82 89	40 90 20 50 69	95 08 30 67 83	28 10 25 78 16
25 80 72 42 60	71 52 97 89 20	72 68 20 73 85	90 72 65 71 66	98 88 40 85 83
06 17 09 79 65	88 30 29 80 41	21 44 34 18 08	68 98 48 36 20	89 74 79 88 82
60 80 85 44 44	74 41 28 11 05	01 17 62 88 38	36 42 11 64 89	18 05 95 10 61
80 94 04 48 93	10 40 83 62 22	80 58 27 19 44	92 63 84 03 33	67 05 41 60 67
19 51 69 01 20	46 75 97 16 43	13 17 75 52 92	21 03 68 28 08	77 50 19 74 27
49 38 65 44 80	23 60 42 35 54	21 78 54 11 01	91 17 81 01 74	29 42 09 04 38
06 31 28 89 40	15 99 56 93 21	47 45 86 48 09	98 18 98 18 51	29 65 18 42 15
60 94 20 03 07	11 89 79 26 74	40 40 56 80 32	96 71 75 42 44	10 70 14 13 93
92 32 99 89 32	78 28 44 63 47	71 20 99 20 61	39 44 89 31 36	25 72 20 85 64
77 93 66 35 74	31 38 45 19 24	85 56 12 96 71	58 13 71 78 20	22 75 13 65 18
38 10 17 77 56	11 65 71 38 97	95 88 95 70 67	47 64 81 38 85	70 66 99 34 06
39 64 16 94 57	91 33 92 25 02	92 61 38 97 19	11 94 75 62 03	19 32 42 05 04
84 05 44 04 55	99 39 66 36 80	67 66 76 06 31	69 18 19 68 45	38 52 51 16 00
47 46 80 35 77	57 64 96 32 66	24 70 07 15 94	14 00 42 31 53	69 24 90 57 47
43 32 13 13 70	28 97 72 38 96	76 47 96 85 62	62 34 20 75 89	08 89 90 59 85
64 28 16 18 26	18 55 56 49 37	13 17 33 33 65	78 85 11 64 99	87 06 41 30 75
66 84 77 04 95	32 35 00 29 85	86 71 63 87 46	26 31 37 74 63	55 38 77 26 81
72 46 13 32 30	21 52 95 34 24	92 58 10 22 62	78 43 86 62 76	18 39 67 35 38
21 03 29 10 50	13 05 81 62 18	12 47 05 65 00	15 29 27 61 39	59 52 65 21 13
95 36 26 70 11	06 65 11 61 36	01 01 60 08 57	55 01 85 63 74	35 82 47 17 08
40 71 29 73 80	10 40 45 54 52	34 03 06 07 26	75 21 11 02 71	36 63 36 84 24
58 27 56 17 64	97 58 65 47 16	50 25 94 63 45	87 19 54 60 92	26 78 76 09 39
89 51 41 17 88	68 22 42 34 17	73 95 97 61 45	30 34 24 02 77	11 04 97 20 49
15 47 25 06 69	48 13 93 67 32	46 87 43 70 88	73 46 50 98 19	58 86 93 52 20
12 12 08 61 24	51 24 74 43 02	60 88 35 21 09	21 43 73 67 86	49 22 67 78 37

Table 9-2. Random numbers (continued)

19 61 27 84 30	11 66 19 47 70	77 60 36 56 69	86 86 81 26 65	30 01 27 59 89
39 14 17 74 00	28 00 06 42 38	73 25 87 17 94	31 34 02 62 56	66 45 33 70 16
64 75 68 04 57	08 74 71 28 36	03 46 95 06 78	03 27 44 34 23	66 67 78 25 56
92 90 15 18 78	56 44 12 29 98	29 71 83 84 47	06 45 32 53 11	07 56 55 37 71
03 55 19 00 70	09 48 39 40 50	45 93 81 81 35	36 90 84 33 21	11 07 35 18 03
98 88 46 62 09	06 83 05 36 56	14 66 35 63 46	71 43 00 49 09	19 81 80 57 07
27 36 98 68 82	53 47 30 75 41	53 63 37 08 63	03 74 81 28 22	19 36 04 90 88
59 06 67 59 74	63 33 52 04 83	43 51 43 74 81	58 27 82 69 67	49 32 54 39 51
91 64 79 37 83	64 16 94 90 22	98 58 80 94 95	49 82 95 90 68	38 83 10 48 38
83 60 59 24 19	39 54 20 77 72	71 56 87 56 73	35 18 58 97 59	44 90 17 42 91
24 89 58 85 30	70 77 43 54 39	46 75 87 04 72	70 20 79 26 75	91 62 36 12 75
35 72 02 65 56	95 59 62 00 94	73 75 08 57 88	34 26 40 17 03	46 83 36 52 48
14 14 15 34 10	38 64 90 63 43	57 25 66 13 42	72 70 97 53 18	90 37 93 75 62
27 41 67 56 70	92 17 67 25 35	93 11 95 60 77	06 88 61 82 44	92 34 43 13 74
82 07 10 74 29	81 00 74 77 49	40 74 45 69 74	23 33 68 88 21	53 84 11 05 36
21 44 58 27 93	24 83 19 32 41	14 19 97 62 68	70 88 36 80 02	03 82 91 74 43
72 51 37 64 00	52 22 59 23 48	62 30 89 84 81	29 74 43 31 65	33 14 16 10 20
71 47 94 50 27	76 16 05 74 11	13 78 01 36 32	52 30 87 77 62	88 87 43 36 97
83 21 05 14 66	09 08 85 03 95	26 74 30 53 06	21 70 67 00 01	99 43 98 07 67
68 74 99 51 48	94 89 77 86 36	96 75 00 90 24	94 53 89 11 43	96 69 36 18 86
05 18 47 57 63	47 07 58 81 58	05 31 35 34 39	14 90 80 88 30	60 09 62 15 51
13 65 16 25 46	96 89 22 52 40	47 51 15 84 83	87 34 27 88 18	07 85 53 92 69
00 56 62 12 20	00 29 22 40 69	25 07 22 95 19	52 54 85 40 91	21 28 22 12 96
50 95 81 76 95	58 07 26 89 90	60 32 99 59 55	71 58 66 34 17	35 94 76 78 07
57 62 16 45 47	46 85 03 79 81	38 52 70 90 37	64 75 60 33 24	04 98 68 36 66
09 28 22 58 44	79 13 97 84 35	35 42 84 35 61	69 79 96 33 14	12 99 19 35 16
23 39 49 42 06	93 43 23 78 36	94 91 92 68 46	02 55 57 44 10	94 91 54 81 99
05 28 03 74 70	93 62 20 43 45	15 09 21 95 10	18 09 41 66 13	78 23 45 00 01
95 49 19 79 76	38 30 63 21 92	82 63 95 46 24	72 43 49 26 06	23 19 17 46 93
78 52 10 01 04	18 24 87 55 83	90 32 65 07 85	54 03 46 62 51	35 77 41 46 92
96 34 54 45 79	85 93 24 40 53	75 70 42 08 40	86 58 38 39 44	52 45 67 37 66
77 96 33 11 51	32 36 49 16 91	47 35 74 03 38	23 43 52 40 65	08 45 89 53 66
07 52 01 12 94	23 23 80 17 48	41 69 06 73 28	54 81 43 77 77	10 05 74 23 32
38 42 30 23 09	70 70 38 57 36	46 14 81 42 58	29 23 61 21 52	05 08 86 58 25
02 46 36 55 33	21 19 96 05 55	33 92 80 18 17	07 39 68 92 15	30 72 22 21 02
15 88 09 22 61	17 29 28 81 90	61 78 14 88 98	92 52 52 12 83	88 58 16 00 98
71 92 60 08 19	59 14 40 02 24	30 57 09 01 94	18 32 90 69 99	26 85 71 92 38
64 42 52 81 08	16 55 41 60 16	00 04 28 32 29	10 33 33 61 68	65 61 79 48 34
79 78 22 39 24	49 44 03 04 32	81 07 73 15 43	95 21 66 48 65	13 65 85 10 81
35 33 77 45 38	44 55 36 46 72	90 96 04 18 49	93 86 54 46 08	93 17 63 48 51
05 24 92 93 29	19 71 59 40 82	14 73 88 66 67	43 70 86 63 54	93 69 22 55 27
56 46 39 93 80	38 79 38 57 74	19 05 61 39 39	46 06 22 76 47	66 14 66 32 10
96 29 63 31 21	54 19 63 41 08	75 81 48 59 86	71 17 11 51 02	28 99 26 31 65
98 38 03 62 69	60 01 40 72 01	62 44 84 63 85	42 17 58 83 50	46 18 24 91 26
52 56 76 43 50	16 31 55 39 69	80 39 58 11 14	54 35 86 45 78	47 26 91 57 47
78 49 89 08 30	25 95 59 92 36	43 28 69 10 64	99 96 99 51 44	64 42 47 73 77
49 55 32 42 41	08 15 08 95 35	08 70 39 10 41	77 32 38 10 79	45 12 79 36 86
32 15 10 70 75	83 15 51 02 52	73 10 08 86 18	23 89 18 74 18	45 41 72 02 68
11 31 45 03 63	26 86 02 77 99	49 41 68 35 34	19 18 70 80 59	76 67 70 21 10
12 36 47 12 10	87 05 25 02 41	90 78 59 78 89	81 39 95 81 30	64 43 90 56 14

Table 9-3. Random normal numbers, $\mu = 0$, $\sigma = 1$*

01	02	03	04	05	06	07	08	09	10
0.464	0.137	2.455	−0.323	−0.068	0.296	−0.288	1.298	0.241	−0.957
0.060	−2.526	−0.531	−0.194	0.543	−1.558	0.187	−1.190	0.022	0.525
1.486	−0.354	−0.634	0.697	0.926	1.375	0.785	−0.963	−0.853	−1.865
1.022	−0.472	1.279	3.521	0.571	−1.851	0.194	1.192	−0.501	−0.273
1.394	−0.555	0.046	0.321	2.945	1.974	−0.258	0.412	0.439	−0.035
0.906	−0.513	−0.525	0 595	0.881	−0.934	1.579	0.161	−1.885	0.371
1.179	−1.055	0.007	0.769	0.971	0.712	1.090	−0.631	−0.255	−0.702
−1.501	−0.488	−0.162	−0.136	1.033	0.203	0.448	0.748	−0.423	−0.432
−0.690	0.756	−1.618	−0.345	−0.511	−2.051	−0.457	−0.218	0.857	−0.465
1.372	0.225	0.378	0.761	0.181	−0.736	0.960	−1.530	−0.260	0.120
−0.482	1.678	−0.057	−1.229	−0.486	0.856	−0.491	−1.983	−2.830	−0.238
−1.376	−0.150	1.356	−0.561	−0.256	−0.212	0.219	0.779	0.953	−0.869
−1.010	0.598	−0.918	1.598	0.065	0.415	−0.169	0.313	−0.973	−1.016
−0.005	−0.899	0.012	−0.725	1.147	−0.121	1.096	0.481	−1.691	0.417
1.393	−1.163	−0.911	1.231	−0.199	−0.246	1.239	−2.574	−0.558	0.056
−1.787	−0.261	1.237	1.046	−0.508	−1.630	−0.146	−0.392	−0.627	0.561
−0.105	−0.357	−1.384	0.360	−0.992	−0.116	−1.698	−2.832	−1.108	−2.357
−1.339	1.827	−0.959	0.424	0.969	−1.141	−1.041	0.362	−1.726	1.956
1.041	0.535	0.731	1.377	0.983	−1.330	1.620	−1.040	0.524	−0.281
0.279	−2.056	0.717	−0.873	−1.096	−1.396	1.047	0.089	−0.573	0.932
−1.805	−2.008	−1.633	0.542	0.250	−0.166	0.032	0.079	0.471	−1.029
−1.186	1.180	1.114	0.882	1.265	−0.202	0.151	−0.376	−0.310	0.479
0.658	−1.141	1.151	−1.210	−0.927	0.425	0.290	−0.902	0.610	2.709
−0.439	0.358	−1.939	0.891	−0.227	0.602	0.873	−0.437	−0.220	−0.057
−1.399	−0.230	0.385	−0.649	−0.577	0.237	−0.289	0.513	0.738	−0.300
0.199	0.208	−1.083	−0.219	−0.291	1.221	1.119	0.004	−2.015	−0.594
0.159	0.272	−0.313	0.084	−2.828	−0.439	−0.792	−1.275	−0.623	−1.047
2.273	0.606	0.606	−0.747	0.247	1.291	0.063	−1.793	−0.699	−1.347
0.041	−0.307	0.121	0.790	−0.584	0.541	0.484	−0.986	0.481	0.996
−1.132	−2.098	0.921	0.145	0.446	−1.661	1.045	−1.363	−0.586	−1.023
0.768	0.079	−1.473	0.034	−2.127	0.665	0.084	−0.880	−0.579	0.551
0.375	−1.658	−0.851	0.234	−0.656	0.340	−0.086	−0.158	−0.120	0.418
−0.513	−0.344	0.210	−0.736	1.041	0.008	0.427	−0.831	0.191	0.074
0.292	−0.521	1.266	−1.206	−0.899	0.110	−0.528	−0.813	0.071	0.524
1.026	2.990	−0.574	−0.491	−1.114	1.297	−1.433	−1.345	−3.001	0.479
−1.334	1.278	−0.568	−0.109	−0.515	−0.566	2.923	0.500	0.359	0.326
−0.287	−0.144	−0.254	0.574	−0.451	−1.181	−1.190	−0.318	−0.094	1.114
0.161	−0.886	−0.921	−0.509	1.410	−0.518	0.192	−0.432	1.501	1.068
−1.346	0.193	−1.202	0.394	−1.045	0.843	0.942	1.045	0.031	0.772
1.250	−0.199	−0.288	1.810	1.378	0.584	1.216	0.733	0.402	0.226
0.630	−0.537	0.782	0.060	0.499	−0.431	1.705	1.164	0.884	−0.298
0.375	−1.941	0.247	−0.491	0.665	−0.135	−0.145	−0.498	0.457	1.064
−1.420	0.489	−1.711	−1.186	0.754	−0.732	−0.066	1.006	−0.798	0.162
−0.151	−0.243	−0.430	−0.762	0.298	1.049	1.810	2.885	−0.768	−0.129
−0.309	0.531	0.416	−1.541	1.456	2.040	−0.124	0.196	0.023	−1.204
0.424	−0.444	0.593	0.993	−0.106	0.116	0.484	−1.272	1.066	1.097
0.593	0.658	−1.127	−1.407	−1.579	−1.616	1.458	1.262	0.736	−0.916
0.862	−0.885	−0.142	−0.504	0.532	1.381	0.022	−0.281	−0.342	1.222
0.235	−0.628	−0.023	−0.463	−0.899	−0.394	−0.538	1.707	−0.188	−1.153
−0.853	0.402	0.777	0.833	0.410	−0.349	−1.094	0.580	1.395	1.298

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Table 9-3. Random normal numbers, $\mu = 0$, $\sigma = 1$ (continued)

11	12	13	14	15	16	17	18	19	20
−1.329	−0.238	−0.838	−0.988	−0.445	0.964	−0.266	−0.322	−1.726	2.252
1.284	−0.229	1.058	0.090	0.050	0.523	0.016	0.277	1.639	0.554
0.619	0.628	0.005	0.973	−0.058	0.150	−0.635	−0.917	0.313	−1.203
0.699	−0.269	0.722	−0.994	−0.807	−1.203	1.163	1.244	1.306	−1.210
0.101	0.202	−0.150	0.731	0.420	0.116	−0.496	−0.037	−2.466	0.794
−1.381	0.301	0.522	0.233	0.791	−1.017	−0.182	0.926	−1.096	1.001
−0.574	1.366	−1.843	0.746	0.890	0.824	−1.249	−0.806	−0.240	0.217
0.096	0.210	1.091	0.990	0.900	−0.837	−1.097	−1.238	0.030	−0.311
1.389	−0.236	0.094	3.282	0.295	−0.416	0.313	0.720	0.007	0.354
1.249	0.706	1.453	0.366	−2.654	−1.400	0.212	0.307	−1.145	0.639
0.756	−0.397	−1.772	−0.257	1.120	1.188	−0.527	0.709	0.479	0.317
−0.860	0.412	−0.327	0.178	0.524	−0.672	−0.831	0.758	0.131	0.771
−0.778	−0.979	0.236	−1.033	1.497	−0.661	0.906	1.169	−1.582	1.303
0.037	0.062	0.426	1.220	0.471	0.784	−0.719	0.465	1.559	−1.326
2.619	−0.440	0.477	1.063	0.320	1.406	−0.701	−0.128	0.518	−0.676
−0.420	−0.287	−0.050	−0.481	1.521	−1.367	0.609	0.292	0.048	0.592
1.048	0.220	1.121	−1.789	−1.211	−0.871	−0.740	0.513	−0.558	−0.395
1.000	−0.638	1.261	0.510	−0.150	0.034	0.054	−0.055	0.639	−0.825
0.170	−1.131	−0.985	0.102	−0.939	−1.457	1.766	1.087	−1.275	2.362
0.389	−0.435	0.171	0.891	1.158	1.041	1.048	−0.324	−0.404	1.060
−0.305	0.838	−2.019	−0.540	0.905	1.195	−1.190	0.106	0.571	0.298
−0.321	−0.039	1.799	−1.032	−2.225	−0.148	0.758	−0.862	0.158	−0.726
1.900	1.572	−0.244	−1.721	1.130	0.495	−0.484	0.014	−0.778	−1.483
−0.778	−0.288	−0.224	−1.324	−0.072	0.890	−0.410	0.752	0.376	−0.224
0.617	−1.718	−0.183	−0.100	1.719	0.696	−1.339	−0.614	1.071	−0.386
−1.430	−0.953	0.770	−0.007	−1.872	1.075	−0.913	−1.168	1.775	0.238
0.267	−0.048	0.972	0.734	−1.408	−1.955	−0.848	2.002	0.232	−1.273
0.978	−0.520	−0.368	1.690	−1.479	0.985	1.475	−0.098	−1.633	2.399
−1.235	−1.168	0.325	1.421	2.652	−0.486	−1.253	0.270	−1.103	0.118
−0.258	0.638	2.309	0.741	−0.161	−0.679	0.336	1.973	0.370	−2.277
0.243	0.629	−1.516	−0.157	0.693	1.710	0.800	−0.265	1.218	0.655
−0.292	−1.455	−1.451	1.492	−0.713	0.821	−0.031	−0.780	1.330	0.977
−0.505	0.389	0.544	−0.042	1.615	−1.440	−0.989	−0.580	0.156	0.052
0.397	−0.287	1.712	0.289	−0.904	0.259	−0.600	−1.635	−0.009	−0.799
−0.605	−0.470	0.007	0.721	−1.117	0.635	0.592	−1.362	−1.441	0.672
1.360	0.182	−1.476	−0.599	−0.875	0.292	−0.700	0.058	−0.340	−0.639
0.480	−0.699	1.615	−0.225	1.014	−1.370	−1.097	0.294	0.309	−1.389
−0.027	−0.487	−1.000	−0.015	0.119	−1.990	−0.687	−1.964	−0.366	1.759
−1.482	−0.815	−0.121	1.884	−0.185	0.601	0.793	0.430	−1.181	0.426
−1.256	−0.567	−0.994	1.011	−1.071	−0.623	−0.420	−0.309	1.362	0.863
−1.132	2.039	1.934	−0.222	0.386	1.100	0.284	1.597	−1.718	−0.560
−0.780	−0.239	−0.497	−0.434	−0.284	−0.241	−0.333	1.348	−0.478	−0.169
−0.859	−0.215	0.241	1.471	0.389	−0.952	0.245	0.781	1.093	−0.240
0.447	1.479	0.067	0.426	−0.370	−0.675	−0.972	0.225	0.815	0.389
0.269	0.735	−0.066	−0.271	−1.439	1.036	−0.306	−1.439	−0.122	−0.336
0.097	−1.883	−0.218	0.202	−0.357	0.019	1.631	1.400	0.223	−0.793
−0.686	1.596	−0.286	0.722	0.655	−0.275	1.245	−1.504	0.066	−1.280
0.957	0.057	−1.153	0.701	−0.280	1.747	−0.745	1.338	−1.421	0.386
−0.976	−1.789	−0.696	−1.799	−0.354	0.071	2.355	0.135	−0.598	1.883
0.274	0.226	−0.909	−0.572	0.181	1.115	0.406	0.453	−1.218	−0.115

Table 9-3. Random normal numbers, $\mu = 0$, $\sigma = 1$ (continued)

21	22	23	24	25	26	27	28	29	30
−1.752	−0.329	−1.256	0.318	1.531	0.349	−0.958	−0.059	0.415	−1.084
−0.291	0.085	1.701	−1.087	−0.443	−0.292	0.248	−0.539	−1.382	0.318
−0.933	0.130	0.634	0.899	1.409	−0.883	−0.095	0.229	0.129	0.367
−0.450	−0.244	0.072	1.028	1.730	−0.056	−1.488	−0.078	−2.361	−0.992
0.512	−0.882	0.490	−1.304	−0.266	0.757	−0.361	0.194	−1.078	0.529
−0.702	0.472	0.429	−0.664	−0.592	1.443	−1.515	−1.209	−1.043	0.278
0.284	0.039	−0.518	1.351	1.473	0.889	0.300	0.339	−0.206	1.392
−0.509	1.420	−0.782	−0.429	−1.266	0.627	−1.165	0.819	−0.261	0.409
−1.776	−1.033	1.977	0.014	0.702	−0.435	−0.816	1.131	0.656	0.061
−0.044	1.807	0.342	−2.510	1.071	−1.220	−0.060	−0.764	0.079	−0.964
0.263	−0.578	1.612	−0.148	−0.383	−1.007	−0.414	0.638	−0.186	0.507
0.986	0.439	−0.192	−0.132	0.167	0.883	−0.400	−1.440	−0.385	−1.414
−0.441	−0.852	−1.446	−0.605	−0.348	1.018	0.963	−0.004	2.504	−0.847
−0.866	0.489	0.097	0.379	0.192	−0.842	0.065	1.420	0.426	−1.191
−1.215	0.675	1.621	0.394	−1.447	2.199	−0.321	−0.540	−0.037	0.185
−0.475	−1.210	0.183	0.526	0.495	1.297	−1.613	1.241	−1.016	−0.090
1.200	0.131	2.502	0.344	−1.060	−0.909	−1.695	−0.666	−0.838	−0.866
−0.498	−1.202	−0.057	−1.354	−1.441	−1.590	0.987	0.441	0.637	−1.116
−0.743	0.894	−0.028	1.119	−0.598	0.279	2.241	0.830	0.267	−0.156
0.779	−0.780	−0.954	0.705	−0.361	−0.734	1.365	1.297	−0.142	−1.387
−0.206	−0.195	1.017	−1.167	−0.079	−0.452	0.058	−1.068	−0.394	−0.406
−0.092	−0.927	−0.439	0.256	0.503	0.338	1.511	−0.465	−0.118	−0.454
−1.222	−1.582	1.786	−0.517	−1.080	−0.409	−0.474	−1.890	0.247	0.575
0.068	0.075	−1.383	−0.084	0.159	1.276	1.141	0.186	−0.973	−0.266
0.183	1.600	−0.335	1.553	0.889	0.896	−0.035	0.461	0.486	1.246
−0.811	−2.904	0.618	0.588	0.533	0.803	−0.696	0.690	0.820	0.557
−1.010	1.149	1.033	0.336	1.306	0.835	1.523	0.296	−0.426	0.004
1.453	1.210	−0.043	0.220	−0.256	−1.161	−2.030	−0.046	0.243	1.082
0.759	−0.838	−0.877	−0.177	1.183	−0.218	−3.154	−0.963	−0.822	−1.114
0.287	0.278	−0.454	0.897	−0.122	0.013	0.346	0.921	0.238	−0.586
−0.669	0.035	−2.077	1.077	0.525	−0.154	−1.036	0.015	−0.220	0.882
0.392	0.106	−1.430	−0.204	−0.326	0.825	−0.432	−0.094	−1.566	0.679
−0.337	0.199	−0.160	0.625	−0.891	−1.464	−0.318	1.297	0.932	−0.032
0.369	−1.990	−1.190	0.666	−1.614	0.082	0.922	−0.139	−0.833	0.091
−1.694	0.710	−0.655	−0.546	1.654	0.134	0.466	0.033	−0.039	0.838
0.985	0.340	0.276	0.911	−0.170	−0.551	1.000	−0.838	0.275	−0.304
−1.063	−0.594	−1.526	−0.787	0.873	−0.405	−1.324	0.162	−0.163	−2.716
0.033	−1.527	1.422	0.308	0.845	−0.151	0.741	0.064	1.212	0.823
0.597	0.362	−3.760	1.159	0.874	−0.794	−0.915	1.215	1.627	−1.248
−1.601	−0.570	0.133	−0.660	1.485	0.682	−0.898	0.686	0.658	0.346
−0.266	−1.309	0.597	0.989	0.934	1.079	−0.656	−0.999	−0.036	−0.537
0.901	1.531	−0.889	−1.019	0.084	1.531	−0.144	−1.920	0.678	−0.402
−1.433	−1.008	−0.990	0.090	0.940	0.207	−0.745	0.638	1.469	1.214
1.327	0.763	−1.724	−0.709	−1.100	−1.346	−0.946	−0.157	0.522	−1.264
−0.248	0.788	−0.577	0.122	−0.536	0.293	1.207	−2.243	1.642	1.353
−0.401	−0.679	0.921	0.476	1.121	−0.864	0.128	−0.551	−0.872	1.511
0.344	−0.324	0.686	−1.487	−0.126	0.803	−0.961	0.183	−0.358	−0.184
0.441	−0.372	−1.336	0.062	1.506	−0.315	−0.112	−0.452	1.594	−0.264
0.824	0.040	−1.734	0.251	0.054	−0.379	1.298	−0.126	0.104	−0.529
1.385	1.320	−0.509	−0.381	−1.671	−0.524	−0.805	1.348	0.676	0.799

Table 9-3. Random normal numbers, $\mu = 0$, $\sigma = 1$ (continued)

31	32	33	34	35	36	37	38	39	40
1.556	0.119	−0.078	0.164	−0.455	0.077	−0.043	−0.299	0.249	−0.182
0.647	1.029	1.186	0.887	1.204	−0.657	0.644	−0.410	−0.652	−0.165
0.329	0.407	1.169	−2.072	1.661	0.891	0.233	−1.628	−0.762	−0.717
−1.188	1.171	−1.170	−0.291	0.863	−0.045	−0.205	0.574	−0.926	1.407
−0.917	−0.616	−1.589	1.184	0.266	0.559	−1.833	−0.572	−0.648	−1.090
0.414	0.469	−0.182	0.397	1.649	1.198	0.067	−1.526	−0.081	−0.192
0.107	−0.187	1.343	0.472	−0.112	1.182	0.548	2.748	0.249	0.154
−0.497	1.907	0.191	0.136	−0.475	0.458	0.183	−1.640	−0.058	1.278
0.501	0.083	−0.321	1.133	1.126	−0.299	1.299	1.617	1.581	2.455
−1.382	−0.738	1.225	1.564	−0.363	−0.548	1.070	0.390	−1.398	0.524
−0.590	0.699	−0.162	−0.011	1.049	−0.689	1.225	0.339	−0.539	−0.445
−1.125	1.111	−1.065	0.534	0.102	0.425	−1.026	0.695	−0.057	0.795
0.849	0.169	−0.351	0.584	2.177	0.009	−0.696	−0.426	−0.692	−1.638
−1.233	−0.585	0.306	0.773	1.304	−1.304	0.282	−1.705	0.187	−0.880
0.104	−0.468	0.185	0.498	−0.624	−0.322	−0.875	1.478	−0.691	−0.281
0.261	−1.883	−0.181	1.675	−0.324	−1.029	−0.185	0.004	−0.101	−1.187
−0.007	1.280	0.568	−1.270	1.405	1.731	2.072	1.686	0.728	−0.417
0.794	−0.111	0.040	−0.536	−0.976	2.192	1.609	−0.190	−0.279	−1.611
0.431	−2.300	−1.081	−1.370	2.943	0.653	−2.523	0.756	0.886	−0.983
−0.149	1.294	−0.580	0.482	−1.449	−1.067	1.996	−0.274	0.721	0.490
−0.216	−1.647	1.043	0.481	−0.011	−0.587	−0.916	−1.016	−1.040	−1.117
1.604	−0.851	−0.317	−0.686	−0.008	1.939	0.078	−0.465	0.533	0.652
−0.212	0.005	0.535	0.837	0.362	1.103	0.219	0.488	1.332	−0.200
0.007	−0.076	1.484	0.455	−0.207	−0.554	1.120	0.913	−0.681	1.751
−0.217	0.937	0.860	0.323	1.321	−0.492	−1.386	−0.003	−0.230	0.539
−0.649	0.300	−0.698	0.900	0.569	0.842	0.804	1.025	0.603	−1.546
−1.541	0.193	2.047	−0.552	1.190	−0.087	2.062	−2.173	−0.791	−0.520
0.274	−0.530	0.112	0.385	0.656	0.436	0.882	0.312	−2.265	−0.218
0.876	−1.498	−0.128	−0.387	−1.259	−0.856	−0.353	0.714	0.863	1.169
−0.859	−1.083	1.288	−0.078	−0.081	0.210	0.572	1.194	−1.118	−1.543
−0.015	−0.567	0.113	2.127	−0.719	3.256	−0.721	−0.663	−0.779	−0.930
−1.529	−0.231	1.223	0.300	−0.995	−0.651	0.505	0.138	−0.064	1.341
0.278	−0.058	−2.740	−0.296	−1.180	0.574	1.452	0.846	−0.243	−1.208
1.428	0.322	2.302	−0.852	0.782	−1.322	−0.092	−0.546	0.560	−1.430
0.770	−1.874	0.347	0.994	−0.485	−1.179	0.048	−1.324	1.061	0.449
−0.303	−0.629	0.764	0.013	−1.192	−0.475	−1.085	−0.880	1.738	−1.225
−0.263	−2.105	0.509	−0.645	1.362	0.504	−0.755	1.274	1.448	0.604
0.997	−1.187	−0.242	0.121	2.510	−1.935	0.350	0.073	0.458	−0.446
−0.063	−0.475	−1.802	−0.476	0.193	−1.199	0.339	0.364	−0.684	1.353
−0.168	1.904	−0.485	−0.032	−0.554	0.056	−0.710	−0.778	0.722	−0.024
0.366	−0.491	0.301	−0.008	−0.894	−0.945	0.384	−1.748	−1.118	0.394
0.436	−0.464	0.539	0.942	−0.458	0.445	−1.883	1.228	1.113	−0.218
0.597	−1.471	−0.434	0.705	−0.788	0.575	0.086	0.504	1.445	−0.513
−0.805	−0.624	1.344	0.649	−1.124	0.680	−0.986	1.845	−1.152	−0.393
1.681	−1.910	0.440	0.067	−1.502	−0.755	−0.989	−0.054	−2.320	0.474
−0.007	−0.459	1.940	0.220	−1.259	−1.729	0.137	−0.520	−0.412	2.847
0.209	−0.633	0.299	0.174	1.975	−0.271	0.119	−0.199	0.007	2.315
1.254	1.672	−1.186	−1.310	0.474	0.878	−0.725	−0.191	0.642	−1.212
−1.016	−0.697	0.017	−0.263	−0.047	−1.294	−0.339	2.257	−0.078	−0.049
−1.169	−0.355	1.086	−0.199	0.031	0.396	−0.143	1.572	0.276	0.027

Table 9-3. Random normal numbers, $\mu = 0, \sigma = 1$ (continued)

41	42	43	44	45	46	47	48	49	50
−0.856	−0.063	0.787	−2.052	−1.192	−0.831	1.623	1.135	0.759	−0.189
−0.276	−1.110	0.752	−1.378	−0.583	0.360	0.365	1.587	0.621	1.344
0.379	−0.440	0.858	1.453	−1.356	0.503	−1.134	1.950	−1.816	−0.283
1.468	0.131	0.047	0.355	0.162	−1.491	−0.739	−1.182	−0.533	−0.497
−1.805	−0.772	1.286	−0.636	−1.312	−1.045	1.559	−0.871	−0.102	−0.123
2.285	0.554	0.418	−0.577	−1.489	−1.255	0.092	−0.597	−1.051	−0.980
−0.602	0.399	1.121	−1.026	0.087	1.018	−1.437	0.661	0.091	−0.637
0.229	−0.584	0.705	0.124	0.341	1.320	−0.824	−1.541	−0.163	2.329
1.382	−1.454	1.537	−1.299	0.363	−0.356	−0.025	0.294	2.194	−0.395
0.978	0.109	1.434	−1.094	−0.265	−0.857	−1.421	−1.773	0.570	−0.053
−0.678	−2.335	1.202	−1.697	0.547	−0.201	−0.373	−1.363	−0.081	0.958
−0.366	−1.084	−0.626	0.798	1.706	−1.160	−0.838	1.462	0.636	0.570
−1.074	−1.379	0.086	−0.331	−0.288	−0.309	−1.527	−0.408	0.183	0.856
−0.600	−0.096	0.696	0.446	1.417	−2.140	0.599	−0.157	1.485	1.387
0.918	1.163	−1.445	0.759	0.878	−1.781	−0.056	−2.141	−0.234	0.975
−0.791	−0.528	0.946	1.673	−0.680	−0.784	1.494	−0.086	−1.071	−1.196
0.598	−0.352	0.719	−0.341	0.056	−1.041	1.429	0.235	0.314	−1.693
0.567	−1.156	−0.125	−0.534	0.711	−0.511	0.187	−0.644	−1.090	−1,281
0.963	0.052	0.037	0.637	−1.335	0.055	0.010	−0.860	−0.621	0.713
0.489	−0.209	1.659	0.054	1.635	0.169	0.794	−1.550	1.845	−0.388
−1.627	−0.017	0.699	0.661	−0.073	0.188	1.183	−1.054	−1.615	−0.765
−1.096	1.215	0.320	0.738	1.865	−1.169	−0.667	−0.674	−0.062	1.378
−2.532	1.031	−0.799	1.665	−2.756	−0.151	−0.704	0.602	−0.672	1.264
0.024	−1.183	−0.927	−0.629	0.204	−0.825	0.496	2.543	0.262	−0.785
0.192	0.125	0.373	−0.931	−0.079	0.186	−0.306	0.621	−0.292	1.131
−1.324	−1.229	−0.648	−0.430	0.811	0.868	0.787	1.845	−0.374	−0.651
−0.726	−0.746	1.572	−1.420	1.509	−0.361	−0.310	−3.117	1.637	0.642
−1.618	1.082	−0.319	0.300	1.524	−0.418	−1.712	0.358	−1.032	0.537
1.695	0.843	2.049	0.388	−0.297	1.077	−0.462	0.655	0.940	−0.354
0.790	0.605	−3.077	1.009	−0.906	−1.004	0.693	−1.098	1.300	0.549
1.792	−0.895	−0.136	−1.765	1.077	0.418	−0.150	0.808	0.697	0.435
0.771	−0.741	−0.492	−0.770	−0.458	−0.021	1.385	−1.225	−0.066	−1.471
−1.438	0.423	−1.211	0.723	−0.731	0.883	−2.109	−2.455	−0.210	1.644
−0.294	1.266	−1.994	−0.730	0.545	0.397	1.069	−0.383	−0.097	−0.985
−1.966	0.909	0.400	0.685	−0.800	1.759	0.268	1.387	−0.414	1.615
−0.999	1.587	1.423	0.937	−0.943	0.090	1.185	−1.204	0.300	−1.354
0.581	0.481	−2.400	0.000	0.231	0.079	−2.842	−0.846	−0.508	−0.516
0.370	−1.452	−0.580	−1.462	−0.972	1.116	−0.994	0.374	−3.336	−0.058
0.834	−1.227	−0.709	−1.039	−0.014	−0.383	−0.512	−0.347	0.881	−0.638
--0.376	−0.813	0.660	−1.029	−0.137	0.371	0.376	0.968	1.338	−0.786
−1.621	0.815	−0.544	−0.376	−0.852	0.436	1.562	0.815	−1.048	0.188
0.163	−0.161	2.501	−0.265	−0.285	1.934	1.070	0.215	−0.876	0.073
1.786	−0.538	−0.437	0.324	0.105	−0.421	−0.410	−0.947	0.700	−1.006
2.140	1.218	−0.351	−0.068	0.254	0.448	−1.461	0.784	0.317	1.013
0.064	0.410	0.368	0.419	−0.982	1.371	0.100	−0.505	0.856	0.890
0.789	−0.131	1.330	0.506	−0.645	−1.414	2.426	1.389	−0.169	−0.194
−0.011	−0.372	−0.699	2.382	−1.395	−0.467	1.256	−0.585	−1.359	−1.804
−0.463	0.003	−1.470	1.493	0.960	0.364	−1.267	−0.007	0.616	0.624
−1.210	−0.669	0.009	1.284	−0.617	0.355	−0.589	−0.243	−0.015	−0.712
−1.157	0.481	0.560	1.287	1.129	−0.126	0.006	1.532	1.328	0.980

Table 9-4. Coefficients of orthogonal polynomials*

k	Polynomial	$X = 1$	2	3	4	5	6	7	8	9	10	$\Sigma\xi'^2$	λ
3	Linear	−1	0	1								2	1
	Quadratic	1	−2	1								6	3
4	Linear	−3	−1	1	3							20	2
	Quadratic	1	−1	−1	1							4	1
	Cubic	−1	3	−3	1							20	10/3
5	Linear	−2	−1	0	1	2						10	1
	Quadratic	2	−1	−2	−1	2						14	1
	Cubic	−1	2	0	−2	1						10	5/6
	Quartic	1	−4	6	−4	1						70	35/12
6	Linear	−5	−3	−1	1	3	5					70	2
	Quadratic	5	−1	−4	−4	−1	5					84	3/2
	Cubic	−5	7	4	−4	−7	5					180	5/3
	Quartic	1	−3	2	2	−3	1					28	7/12
7	Linear	−3	−2	−1	0	1	2	3				28	1
	Quadratic	5	0	−3	−4	−3	0	5				84	1
	Cubic	−1	1	1	0	−1	−1	1				6	1/6
	Quartic	3	−7	1	6	1	−7	3				154	7/12
8	Linear	−7	−5	−3	−1	1	3	5	7			168	2
	Quadratic	7	1	−3	−5	−5	−3	1	7			168	1
	Cubic	−7	5	7	3	−3	−7	−5	7			264	2/3
	Quartic	7	−13	−3	9	9	−3	−13	7			616	7/12
	Quintic	−7	23	−17	−15	15	17	−23	7			2184	7/10
9	Linear	−4	−3	−2	−1	0	1	2	3	4		60	1
	Quadratic	28	7	−8	−17	−20	−17	−8	7	28		2772	3
	Cubic	−14	7	13	9	0	−9	−13	−7	14		990	5/6
	Quartic	14	−21	−11	9	18	9	−11	−21	14		2002	7/12
	Quintic	−4	11	−4	−9	0	9	4	−11	4		468	3/20
10	Linear	−9	−7	−5	−3	−1	1	3	5	7	9	330	2
	Quadratic	6	2	−1	−3	−4	−4	−3	−1	2	6	132	1/2
	Cubic	−42	14	35	31	12	−12	−31	−35	−14	42	8580	5/3
	Quartic	18	−22	−17	3	18	18	3	−17	−22	18	2860	5/12
	Quintic	−6	14	−1	−11	−6	6	11	1	−14	6	780	1/10

* From B. J. Winer, "Statistical Principles in Experimental Design," McGraw-Hill Book Company, New York, 1962.

Table 9-5. Arcsin transformation ($\phi = 2 \arcsin \sqrt{X}$)*

X	ϕ	X	ϕ	X	ϕ	X	ϕ	X	ϕ
.001	.0633	.041	.4078	.36	1.2870	.76	2.1177	.971	2.7993
.002	.0895	.042	.4128	.37	1.3078	.77	2.1412	.972	2.8053
.003	.1096	.043	.4178	.38	1.3284	.78	2.1652	.973	2.8115
.004	.1266	.044	.4227	.39	1.3490	.79	2.1895	.974	2.8177
.005	.1415	.045	.4275	.40	1.3694	.80	2.2143	.975	2.8240
.006	.1551	.046	.4323	.41	1.3898	.81	2.2395	.976	2.8305
.007	.1675	.047	.4371	.42	1.4101	.82	2.2653	.977	2.8371
.008	.1791	.048	.4418	.43	1.4303	.83	2.2916	.978	2.8438
.009	.1900	.049	.4464	.44	1.4505	.84	2.3186	.979	2.8507
.010	.2003	.050	.4510	.45	1.4706	.85	2.3462	.980	2.8578
.011	.2101	.06	.4949	.46	1.4907	.86	2.3746	.981	2.8650
.012	.2195	.07	.5355	.47	1.5108	.87	2.4039	.982	2.8725
.013	.2285	.08	.5735	.48	1.5308	.88	2.4341	.983	2.8801
.014	.2372	.09	.6094	.49	1.5508	.89	2.4655	.984	2.8879
.015	.2456	.10	.6435	.50	1.5708	.90	2.4981	.985	2.8960
.016	.2537	.11	.6761	.51	1.5908	.91	2.5322	.986	2.9044
.017	.2615	.12	.7075	.52	1.6108	.92	2.5681	.987	2.9131
.018	.2691	.13	.7377	.53	1.6308	.93	2.6062	.988	2.9221
.019	.2766	.14	.7670	.54	1.6509	.94	2.6467	.989	2.9315
.020	.2838	.15	.7954	.55	1.6710	.95	2.6906	.990	2.9413
.021	.2909	.16	.8230	.56	1.6911	.951	2.6952	.991	2.9516
.022	.2978	.17	.8500	.57	1.7113	.952	2.6998	.992	2.9625
.023	.3045	.18	.8763	.58	1.7315	.953	2.7045	.993	2.9741
.024	.3111	.19	.9021	.59	1.7518	.954	2.7093	.994	2.9865
.025	.3176	.20	.9273	.60	1.7722	.955	2.7141	.995	3.0001
.026	.3239	.21	.9521	.61	1.7926	.956	2.7189	.996	3.0150
.027	.3301	.22	.9764	.62	1.8132	.957	2.7238	.997	3.0320
.028	.3363	.23	1.0004	.63	1.8338	.958	2.7288	.998	3.0521
.029	.3423	.24	1.0239	.64	1.8546	.959	2.7338	.999	3.0783
.030	.3482	.25	1.0472	.65	1.8755	.960	2.7389		
.031	.3540	.26	1.0701	.66	1.8965	.961	2.7440		
.032	.3597	.27	1.0928	.67	1.9177	.962	2.7492		
.033	.3654	.28	1.1152	.68	1.9391	.963	2.7545		
.034	.3709	.29	1.1374	.69	1.9606	.964	2.7598		
.035	.3764	.30	1.1593	.70	1.9823	.965	2.7652		
.036	.3818	.31	1.1810	.71	2.0042	.966	2.7707		
.037	.3871	.32	1.2025	.72	2.0264	.967	2.7762		
.038	.3924	.33	1.2239	.73	2.0488	.968	2.7819		
.039	.3976	.34	1.2451	.74	2.0715	.969	2.7876		
.040	.4027	.35	1.2661	.75	2.0944	.970	2.7934		

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* From B. J. Winer, "Statistical Principles in Experimental Design," McGraw-Hill Book Company, New York, 1962.

Table 9-6. Entropy of a discrete binary source*

P	$-\text{Log } P$	$-P \text{ Log } P$	H	$-Q \text{ Log } Q$	$-\text{Log } Q$
0.0001	13.287712	0.001329	0.001473	0.000144	0.000144
0.0005	10.965784	0.005483	0.006204	0.000721	0.000722
0.0010	9.965784	0.009966	0.011408	0.001442	0.001443
0.0015	9.380822	0.014071	0.016234	0.002162	0.002166
0.0020	8.965784	0.017932	0.020814	0.002882	0.002888
0.0025	8.643856	0.021610	0.025212	0.003602	0.003611
0.0030	8.380822	0.025142	0.029464	0.004322	0.004335
0.0035	8.158429	0.028555	0.033595	0.005041	0.005058
0.0040	7.965784	0.031863	0.037622	0.005759	0.005782
0.0045	7.795859	0.035081	0.041559	0.006477	0.006507
0.0050	7.643856	0.038219	0.045415	0.007195	0.007232
0.0055	7.506353	0.041285	0.049198	0.007913	0.007957
0.0060	7.380822	0.044285	0.052915	0.008630	0.008682
0.0065	7.265345	0.047225	0.056572	0.009347	0.009408
0.0070	7.158429	0.050109	0.060172	0.010063	0.010134
0.0075	7.058894	0.052942	0.063721	0.010780	0.010861
0.0080	6.965784	0.055726	0.067222	0.011495	0.011588
0.0085	6.878321	0.058466	0.070676	0.012211	0.012315
0.0090	6.795859	0.061163	0.074088	0.012926	0.013043
0.0095	6.717857	0.063820	0.077460	0.013640	0.013771
0.0100	6.643856	0.066439	0.080793	0.014355	0.014500
0.0110	6.506353	0.071570	0.087352	0.015782	0.015958
0.0120	6.380822	0.076570	0.093778	0.017208	0.017417
0.0130	6.265345	0.081449	0.100082	0.018633	0.018878
0.0140	6.158429	0.086218	0.106274	0.020056	0.020340
0.0150	6.058894	0.090883	0.112361	0.021477	0.021804
0.0160	5.965784	0.095453	0.118350	0.022897	0.023270
0.0170	5.878321	0.099931	0.124248	0.024316	0.024737
0.0180	5.795859	0.104325	0.130059	0.025733	0.026205
0.0190	5.717857	0.108639	0.135788	0.027149	0.027675
0.0200	5.643856	0.112877	0.141441	0.028563	0.029146
0.0210	5.573467	0.117043	0.147019	0.029976	0.030619
0.0220	5.506353	0.121140	0.152527	0.031388	0.032094
0.0230	5.442222	0.125171	0.157969	0.032797	0.033570
0.0240	5.380822	0.129140	0.163346	0.034206	0.035047
0.0250	5.321928	0.133048	0.168661	0.035613	0.036526
0.0260	5.265345	0.136899	0.173917	0.037018	0.038006
0.0270	5.210897	0.140694	0.179116	0.038422	0.039488
0.0280	5.158429	0.144436	0.184261	0.039825	0.040972
0.0290	5.107803	0.148126	0.189352	0.041226	0.042457

* From F. M. Reza, "An Introduction to InformationTheory," McGraw-Hill Book Company, New York, 1961.

Table 9-6. Entropy of a discrete binary source (continued)

P	$-\operatorname{Log} P$	$-P \operatorname{Log} P$	H	$-Q \operatorname{Log} Q$	$-\operatorname{Log} Q$
0.0300	5.058894	0.151767	0.194392	0.042625	0.043943
0.0310	5.011588	0.155359	0.199382	0.044023	0.045431
0.0320	4.965784	0.158905	0.204325	0.045420	0.046921
0.0330	4.921390	0.162406	0.209220	0.046815	0.048412
0.0340	4.878321	0.165863	0.214071	0.048208	0.049905
0.0350	4.836501	0.169278	0.218878	0.049600	0.051399
0.0360	4.795859	0.172651	0.223642	0.050991	0.052895
0.0370	4.756331	0.175984	0.228364	0.052380	0.054392
0.0380	4.717857	0.179279	0.233046	0.053767	0.055891
0.0390	4.680382	0.182535	0.237688	0.055153	0.057392
0.0400	4.643856	0.185754	0.242292	0.056538	0.058894
0.0410	4.608232	0.188938	0.246858	0.057921	0.060397
0.0420	4.573467	0.192086	0.251388	0.059303	0.061902
0.0430	4.539519	0.195199	0.255882	0.060683	0.063409
0.0440	4.506353	0.198280	0.260341	0.062061	0.064917
0.0450	4.473931	0.201327	0.264765	0.063438	0.066427
0.0460	4.442222	0.204342	0.269156	0.064814	0.067939
0.0470	4.411195	0.207326	0.273514	0.066188	0.069452
0.0480	4.380822	0.210279	0.277840	0.067560	0.070967
0.0490	4.351074	0.213203	0.282134	0.068931	0.072483
0.0500	4.321928	0.216096	0.286397	0.070301	0.074001
0.0510	4.293359	0.218961	0.290630	0.071668	0.075520
0.0520	4.265345	0.221798	0.294833	0.073035	0.077041
0.0530	4.237864	0.224607	0.299007	0.074400	0.078564
0.0540	4.210897	0.227388	0.303152	0.075763	0.080088
0.0550	4.184425	0.230143	0.307268	0.077125	0.081614
0.0560	4.158429	0.232872	0.311357	0.078485	0.083141
0.0570	4.132894	0.235575	0.315419	0.079844	0.084670
0.0580	4.107803	0.238253	0.319454	0.081201	0.086201
0.0590	4.083141	0.240905	0.323462	0.082557	0.087733
0.0600	4.058894	0.243534	0.327445	0.083911	0.089267
0.0625	4.000000	0.250000	0.337290	0.087290	0.093109
0.0650	3.943416	0.256322	0.346981	0.090659	0.096962
0.0675	3.888969	0.262505	0.356524	0.094019	0.100824
0.0700	3.836501	0.268555	0.365924	0.097369	0.104697
0.0725	3.785875	0.274476	0.375185	0.100709	0.108581
0.0750	3.736966	0.280272	0.384312	0.104039	0.112475
0.0775	3.689660	0.285949	0.393308	0.107360	0.116379
0.0800	3.643856	0.291508	0.402179	0.110671	0.120294
0.0825	3.599462	0.296956	0.410927	0.113972	0.124220

Table 9-6. Entropy of a discrete binary source (continued)

P	$-\text{Log } P$	$-P \text{ Log } P$	H	$-Q \text{ Log } Q$	$-\text{Log } Q$
0.0850	3.556393	0.302293	0.419556	0.117263	0.128156
0.0875	3.514573	0.307525	0.428070	0.120544	0.132104
0.0900	3.473931	0.312654	0.436470	0.123816	0.136062
0.0925	3.434403	0.317682	0.444760	0.127078	0.140030
0.0950	3.395929	0.322613	0.452943	0.130329	0.144010
0.0975	3.358454	0.327449	0.461020	0.133571	0.148001
0.1000	3.321928	0.332193	0.468996	0.136803	0.152003
0.1025	3.286304	0.336846	0.476871	0.140024	0.156016
0.1050	3.251539	0.341412	0.484648	0.143236	0.160040
0.1075	3.217591	0.345891	0.492329	0.146438	0.164076
0.1100	3.184425	0.350287	0.499916	0.149629	0.168123
0.1125	3.152003	0.354600	0.507411	0.152811	0.172181
0.1150	3.120294	0.358834	0.514816	0.155982	0.176251
0.1175	3.089267	0.362989	0.522132	0.159143	0.180332
0.1200	3.058894	0.367067	0.529361	0.162294	0.184425
0.1225	3.029146	0.371070	0.536505	0.165434	0.188529
0.1250	3.000000	0.375000	0.543564	0.168564	0.192645
0.1275	2.971431	0.378857	0.550542	0.171684	0.196773
0.1300	2.943416	0.382644	0.557438	0.174794	0.200913
0.1325	2.915936	0.386361	0.564255	0.177893	0.205064
0.1350	2.888969	0.390011	0.570993	0.180982	0.209228
0.1375	2.862496	0.393593	0.577654	0.184061	0.213404
0.1400	2.836501	0.397110	0.584239	0.187129	0.217591
0.1425	2.810966	0.400563	0.590749	0.190186	0.221791
0.1450	2.785875	0.403952	0.597185	0.193233	0.226004
0.1475	2.761213	0.407279	0.603549	0.196270	0.230228
0.1500	2.736966	0.410545	0.609840	0.199295	0.234465
0.1525	2.713119	0.413751	0.616061	0.202311	0.238715
0.1550	2.689660	0.416897	0.622213	0.205315	0.242977
0.1575	2.666576	0.419986	0.628295	0.208309	0.247251
0.1600	2.643856	0.423017	0.634310	0.211293	0.251539
0.1625	2.621488	0.425992	0.640257	0.214265	0.255839
0.1650	2.599462	0.428911	0.646138	0.217227	0.260152
0.1675	2.577767	0.431776	0.651954	0.220178	0.264478
0.1700	2.556393	0.434587	0.657705	0.223118	0.268817
0.1725	2.535332	0.437345	0.663392	0.226047	0.273169
0.1750	2.514573	0.440050	0.669016	0.228966	0.277534
0.1775	2.494109	0.442704	0.674577	0.231873	0.281912
0.1800	2.473931	0.445308	0.680077	0.234769	0.286304
0.1825	2.454032	0.447861	0.685516	0.237655	0.290709

Table 9-6. **Entropy of a discrete binary source (continued)**

P	$-\operatorname{Log} P$	$-P \operatorname{Log} P$	H	$-Q \operatorname{Log} Q$	$-\operatorname{Log} Q$
0.1850	2.434403	0.450365	0.690894	0.240529	0.295128
0.1875	2.415037	0.452820	0.696212	0.243393	0.299560
0.1900	2.395929	0.455226	0.701471	0.246245	0.304006
0.1925	2.377070	0.457586	0.706672	0.249086	0.308466
0.1950	2.358454	0.459899	0.711815	0.251916	0.312939
0.1975	2.340075	0.462165	0.716900	0.254735	0.317427
0.2000	2.321928	0.464386	0.721928	0.257542	0.321928
0.2050	2.286304	0.468692	0.731816	0.263124	0.330973
0.2100	2.251539	0.472823	0.741483	0.268660	0.340075
0.2150	2.217591	0.476782	0.750932	0.274150	0.349235
0.2200	2.184425	0.480573	0.760167	0.279594	0.358454
0.2250	2.152003	0.484201	0.769193	0.284992	0.367732
0.2300	2.120294	0.487668	0.778011	0.290344	0.377070
0.2350	2.089267	0.490978	0.786626	0.295648	0.386468
0.2400	2.058894	0.494134	0.795040	0.300906	0.395929
0.2450	2.029146	0.497141	0.803257	0.306116	0.405451
0.2500	2.000000	0.500000	0.811278	0.311278	0.415037
0.2550	1.971431	0.502715	0.819107	0.316392	0.424688
0.2600	1.943416	0.505288	0.826746	0.321458	0.434403
0.2650	1.915936	0.507723	0.834198	0.326475	0.444184
0.2700	1.888969	0.510022	0.841465	0.331443	0.454032
0.2750	1.862496	0.512187	0.848548	0.336362	0.463947
0.2800	1.836501	0.514220	0.855451	0.341230	0.473931
0.2850	1.810966	0.516125	0.862175	0.346049	0.483985
0.2900	1.785875	0.517904	0.868721	0.350817	0.494109
0.2950	1.761213	0.519558	0.875093	0.355535	0.504305
0.3000	1.736966	0.521090	0.881291	0.360201	0.514573
0.3050	1.713119	0.522501	0.887317	0.364816	0.524915
0.3100	1.689660	0.523795	0.893173	0.369379	0.535332
0.3150	1.666576	0.524972	0.898861	0.373890	0.545824
0.3200	1.643856	0.526034	0.904381	0.378347	0.556393
0.3250	1.621488	0.526984	0.909736	0.382752	0.567041
0.3300	1.599462	0.527822	0.914926	0.387104	0.577767
0.3350	1.577767	0.528552	0.919953	0.391402	0.588574
0.3400	1.556393	0.529174	0.924819	0.395645	0.599462
0.3450	1.535332	0.529689	0.929523	0.399834	0.610433
0.3500	1.514573	0.530101	0.934068	0.403967	0.621488
0.3550	1.494109	0.530409	0.938454	0.408046	0.632629
0.3600	1.473931	0.530615	0.942683	0.412068	0.643856
0.3650	1.454032	0.530722	0.946755	0.416034	0.655171

Table 9-6. Entropy of a discrete binary source (continued)

P	$-\text{Log } P$	$-P \text{ Log } P$	H	$-Q \text{ Log } Q$	$-\text{Log } Q$
0.3700	1.434403	0.530729	0.950672	0.419943	0.666576
0.3750	1.415037	0.530639	0.954434	0.423795	0.678072
0.3800	1.395929	0.530453	0.958042	0.427589	0.689660
0.3850	1.377070	0.530172	0.961497	0.431325	0.701342
0.3900	1.358454	0.529797	0.964800	0.435002	0.713119
0.3950	1.340075	0.529330	0.967951	0.438621	0.724993
0.4000	1.321928	0.528771	0.970951	0.442179	0.736966
0.4050	1.304006	0.528122	0.973800	0.445678	0.749038
0.4100	1.286304	0.527385	0.976500	0.449116	0.761213
0.4150	1.268817	0.526559	0.979051	0.452493	0.773491
0.4200	1.251539	0.525646	0.981454	0.455808	0.785875
0.4250	1.234465	0.524648	0.983708	0.459061	0.798366
0.4300	1.217591	0.523564	0.985815	0.462251	0.810966
0.4350	1.200913	0.522397	0.987775	0.465378	0.823677
0.4400	1.184425	0.521147	0.989588	0.468441	0.836501
0.4450	1.168123	0.519815	0.991254	0.471439	0.849440
0.4500	1.152003	0.518401	0.992774	0.474373	0.862496
0.4550	1.136062	0.516908	0.994149	0.477241	0.875672
0.4600	1.120294	0.515335	0.995378	0.480043	0.888969
0.4650	1.104697	0.513684	0.996462	0.482778	0.902389
0.4700	1.089267	0.511956	0.997402	0.485446	0.915936
0.4750	1.074001	0.510150	0.998196	0.488046	0.929611
0.4800	1.058894	0.508269	0.998846	0.490577	0.943416
0.4850	1.043943	0.506313	0.999351	0.493038	0.957356
0.4900	1.029146	0.504282	0.999711	0.495430	0.971431
0.4950	1.014500	0.502177	0.999928	0.497751	0.985645
0.5000	1.000000	0.500000	1.000000	0.500000	1.000000

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EDGE INDEX

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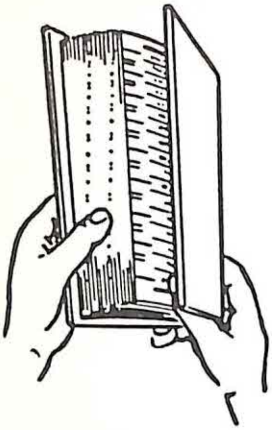

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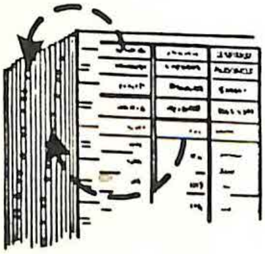

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